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THE

ŚULVASÚTRAS.

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ON THE S'ULVASUTRAS.

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It is well known that not only Indian life with all its social and political institutions has been at all times under the mighty sway of religion, but that we are also led back to religious belief and worship when we try to account for the origin of research in those departments of knowledge which the Indians have cultivated with such remarkable success. At first sight, few traces of this origin may be visible in the S'ástras of later times, but looking closer we may always discern the connecting thread. The want of some norm by which to fix the right time for the sacrifices, gave the first impulse to astronomical observations; urged by this want, the priests remained watching night after night the advance of the moon through the circle of the nakshatras and day after day the alternate progress of the sun towards the north and the south. The laws of phonetics were investigated, because the wrath of the gods followed the wrong pronunciation of a single letter of the sacrificial formulas; grammar and etymology had the task of securing the right understanding of the holy texts. The close connexion of philosophy and theology—so close that it is often impossible to decide where the one ends and the other begins—is too well known to require any comment.

These facts have a double interest. They are in the first place valuable for the history of the human mind in general; they are in the second place important for the mental history of India and for answering the question relative to the originality of Indian science. For whatever is closely connected with the ancient Indian religion must be considered as having sprung up among the Indians themselves, unless positive evidence of the strongest kind point to a contrary conclusion.

We have been long acquainted with the progress which the Indians made in later times in arithmetic, algebra, and geometry; but as the influence of Greek science is clearly traceable in the development of their astronomy, and as their treatises on algebra, &c., form but parts of astronomical text books, it is possible that the Indians may have received from the Greeks also communications regarding the methods of calculation. I merely say possible, because no direct evidence of such influence has been brought forward as yet, and because the general impression we receive from a comparison of the methods employed by Greeks and Indians respectively seems rather to point to an entirely independent growth of this branch of Indian science. The whole question is still unsettled, and new researches are required before we can arrive at a final decision.

While therefore unable positively to assert that the treasure of mathematical knowledge contained in the Lilávatí, the Víjagaṇita, and similar treatises, has been accumulated by the Indians without the aid of foreign nations, we must search whether there are not any traces left pointing to a purely Indian origin of these sciences. And such traces we find in a class of writings, commonly called S'ulvasútras, that means "sútras of the cord," which prove that the earliest geometrical and mathematical investigations among the Indians arose from certain requirements of their sacrifices. "S'ulvasútras" is the name given to those portions or supplements of the Kalpasútras, which treat of the measurement and construction of the different vedis, or altars, the word "s'ulva" referring to the cords which were employed for those measurements. (I may remark at once that the sútras themselves do not make use of the term "s'ulva"; a cord is regularly called by them "rajju".) It appears that a s'ulva-adhyáya or, pras'na or, instead of that, a s'ulvaparis'ishṭa belonged to all Kalpasútras. Among the treatises belonging to this class which are known to me, the two most important are the S'ulvasútras of Baudháyana and of A'pastamba. The former, entitled to the first place by a clearer and more extensive treatment of the topics in question, very likely forms a part of Baudháyana's Kalpasútra; the want of complete manuscripts of this latter work prevents me from being positive on this point. The same remark applies to the S'ulvasútra of A'pastamba.

Two smaller treatises, a Mánava S'ulvasútra and a Maitráyaníya S'ulvasútra, bear the stamp of a later time, compared with the works of Baudháyana and A'pastamba. The literature of the white Yajur Veda possesses a S'ulvaparis'ishṭa, ascribed to Kátyáyana, and there is no sufficient reason for doubting that it was really composed by the author of the Kalpasútra.

The first to direct attention to the importance of the S'ulvasútras was Mr. A. C. Burnell, who in his "Catalogue of a Collection of Sanscrit Manuscripts," p. 29, remarks that "we must look to the S'ulva portions of the Kalpasútras for the earliest beginnings of geometry among the Bráhmans."

I have begun the publication of Baudháyana's S'ulvasútra, with the commentary by Dvárakánáthayajvan and a translation, in the May number of the "Paṇḍit, a monthly Journal of the Benares College, etc.", and intend as soon as I have finished Baudháyana, to publish all other ancient S'ulva works of which I shall be able to procure sufficiently correct manuscripts. In the following pages I shall extract and fully explain the most important sútras, always combining the rules given in the three most important s'ulva treatises, those of Baudháyana, A'pastamba, and Kátyáyana, and so try to exhibit in some systematic order the knowledge embodied in these ancient sacrificial tracts.

The sútras begin with general rules for measuring; the greater part of these rules, in which the chief interest of this class of writings is concentrated, will be given further on. In the next place they teach how to fix the right places for the sacred fires, and how to measure out the vedis of the different sacrifices, the saumikí vedi, the paitṛikí vedi ,and so on.

The remainder of the sútras contains the detailed description of the construction of the "agni", the large altar built of bricks, which was required at the great soma sacrifices.

This altar could be constructed in different shapes, the earliest enumeration of which we find in the Taittiríya Saṃhitá, V. 4. 11.

Following this enumeration Baudháyana and A'pastamba furnish us with full particulars about the shape of all these different chitis and the bricks which had to be employed for their construction. The most ancient and primitive form is the chaturasras'yenachit, so called because it rudely imitates the form of a falcon, and because the bricks out of which it is composed are all of a square shape. It had to be employed whenever there was no special reason for preferring another shape of the agni; and all rules given by bráhmaṇas and sútras for the agnichayana refer to it in first line. A full description of the construction of this agni according to the ritual of the white Yajur Veda and of all accompanying ceremonies has been given by Professor A. Weber in the 13th volume of the "Indische Studien." A nearer approach to the real shape of a falcon or—as the sútras have it—of the shadow of a falcon about to take wing is made in the s'yena vakrapaksha vyastapuchchha, the falcon with curved wings and outspread tail.* The kañkachit, the agni constructed in the form of a heron, or according to Burnell (Catalogue, p. 29) of a carrion-kite, is but a slight variation of the s'yenachiti; it is distinguished from it by the addition of the two feet. The alajachit again is very little different from the kañkachit, showing only a slight variation in the outline of the wings. What particular bird was denoted by the word alaja, the commentators are unable to inform us; in the commentary to Taittir. Saṃh. V. 5. 20 it is explained as "bhása", which does not advance us very much, as the meaning of bhása itself is doubtful. Next comes the praügachit, the construction imitating the form of the praüga, the forepart of the poles of a chariot, an equilateral acutangular triangle and the ubhayatah-praügachit made out of two such triangles joined with their bases. Then follows the rathachakrachit, the altar constructed in the form of a wheel; in the first place the simple rathachakrachit, a massive wheel without spokes, and secondly, the more elaborate sárarathachakrachit, representing a wheel with sixteen spokes. The droṇachit represents a droṇa, a particular kind of tub or vessel; it could be constructed in two shapes, either square or circular (chaturasradroṇachit and parimaṇḍala-droṇachit). The paricháyyachit, which is mentioned in the next place, is in its circular outline equal to the rathachakrachit, but it differs from it in the arrangement of the bricks, which are to be placed in six concentric circles. The samúhyachit has likewise a circular shape; its characteristic feature was that loose earth was employed for its construction instead of the bricks. Of the s'mas'ánachit a full description together with the necessary diagrams will be given further on. The last chiti mentioned is the kúrmachit, the altar representing a tortoise; the tortoise may be either vakránga, of an angular shape, or parimaṇḍala, circular.

Every one of these altars had to be constructed out of five layers of bricks, which reached together to the height of the knee; for some cases ten or fifteen layers and a correspondingly increased height of the altar were prescribed. Every layer in its turn was to consist of two hundred bricks, so that the whole agni contained a thousand; the first, third, and fifth layers were divided into two hundred parts in exactly the same manner; a different division was adopted for the second and the fourth, so that one brick was never lying upon another brick of the same size and form.

Regarding the reasons which may have induced the ancient Indians to devise all these strange shapes, the Saṃhitás and Bráhmaṇas give us

* The plates accompanying this paper contain the diagrams of three different chitis; diagrams of all the remaining chitis will be given in the 'Paṇḍit' in the proper places.

but little information. Thus we read for instance in the Taittiríya Saṃhitá:

S'yenachitaṃ chinvíta suvargakámah, s'yeno vai vayasáṃ patishṭhah, s'yena eva bhútvá suvargaṃ lokaṃ patati.

"He who desires heaven, may construct the falcon-shaped altar; for the falcon is the best flyer among the birds; thus he (the sacrificer) having become a falcon himself flies up to the heavenly world."

In the same place the droṇachiti is brought into connexion with the acquiring of food; the praüga and rathachakra are described as thunderbolts which the sacrificer hurls on his enemies, and so on. Here as in many other cases we may doubt if the symbolical meaning which the authors of the bráhmaṇas find in the sacrificial requísites and ceremonies is the right one; still we cannot propose anything more satisfactory.

But the chief interest of the matter does not lie in the superstitious fancies in which the wish of varying the shape of the altars may have originated, but in the geometrical operations without which these variations could not be accomplished. The old yájnikas had fixed for the most primitive chiti, the chaturasras'yenachit, an area of seven and a half square purushas, that means seven and a half squares, the side of which was equal to a purusha, *i. e.*, the height of a man with uplifted arms. This rule was valid at least for the case of the agni being constructed for the first time; on each subsequent occasion the area had to be increased by one square purusha.

Looking at the sketch of the chaturas'ra s'yena we easily understand why just $7\frac{1}{2}$ square purushas were set down for the agni. Four of them combined into a large square form the átman, or body of the bird, three are required for the two wings and the tail, and lastly, in order that the image might be a closer approach to the real shape of a bird, wings and tail were lengthened, the former by one fifth of a purusha each, the latter by one tenth. The usual expression used in the sútras to denote the agni of this area is "agnih saptavidhah sáratniprádes'ah, the sevenfold agni with aratni and prádes'a," the aratni being the fifth (= 24 aṅgulis), and the prádes'a, the tenth of a purusha (= 12 aṅgulis).

Now when for the attainment of some special purpose, one of the variations enumerated above was adopted instead of the primitive shape of the agni, the rules regulating the size of the altar did not cease to be valid, but the area of every chiti whatever its shape might be—falcon with curved wings, wheel, praüga, tortoise, etc.—had to be equal to $7\frac{1}{2}$ square purushas. On the other hand, when at the second construction of the altar one square purusha had to be added to the seven and a half constituting the first chiti, and when for the third construction two square purushas more were required the shape of the whole, the relative proportions of the single parts had to remain unchanged. A look at the outlines of the differen' chitis is sufficient to show that all this could not be accomplished without a certain amount of geometrical knowledge. Squares had to be found which would be equal to two or more given squares, or equal to the difference of two given squares; oblongs had to be turned into squares and squares into oblongs; triangles had to be constructed equal to given squares or oblongs, and so on. The last task and not the least was that of finding a circle, the area of which might equal as closely as possible that of a given square.

Nor were all these problems suggested only by the substitution of the more complicated forms of the agni for the primitive chaturasras'yena, although this operation doubtless called for the greatest exertion of ingenuity; the solution of some of them was required for the simplest sacrificial constructions. Whenever a figure with right angles, square or oblong, had to be drawn on the ground, care had to be taken that the sides really stood at right angles on each other; for would the áhavaníya fire have carried up the offerings of the sacrificer to the gods if its hearth had not the shape of a perfect square? There was an ancient precept that the vedi at the sautrámaṉí sacrifice was to be the third part of the vedi at the soma sacrifices, and the vedi at the piṭriyajna its ninth part; consequently a method had to be found out by which it was possible to get the exact third and ninth part of a given figure. And when, according to the opinion of some theologians, the gárhapatya had to be constructed in a square shape, according to the opinion of others as a circle, the difference of the opinions referred only to the shape, not to the size, and consequently there arose the want of a rule for turning a square into a circle.

The results of the endeavours of the priests to accomplish tasks of this nature are contained in the paribháshá sútras of the S'ulvasútras. The most important among these is, to use our terms, that referring to the hypotenuse of the rectangular triangle. The geometrical proposition, the discovery of which the Greeks ascribed to Pythagoras, was known to the old ácháryas, in its essence at least. They express it, it is true, in words very different from those familiar to us; but we must remember that they were interested in geometrical truths only as far as they were of practical use, and that they accordingly gave to them the most practical expression. What they wanted was, in the first place, a rule enabling them to draw a square of double the size of another square, and in the second place a rule teaching how to draw a square equal to any two given squares, and according to that want they worded their knowledge. The result is, that we have two propositions instead of one, and that these propositions speak of squares and oblongs instead of the rectangular triangle.

These propositions are as follows :

Baudháyana :

समचतुरश्रस्याक्ष्णयारज्जुर्द्विस्तावतीं भूमिं करोति ।

The cord which is stretched across—in the diagonal of—a square produces an area of double the size.

That is: the square of the diagonal of a square is twice as large as that square.

Ápastamba :

चतुरश्रस्याक्ष्णयारज्जुर्द्विस्तावतीं भूमिं करोति ।

Kátyáyana :

समचतुरश्रस्याक्ष्णयारज्जुर्द्विकरणी ।

The cord in the diagonal of a square is the cord (the line) producing the double (area).

"Samachaturasra" is the term employed throughout in the S'ulva-sútras to denote a square, the "sama" referring to the equal length of the four sides and the chaturasra implying that the four angles are right angles. The more accurate terminology of later Indian geometry distinguishes two classes of samachaturas'ras, or samachaturbhujas, *viz.* the samakarṇa samachaturbhuja and the vishamakarṇa samachaturbhuja; the S'ulvasútras, having to do only with the former one, make no such distinction. Akshṇayárajju is the ancient term, representing the later "karṇarajju" or simply "karna." "Area" is here denoted by "bhúmi," while in later times "kshetra" expressed this idea, and "bhúmi" became one of the words for the base of a triangle or any other plane figure.

The side of a square is said to produce that square (karoti), a way of speaking apparently founded on the observation that the square is found by multiplying the number which expresses the measure of the side by itself; if the side was five feet long, the square was found to consist of 5 × 5 little squares, &c. The expression was not applicable to other plane figures, to an oblong for instance; for there the area is the product of two sides of different length, neither of which can be said to produce the figure by itself.

The side of a square, or originally the cord forming the side of a square, is therefore called the "karaṇí" of the square. That "rajju" is to be supplied to "karaṇí", is explicitly stated by Kátyáyana:

करणी तत्करणी तिर्यङ्मानी पार्श्वमान्यक्ष्णयेति रज्जवः ।

By the expressions: karaṇí, karaṇí of that (of any square) &c., we mean cords.

The side of a square being called its karaṇí, the side of a square of double the size was the "dvikaraṇí", the line producing the double (I shall for convenience sake often employ the terms "side" or "line" instead of "cord"); this was therefore the name for the diagonal of a square. Other compounds with karaṇí will occur further on; the change of meaning which the word has undergone in later times will be considered at the end of this paper.

The authors of the sútras do not give us any hint as to the way in which they found their proposition regarding the diagonal of a square; but we may suppose that they, too, were observant of the fact that the square on the diagonal is divided by its own diagonals into four triangles, one of which is equal to half the first square. This is at the same time an immediately convincing proof of the Pythagorean proposition as far as squares or equilateral rectangular triangles are concerned.

The second proposition is the following:

Baudháyana:

दीर्घचतुरश्रस्याक्ष्णयारज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतस्तदुभयं करोति ।

The cord stretched in the diagonal of an oblong produces both (areas) which the cords forming the longer and the shorter side of an oblong produce separately.

That is: the square of the diagonal of an oblong is equal to the square of both its sides.

Ápastamba:

दीर्घस्याक्ष्णयारज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतस्तदुभयं करोति ।

Kátyáyana gives the rule in the same words as Baudháyana.

The remark made about the term samachaturasra applies also to "dírghachaturasra" "the long quadrangle" meaning the long quadrangle with four right angles. "Párs'vamání (rajju)" is the cord measuring the párs'va or the long side of the oblong or simply this side itself; tiryañmání, the cord measuring the horizontal extent or the breadth of the oblong, in other words its shorter side, which stands at right angles to the longer side. Noteworthy is the expression "prithagbhúte;" for as one of the commentators observes it is meant as a caution against taking the square of the sum of the two sides instead of the sum of their squares (prithaggrahaṇaṃ samsargo má bhúd ity evamartham).

It is apparent that these two propositions about the diagonal of a square and an oblong, when taken together, express the same thing that is enunciated in the proposition of Pythagoras.

But how did the sútrakáras satisfy themselves of the general truth of their second proposition regarding the diagonal of rectangular oblongs?

Here there was no such simple diagram as that which demonstrates the truth of the proposition regarding the diagonal of a square, and other means of proof had to be devised.

Baudháyana :

त्रिकचतुष्कयोर्द्वादशिकपञ्चिकयोः पञ्चदशिकाष्टिकयोः सप्तिकचतुर्विंशिकयोर्द्वादशिकपञ्चत्रिंशिकयोः पञ्चदशिकषट्त्रिंशिकयोरित्येतासूपलब्धिः ।

This (*viz.* that the diagonal of an oblong produces by itself, &c.,) is seen in those oblongs the sides of which are three and four, twelve and five, fifteen and eight, seven and twenty-four, twelve and thirty-five, fifteen and thirty-six (literally, the sides of which consist of three parts and four parts, &c.)

This sútra contains the enumeration of, as we should say, five Pythagorean triangles, *i. e.*, rectangular triangles, the three sides of which can be expressed in integral numbers. (Baudháyana enumerates six ; but the last is essentially the same with the second, 15 and 36 being 3×5 and 3×12.) Baudháyana does not give the numbers expressing the length of the diagonals of his oblongs or the hypotenuses of the rectangular triangles, and I subjoin therefore some rules from A'pastamba, which supply this want, while they show at the same time the practical use, to which the knowledge embodied in Baudháyana's sútra could be turned.

The vedi or altar employed in the soma sacrifices was to have the dimensions specified in the following :

त्रिंशत्पदानि प्रक्रमा वा पश्चात्तिरश्ची भवति षट्त्रिंशत् प्राची चतुर्विंशतिः पुरस्तात्तिरश्चीति सौमिक्या वेदेर्विज्ञायते ।

The western side is thirty padas or prakramas long, the práchí or east line (*i. e.*, the line drawn from the middle of the western side to the middle of the eastern side of the vedi) is thirty-six padas or prakramas long ; the eastern side twenty-four ; this is the tradition for the vedi at the soma sacrifices.

Now follow the rules for the measurement of the area of this vedi :

षट्त्रिंशिकायामष्टादशोपसमस्यापरस्मादन्ताद् द्वादशसु लक्षणं पञ्चदशसु लक्षणं पृष्ठ्यान्तयोरन्तौ नियम्य पञ्चदशिकेन दक्षिणापायम्य शङ्कुं निहन्त्येवमुत्तरतस्ते श्रोणी विपर्यस्यांसौ पञ्चदशिकेनैवापायम्य द्वादशिक शङ्कुं निहन्त्येवमुत्तरतस्तावंसा तदेकरज्ज्वा विहरणम् ।

Add to the length of thirty-six (*i. e.*, to a cord of the length of thirty-six either padas or prakramas) eighteen (the whole length of the cord is then 54), and make two marks on the cord, one at twelve, the other at fifteen, beginning from the western end ; tie the ends of the cord to the ends of the pṛishṭhyá line (the pṛishṭhyá is the same as the práchí, the line directed exactly towards the east and west points, and going through the centre of the vedi. The fixing of the práchí was the first thing to be done when any altar had to be measured out. The methods devised for this end will not be discussed here, as they are based on astronomical observations ; for our purpose it is sufficient to know that a line of 36 padas length and running from the east towards the west had been drawn on the ground. On both ends of this line a pole was fixed and the ends of the cord of 54 padas length tied to these poles) and taking it by the sign at fifteen, draw it towards the south; (at the place reached by the mark, after the cord has been well stretched) fix a pole. Do the same on the northern side (*i. e.*, draw the cord towards the north as you have drawn it just now towards the south). By this process the two s'roṇís, the southwest corner and the southeast corner of the vedi are fixed. After that exchange (the ends of the cord; *i. e.*, tie that end which had been fastened at the pole on the east end of the práchí to the pole on its west end and *vice versâ*), and fix the two aṃsas ("shoulders" of the vedi, *i. e.*, the southeast corner and the northeast corner). This is done by stretching the cord towards the south having taken it by the mark at fifteen and by fixing a pole on the spot reached by the mark at twelve; and by repeating the same operation on the northern side. The result are the two aṃsas. This is the measurement of the vedi by means of one cord (the measurements described further on require two cords each). (See diagram 1.)

The whole process described in the preceding is founded on the knowledge that a triangle, the three sides of which are equal to 15, 36, 39, is rectangular.

The end aimed at was to draw the east and the west side of the vedi at right angles on the práchí. Accordingly, the práchí a b being 36 feet long, a cord a c b (= 54) was divided by a mark into two parts a c = 39 and b c = 15 and fastened at a and b. If then this cord was taken at c, and stretched towards the right, the angle a b c could not but be a right angle. The same applies to the angles a b d, b a e, and b a f. In fixing the two east corners, both marks on the cord had to be employed, the mark at fifteen being used for constructing the right angle, the mark at 12 giving to the east side of the vedi the prescribed length (24 padas).

त्रिकचतुष्कयोः पञ्चिकाक्ष्णयारज्जुः ।

The diagonal cord of an oblong, the side cords of which are three and four, is five.

ताभिस्त्रिरभ्यस्ताभिरंसौ ।

With these cords increased three times (by itself; *i. e.*, multiplied by four) the two eastern corners of the vedi are fixed.

The proceeding is as follows: (See diagram 2.)

At c, at a distance of 16 padas from a, the east end of the práchí, a pole is fixed and then a cord of 32 feet length tied to the poles at a and c. The cord is marked at a distance of 12 padas from a, and then taken by the mark and drawn towards the south until it reaches the position a e c. Thus a triangle is formed, the sides of which are 12, 16, 20 and this triangle is a rectangular one; a e stands at right angles on a c, and as it is just 12 padas long, e marks the place of the southeast corner of the vedi. The north east corner d is found in the same way.

चतुरभ्यस्ताभिः श्रोणी ।

With the same cords increased four times (*i. e.*, their length multiplied by five) the two western corners of the vedi are found.

In this case a cord of 40 padas length is tied to the poles at c and b, and marked at the distance of 15 padas from b. Then it is taken by the mark and drawn towards the south into the position b g c. The result is a rectangular triangle as above; g marks the place of the southwest corner. The same operation repeated on the north side gives f as the place of the northwest corner of the vedi.

Another method for the measurement of the vedi follows:

द्वादशिकपञ्चिकयोस्त्रयोदशिकाक्ष्णयारज्जुस्ताभिरंसौ ।

The diagonal cord of an oblong, the sides of which are twelve and five, is thirteen; with these cords the two east corners are fixed.

(See diagram III.)

A pole is fixed at the distance of five padas from the east end of the práchí, a cord of twenty-five padas length fastened at a and c, marked at the distance of 12 padas from a, drawn towards the south &c., as above.

द्विरभ्यस्ताभिः श्रोणी ।

With these cords increased twice (multiplied by three) the two western corners are fixed.

The requisite rectangular triangle is here formed by the whole práchí = 36, and by a cord of 54, divided by a mark into two pieces of 15 and 39.

Another method follows:

पञ्चदशिकाष्टिकयोः सप्तदशिकाक्ष्णयारज्जुस्ताभिः श्रोणी ।

The diagonal cord of an oblong, the sides of which are fifteen and eight, is seventeen; with these cords the two western corners are fixed.

(See diagram 4.)

A pole b is fixed at the distance of eight padas from d, a cord of 32 padas tied to b and d, &c.

द्वादशिकपञ्चत्रिंशिकयोः सप्तत्रिंशिकाक्ष्णयारज्जुस्ताभिरंसौ ।

The diagonal cord of an oblong, the sides of which are twelve and thirty-five is thirty-seven; with these cords the two eastern corners are fixed.

A pole is fixed at c, thirty-five padas to the west from a; a cord of forty-nine padas tied to a and c, &c.

एतावन्ति विज्ञेयानि वेदिविहरणानि भवन्ति ।

So many "cognizable" measurements of the vedi exist.

That means: these are the measurements of the vedi effected by oblongs, of which the sides and the diagonal can be known, *i. e.*, can be expressed in integral numbers.

In this manner A'pastamba turns the Pythagorean triangles known to him to practical use (the fourth of those which Baudháyana enumerates is not mentioned, very likely because it was not quite convenient for the measurement of the vedi), but after all Baudháyana's way of mentioning these triangles as proving his proposition about the diagonal of an oblong is more judicious. It was no practical want which could have given the impulse to such a research—for right angles could be drawn as soon as one of the "vijneya" oblongs (for instance that of 3, 4, 5) was known—but the want of some proof which might establish a firm conviction of the truth of the proposition.

The way in which the Sútrakáras found the cases enumerated above, must of course be imagined as a very primitive one. Nothing in the sútras would justify the assumption that they were expert in long calculations. Most likely they discovered that the square on the diagonal of an oblong, the sides of which were equal to three and four, could be divided into twenty-five small squares, sixteen of which composed the square on the longer side of the oblong, and nine of which formed the area of the square on the shorter side. Or, if we suppose a more convenient mode of trying, they might have found that twenty-five pebbles or seeds, which could be arranged in one square, could likewise be arranged in two squares of sixteen and of nine. Going on in that way they would form larger squares, always trying if the pebbles forming one of these squares could not as well be arranged in two smaller squares. So they would form a square of 36, of 49, of 64, &c. Arriving at the square formed by $13 \times 13 = 169$ pebbles, they would find that 169 pebbles could be formed in two squares, one of 144 the other of 25. Further on 625 pebbles could again be arranged in two squares of 576 and 49, and so on. The whole thing required only time and patience, and after all the number of cases which they found is only a small one.

Having found that, in certain cases at least, it was possible to express the sides and the diagonal of an oblong in numbers, the Sútrakáras naturally asked themselves if it would not be possible to do the same thing for a square. As the side and the diagonal of a square are in reality incommensurable quantities we can of course only expect an approximative value; but their approximation is a remarkably close one.

Baudháyana:

प्रमाणं तृतीयेन वर्धयेत्तच्च चतुर्थेनात्मचतुस्त्रिंशोनेन । सविशेषः ।

Increase the measure by its third part and this third by its own fourth less the thirty-fourth part of that fourth; (the name of this increased measure) is savis'esha.

Ápastamba gives the rule in the same words.

Kátyáyana:

करणीं तृतीयेन वर्धयेत्तच्च स्वचतुर्थेनात्मचतुस्त्रिंशोनेन सविशेष इति विशेषः।

The sútras themselves are of an enigmatical shortness, and do not state at all what they mean by this increasing of the measure; but the commentaries leave no doubt about the real meaning; the measure is the karaṇí, the side of a square and the increased measure the diagonal, the dvikaraṇí. If we take 1 for the measure, and increase it as directed, we get the following expression: $1 + \frac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 \times 34}$ and this turned into a decimal fraction gives: 1·4142156 ...... Now the side of a square being put equal to 1, the diagonal is equal to $\sqrt{2} = 1{\cdot}414213$ .. Comparing this with the value of the savis'esha we cannot fail to be struck by the accuracy of the latter.

The question arises: how did Baudháyana or Ápastamba or whoever may have the merit of the first investigation, find this value? Certainly they were not able to extract the square root of 2 to six places of decimals; if they had been able to do so, they would have arrived at a still greater degree of accuracy. I suppose that they arrived at their result by the following method which accounts for the exact degree of accuracy they reached.

Endeavouring to discover a square the side and diagonal of which might be expressed in integral numbers they began by assuming two as the measure of a square's side. Squaring two and doubling the result they got the square of the diagonal, in this case = eight. Then they tried to arrange eight, let us say again, eight pebbles, in a square; as we should say, they tried to extract the square root of eight. Being unsuccessful in this attempt, they tried the next number, taking three for the side of a square; but eighteen yielded a square root no more than eight had done. They proceeded in consequence to four, five, &c. Undoubtedly they arrived soon at the conclusion that they would never find exactly what they wanted, and had to be contented with an approximation. The object was now to single out a case in which the number expressing the square of the diagonal approached as closely as possible to a real square number. I subjoin a list, in which the numbers in the first column express the side of the squares which they subsequently tried, those in the second column the square of the diagonal, those in the third the nearest square number.

1.	2.	1.	11.	242.	256.
2.	8.	9.	12.	288.	289.
3.	18.	16.	13.	338.	324.
4.	32.	36.	14.	392.	400.
5.	50.	49.	15.	450.	441.
6.	72.	64.	16.	512.	529.
7.	98.	100.	17.	578.	576.
8.	128.	121.	18.	648.	625.
9.	162.	169.	19.	722.	729.
10.	200.	196.	20.	800.	784.

How far the Sútrakáras went in their experiments we are of course unable to say; the list up to twenty suffices for our purposes. Three cases occur in which the number expressing the square of the diagonal of a square differs only by one from a square-number; 8 — 9; 50 — 49; 288 — 289; the last case being the most favourable, as it involves the largest numbers. The diagonal of a square, the side of which was equal to twelve, was very little shorter than seventeen ($\sqrt{289} = 17$). Would it then not be possible to reduce 17 in such a way as to render the square of the reduced number equal or almost equal to 288?

Suppose they drew a square the side of which was 17 padas long, and divided it into $17 \times 17 = 289$ small squares. If the side of the square could now be shortened by so much, that its area would contain not 289, but only 288 such small squares, then the measure of the side would be the exact measure of the diagonal of the square, the side of which is equal to 12 ($12^2 + 12^2 = 288$). When the side of the square is shortened a little, the consequence is that from two sides of the square a stripe is cut off; therefore a piece of that length had to be cut off from the side that the area of the two stripes would be equal to one of the 289 small squares. Now, as the square is composed of 17×17 squares, one of the two stripes cuts off a part of 17 small squares and the other likewise of 17, both together of 34 and since these 34 cut-off pieces are to be equal to one of the squares, the length of the piece to be cut off from the side is fixed thereby: it must be the thirty-fourth part of the side of one of the 289 small squares.

The thirty-fourth part of thirty-four small squares being cut off, one whole small square would be cut off and the area of the large square reduced exactly to 288 small squares; if it were not for one unavoidable circumstance. The two stripes which are cut off from two sides of the square, let us say the east side and the south side, intersect or overlap each other in the south-east corner and the consequence is, that from the small square in that corner not $\frac{2}{34}$ are cut off, but only $\frac{2}{34} - \frac{1}{34 \times 34}$. Thence the error in the determination of the value of the savis'esha. When the side of a square was reduced from 17 to $16\frac{33}{34}$ the area of the square of that reduced side was not 288, but $288 + \frac{1}{34 + 34}$. Or putting it in a different way: taking 12 for the side of a square, dividing each of the 12 parts into 34 parts (altogether 408) and dividing the square into the corresponding small squares, we get $408 \times 408 = 166464$. This doubled is 332928. Then taking the savis'esha-value of $16\frac{33}{34}$ for the diagonal and dividing the square of the diagonal into the small squares just described, we get $577 \times 577 = 332929$ such small squares. The difference is slight enough.

The relation of $16\frac{33}{34}$ to 12 was finally generalized into the rule: increase a measure by its third, this third by its own fourth less the thirty-fourth part of this fourth $\left(16\frac{33}{34} = 12 + \frac{12}{3} + \frac{12}{3 \times 4} - \frac{12}{3 \times 4 \times 34}.\right)$ The example of the savis'esha given by commentators is indeed $16\frac{33}{34}$: 12; the case recommended itself by being the first in which the third part of a number and the fourth part of the third part were both whole numbers.

Regarding the practical use of the savis'esha, there is in Baudháyana or rather, as far as I am able to see, in all s'ulvasútras only one operation, for which it was absolutely necessary; this is, as we shall see later, the turning of a circle into a square, when the intention was to connect the rule for this operation with the rule for turning a square into a circle. A'pastamba employs (see further on) the savis'esha for the construction of right angles, but there were better methods for that purpose. The commentators indeed make the most extended use of the savis'esha, calculating by means of it the diagonals wherever diagonals come into question; this proceeding, however, is not only useless, but positively wrong, as in all such cases calculation cannot vie in accuracy with geometrical construction.

At the commencement of his sútras, Baudháyana defining the measures he is going to employ, divides the añguli into eight yavas, barley grains, or into thirty-four tilas (seeds of the sesame). I have no doubt that the second division which I have not elsewhere met, owns its origin to the savis'esha. The añguli being the measure most in use, it was convenient to have a special word for its thirty-fourth part, and to be able to say "sixteen añgulis, thirty-three tilas", instead of "sixteen añgulis, and thirty-three thirty-fourths of an añguli." Therefore some plant was searched for of which thirty-four seeds might be considered as equal in length to one añguli; if the tilas really had that exact property, was after all a matter of little relevancy.

Having once acquired the knowledge of the Pythagorean proposition, it was easy to perform a great number of the required geometrical operations. The diagonal of a square being the side of a square of double the size, was, as we have seen, called dvikaraṇí; by forming with this dvikaraṇí and the side of the square an oblong and drawing the diagonal of this oblong, they got the trikaraṇí or the side of a square the area of which was equal to three squares of the first size.

Baudh. A'past. Káty.

प्रमाणं तिर्यग्द्विकरण्यायामस्तस्याक्ष्णयारज्जुस्त्रिकरणी ।

Take the measure (the side of a square) for the breadth, the diagonal for the length (of an oblong); the diagonal cord is the trikaraṇí.

By continuing to form new oblongs and to draw their diagonals, squares could be constructed, equal in area to any number of squares of the first size. Often the process could be shortened by skilful combination of different karaṇís. Kátyáyana furnishes us with some examples.

पदं तिर्यङ्मानी त्रिपदा पार्श्वमानी तस्याक्ष्णयारज्जुर्दशकरणी ।

Take a pada for the breadth, three padas for the length of on oblong; the diagonal is the das'akaraṇí (the square of the diagonal comprises ten square padas, for it combines the square of the karaṇí of one pada and of the navakaraṇí which is three padas long).

द्विपदा तिर्यङ्मानी षट्पदा पार्श्वमानी तस्याक्ष्णयारज्जुश्चत्वारिंशत्करणी ।

Take two padas for the breadth, six padas for the length of an oblong; the diagonal is the chatvárims'at-karaṇí, the side of a square of forty square padas ($2^2 + 6^2 = 40$).

On the other hand, any part of a given square could be found by similar proceedings.

Baudháyana, after the rule for the trikaraṇí:

तृतीयकरण्येतेन व्याख्याता नवमस्तु भूमेर्भागो भवतीति ।

Thereby is explained the tritíyakaraṇí, the side of a square the area of which is the third part of the area of a given square; it is the ninth part of the area.

A'pastamba:

तृतीयकरण्येतेन व्याख्याता विभागस्तु नवधा ।

Kátyáyana:

तृतीयकरण्येतेन व्याख्याता प्रमाणविभागस्तु नवधा । करणीतृतीयं नवभागो नवभागत्रयस्तृतीयकरणी ।

Baudháyana's and A'pastamba's commentators disagree in the explanation of the sútra; the methods they teach are, however, both legitimate. Dvárakánáthayajvan directs us to divide the given square into nine small squares by dividing the side into three parts, and to form with the side and the diagonal of one of these small squares an oblong; the diagonal of this oblong is the ṭritíyakaraṇí.

Kapardisvámin proposes to find the trikaraṇí of the given square and to divide it into three parts; one of these parts is the ṭritíyakaraṇí; for its square is the ninth part of a square of three times the area of the given square, and therefore the third part of the given square. This explanation seems preferable, as it preserves better the connexion of the rule with the preceding rule for the trikaraṇí.

The fourth, fifth, &c., parts of a square were found in the same way.

A'pastamba and Kátyáyana give some special examples illustrating the manner in which the increase or decrease of the side affects the increase and decrease of the square.

A'pastamba:

अध्यर्धपुरुषा रज्जुर्द्वे सपादे करोत्यर्धतृतीयपुरुषा षट् सपादान् ।

A cord of the length of one and a half purusha produces two square purushas and a quarter; and a cord of the length of two purushas and a half produces six square-purushas and a quarter.

Kátyáyana:

द्विः प्रमाणा चतुष्करणी त्रिः प्रमाणा नवकरणी चतुःप्रमाणा षोडशकरणी ।

A cord of double the length produces four (squares); one of three times the length produces nine, and one of four times the length produces sixteen.

A'pastamba and Kátyáyana:

अर्धप्रमाणेन पादप्रमाणं विधीयते ।

By a measure of half the length a square is produced equal to the fourth part of the original square.

A'pastamba:

तृतीयेन नवमी कला ।

Kátyáyana:

तृतीयेन नवमोऽंशः ।

By the third part the ninth part is produced.

Kátyáyana:

चतुर्थेन षोडशी कला ।

The sixteenth part is produced by the fourth part.

Next follow the rules for squares of different size.

A'pastamba:

तुल्ययोश्चतुरश्रयोः समासः । नानाप्रमाणयोश्चतुरश्रयोः समासः । ह्रसीयसः करण्या वर्षीयसो वृध्रमुल्लिखेत् । वृध्रस्याक्ष्णयारज्जुरुभे समस्यति ।

Baudháyana:

नानाचतुरश्रे समस्यन्कनीयसः करण्या वर्षीयसो वृध्रमुल्लिखेद्वृध्रस्याक्ष्णयारज्जुः समस्तयोः पार्श्वमानी भवति ।

For a literal translation of this difficult sútra and a discussion of the word "vṛidhra", see the 'Paṇḍit' of June 1st, 1875, p. 17. The sense is as follows:

A'pastamba: The combining of two squares of equal size has been taught; the following is the method for combining two squares of different sizes. Cut off from the larger square an oblong with the side of the smaller square (*i. e.*, an oblong one side of which is formed by the side of the larger square, the other by that of the smaller square); the diagonal of this oblong combines both squares (is the side of a square the area of which is equal to the area of both the given squares together).

Baudháyana:

If you wish to combine two squares of different size, cut off an oblong from the larger square with the side of the smaller one; the diagonal of that oblong is the side of both squares combined.

Kátyáyana:

समचतुरश्राणामुक्तः समासो नानाप्रमाणसमासे ह्रसीयसः करण्या वर्षीयसोऽपच्छिन्द्यात्तस्याक्ष्णयारज्जुरुभे समस्यतीति समासः ।

The method needs no further explanation; it is in fact the same we employ for the same purpose.

We proceed to the rule for deducting one square from another.

Baudháyana, A'pastamba:

चतुरश्राच्चतुरश्रं निर्जिहीर्षन्यावन्निर्जिहीर्षेत्तस्य करण्या वर्षीयसो वृध्रमुल्लिखेद्वृध्रस्य पार्श्वमानीमक्ष्णयेतरत्पार्श्वमुपसंहरेत्सा यत्र निपतेत्तदपच्छिन्द्याच्छिन्नया निरस्तम् ।

See the 'Paṇḍit', *loc. cit.*

If you wish to deduct one square from another, cut off from the larger one an oblong with the side of the smaller one; draw one of the sides of that oblong across to the other side; where it touches the other side, that piece cut off; by it the deduction is made.

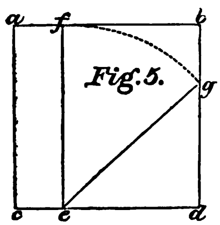


Fig. 5.

a b c d = the larger square; cut off from it the oblong b d e f, in which e d and b f are equal to the side of the smaller square which is to be deducted. Fasten a cord e f at e, and draw it across the oblong into the position e g; then d g is the side of a square the area of which is equal to the difference of the two given squares. ($dg^2 = eg^2 - ed^2$).

Kátyáyana words his rule as follows:

चतुरश्राच्चतुरश्रं निर्जिहीर्षन्यावन्निर्जिहीर्षेत्तावदुभयतोऽपच्छिद्य मध्ये निधाय पार्श्व-

मानीं कृत्वा पार्श्वमानीसंमितामक्ष्णया तयोपसँहरति स समासेऽपच्छेदः स करणेन निर्ह्रासः ।

A'pastamba illustrates the rule by an example:

उपसँहृताक्ष्णयारज्जुः सा चतुःकरणी । छिन्ना चेतरा च यत्स्थसम्भूते कुरुतस्तदुभयं करोति । तिर्यङ्मानी पुरुषँ शेषस्त्रीन् ।

The question is about a square of four square purushas, from which a square of one square purusha is to be deducted. The diagonal (e g), which has been drawn across the oblong, is the side of a square of four purushas, and produces by itself as much as the cut-off side (g d) and the other side (e d) produce separately. The breadth of the oblong (e d) is the side of one square purusha; the rest—the other side, d g—the side of three square purushas.

In order to combine oblongs with squares, a rule was wanted for turning oblongs into squares.

Baudháyana:

दीर्घचतुरश्रँ समचतुरश्रं चिकीर्षँस्तिर्यङ्मानीं करणीं कृत्वा शेषं द्वेधा विभज्य विपर्यस्येतरच्चोपदध्यात् खण्डमावापेन तत्संपूरयेत्तस्य निर्हार उक्तः ।

In order to turn an oblong into a square, take the breadth of the oblong for the side of the square; divide the rest of the oblong into two parts, and inverting their places join those two parts to two sides of the square. Fill the empty place with an added piece. The deduction of this has been taught.

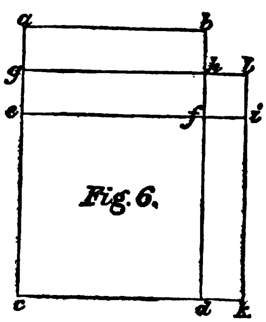


Fig. 6.

That means: if you wish to turn the oblong a b c d into a square, cut off from the oblong the square c d e f, the side of which is equal to the breadth of the oblong; divide a b e f, the rest of the oblong, into two parts, a b g h and g h e f; take a b g h, and place it into the position d f i k; fill up the empty place in the corner by the small square f h l i; then deduct by samachaturasranirhára the small square f h l i from the large square g l k c; the square you get by this deduction will be equal to the oblong a b c d.

A'pastamba gives the same rule:

दीर्घचतुरश्रँ समचतुरश्रं चिकीर्षंस्तिर्यङ्मान्यापच्छिद्य शेषं विभज्योभयत उपदध्यात् । खण्डमागन्तुना संपूरयेत् । तस्य निर्ह्रास उक्तः ।

And Kátyáyana:

दीर्घचतुरश्रँ समचतुरश्रं चिकीर्षन्मध्ये तिर्यगपच्छिद्यान्यतरद्विभज्येतरत्पुरस्ताद्दक्षिणतश्चोपदध्याच्छेषमागन्तुना पूरयेत्तस्योक्तो निर्ह्रासः ।

When one side of the oblong which had to be turned into a square, was more than double the length of the other, it was not sufficient to cut off a square once, but this had to be done several times, according to the length of the oblong, and finally all squares had to be combined into one.

Kátyáyana has a rule to this purpose:

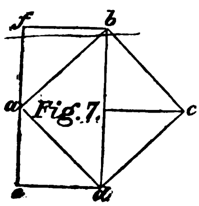
अतिदीर्घं चेत् तिर्यङ्मान्यापच्छिद्यापच्छिद्यैकसमासेन समस्य शेषं यथायोगमुपसंहरेत्।

I add the rules for the reverse process, the turning of a square into an oblong.

Baudháyana:

समचतुरश्रं दीर्घचतुरश्रं चिकीर्षन्नक्ष्णयापच्छिद्य भागं द्वेधा विभज्य पार्श्वयोरुपदध्याद्यथायोगम्।

If you wish to turn a square into an oblong, divide it by the diagonal; divide again one of the two halves into two parts, and join these two parts to the two sides (those two sides of the other half which form the right angle) as it fits (when joining them, join those sides which fit together).



Proceeding as directed, we turn the square a b c d into the oblong b d e f. This rule is, of course, very imperfect as it enables us to turn the square into one oblong only.

Kátyáyana has the following:

समचतुरश्रं दीर्घचतुरश्रं चिकीर्षन्नध्योऽक्ष्णयापच्छिद्य विभज्येतरत्पुरस्तादुत्तरतश्चोपदध्यात्।

A'pastamba's rule helps us somewhat further:

समचतुरश्रं दीर्घचतुरश्रं चिकीर्षन्यावच्चिकीर्षेत्तावतीं पार्श्वमानीं कृत्वा यदधिकं स्यात्तद्यथायोगमुपदध्यात्।

In order to turn a square into an oblong, make a side as long as you wish the oblong to be (*i. e.*, cut off from the square an oblong one side of which is equal to one side of the desired oblong); then join to that the remaining portion as it fits.

Given for instance a square the side of which is equal to five, and required an oblong one side of which is equal to three. Cut off from the square an oblong the sides of which are five and three. There remains an oblong the sides of which are five and two; from this we cut off an oblong of three by two, and join it to the oblong of five by three. There remains a square of two by two, instead of which we take an oblong of 3 by $1\frac{1}{3}$. Joining this oblong to the two oblongs joined previously we get altogether an oblong of 3 by $8\frac{1}{3}$, the area of which is equal to the area of the square 5 by 5.

In this way the sútra, as it appears from the commentaries, must be explained. The method taught in it was no doubt sufficient for most cases, but it cannot be called a really geometrical method.

I subjoin the description of a method for turning squares into oblongs, which is given by Baudháyana's commentator, although it is not founded on the text of the sútras. He, after having explained Baudháyana's way of proceeding, continues—

अन्यश्च प्रकारः। यावदिच्छं पार्श्वमान्यौ प्राच्यौ वर्धयित्वा उत्तरपूर्वां कर्णरज्जुमायच्छे-
त्सा दीर्घचतुरश्रमध्यस्थायां समचतुरश्रतिर्यङ्मान्यां यत्र निपतति तत उत्तरं छित्त्वा दक्षि-
णाऽग्रं तिर्यङ्मानीं कुर्यात्। तद्दीर्घचतुरश्रं भवति।

And there is another method. Lengthen the north side and the south side of the square towards east by as much as you want (*i. e.*, give to them the length of the oblong you wish to construct) and stretch (through the oblong formed by the two lengthened sides and the lines joining their ends) a cord in the diagonal from the north-east to the south-west corner. This diagonal cuts the east side of the square, which (side) runs through the middle of the oblong. Putting aside that part of the cut line which lies to the north of the point of intersection, take the southern part for the breadth; this is the required oblong.

For example:

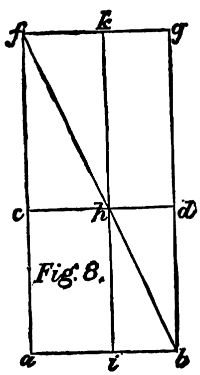


Fig. 8.

Given the square a b c d and required an oblong of the same area and of the length b g. Lengthen a c and b d into a f and b g; draw f g parallel to c d; draw the diagonal f b, which cuts c d at h; draw i k parallel to a f and b g; then b g i k is the desired oblong.

This method is purely geometrical and perfectly satisfactory; for a b f = b f g, and b d h = b h i and c f h = f h k; therefore a c h i = d g h k, and consequently a b c d = b g k i. Q. E. D.

In this place now we have to mention the rules which are given at the beginning of the sútras, the rules, as they call it, for making a square, in reality for drawing one line at right angles upon another. Their right place is here, after the general propositions about the diagonal of squares and oblongs, upon which they are founded.

Baudháyana:

प्रमाणाद् द्विगुणाꣳ रज्जुमुभयतः पाशां कृत्वा मध्ये लक्षणं करोति स प्राच्यर्थः
अपरस्मिन्नर्धे चतुर्भागोने लक्षणं करोति। तन्न्यञ्छनम्। अर्धेऽꣳसार्थम्। पृष्ठ्यान्तयोः पाशौ
प्रतिमुच्य न्यञ्छनेन दक्षिणापायम्यार्धेनार्धेन श्रोण्यꣳसान्निर्हरेत्।

Make two ties at the ends of a cord the length of which is double the measure (of the side of the required square) and a mark at its middle. This piece of the cord (*i. e.*, its half) gives us the práchí (of the required square; the práchí of a square has the same length as its side). Then make a mark at the western half of the cord less the fourth part (of the half. If we wish, for instance, to make a square the side of which is twelve padas long, we take a cord twenty-four padas long; stretching this cord on the ground from the west towards the east, we find its middle by a measurement beginning from the western end, and having fixed the point which lies at the distance of twelve padas from both ends, we measure three padas back, towards the west, and make at the point we arrive at a mark; this mark divides the cord into two parts of 15 and 9 padas length). The name of this mark is nyañchhana. Then another mark is to be made at the half (of the western half of the cord), in order to fix by it the four corners of the square. (This second sign is at a distance of 18 padas from the eastern end of the cord.) Having fastened the two ties at the ends of the prishṭhyá line, we take the cord at the nyañchhana mark and stretch it towards the south; the four corners of the square are then fixed by the half (of the cord).

The same method is known to A'pastamba:

आयामं वाभ्यस्यागन्तुचतुर्थमायामश्चाक्ष्णयारज्जुस्तिर्यङ्मानी शेषः।

Or the length of the práchí of the desired square, is to be doubled; the length and the fourth part of the added piece form the diagonal cord; the rest, *i. e.* three quarters of the added piece form the breadth (the shorter side of the oblong).

And the S'ulvaparis'ishṭa:

प्रमाणमभ्यस्याभ्यासचतुर्थं लक्षणं करोति तन्निरञ्छनमक्ष्णया तिर्यङ्मानी शेषः।

These rules make use of one of the Pythagorean triangles which were, as we have seen above, known to the Sútrakáras, *viz.* of that one the sides of which are equal to three, four, and five. It recommended itself by the ease with which the three sides can be expressed in terms of each other, 3 + 5 being the double of 4, and 3 being equal to half the sum of 3 and 5, minus one quarter of half that sum.

Of course any other oblong with measurable sides and diagonal could be employed for the same purpose, and so we find in A'pastamba a rule for chaturasrakaraṇa abstracted from the dírghachaturasra, of which the sides are five and twelve and the diagonal thirteen.

यावदायामं प्रमाणं तदर्धमभ्यस्यापरस्मिँस्तृतीये षड्भागोने लक्षणं करोति। पृष्ठ्यान्तयोरन्तौ नियम्य लक्षणेन दक्षिणापायम्य निमित्तं करोति। एवमुत्तरतः। विपर्यस्येतरतस् समाधिः।

Take a measure equal to the length (of the side and práchí of the desired square) and increase it by its half. Make a mark at the western third less its sixth part. Fasten the ends of the cord, &c.

Increase 12 by 6; result 18; make a mark at a third, (reckoning from 18; that would be at 12) less the sixth part of that third (*i. e.*, a sixth part before the third) *i. e.*, at 13. Thus we get a rectangular triangle of 5, 12, 13.

The same rule in the S'ulvaparis'ishṭa:

प्रमाणार्धं वाभ्यस्याभ्यासषष्ठे लक्षणं करोति तस्मिंश्छ्रमसच्छ्रोण्या तिर्यङ्मानी शेषः ।

Here, as in many other places, the paris'ishṭa is much clearer and more practical in the wording of its rules than the more ancient sútras. The mark is, according to its expression, to be made not at the western third less its sixth part, but simply at a sixth of the added piece (6 is added to 12; the mark is made at 13).

Another method for chaturasrakaraṇa, taught by A'pastamba only, makes use of the above-mentioned savis'esha.

पृष्ठ्यान्तयोर्मध्ये च शङ्कुं निहत्यार्धं तद्विशेषमभ्यस्य लक्षणं कृत्वार्धमागमयेदन्तयोः पाशौ कृत्वा मध्यमे सविशेषं प्रतिमुच्य पूर्वस्मिन्नितरं लक्षणेन दक्षिणमꣳसमायच्छेदुन्मुच्य पूर्वस्मादपरस्मिन्प्रतिमुच्य लक्षणेनैव दक्षिणाꣳ श्रोणिमायच्छेदेवमुत्तरौ श्रोण्यꣳसौ ।

Fix poles on both ends and the middle of the pṛishṭhyá line, add to a cord of half the length (of the pṛishṭhyá) its vis'esha, *i. e.*, its third plus the fourth part of the third minus the thirty-fourth part of that fourth part, and add moreover a piece of the length of half the pṛishṭhyá, after having made a mark (to separate the two parts of the cord). Then tie the savis'esha part of the cord to the middle pole, the other part to the eastern pole, and fix the south-east corner of the square by stretching the cord (towards the south), having taken it at the mark. Untie the end of the cord from the eastern pole, &c.

This method is of course inferior to those described above and certainly unnecessary; Baudháyana does not mention it.

I subjoin the remaining methods for chaturasṛakaraṇa, which do not presuppose the knowledge of the Pythagorean theorem.

Apastamba:

प्रमाणाद्वीमꣳ रज्जुमुभयतः पाशां करोति । मध्ये लक्षणमर्धमध्ययोश्च । पृष्ठ्यायाꣳ रज्जुमायम्य पाशयोर्लक्षणेष्विति शङ्कून्निहन्त्युपान्त्ययोः पाशौ प्रतिमुच्य मध्यमेन लक्षणेन दक्षिणापायम्य निमित्तं करोति मध्यमे पाशौ प्रतिमुच्योपर्युपरि निमित्तं मध्यमेन लक्षणेन दक्षिणापायम्य शङ्कुं निहन्ति तस्मिन्पाशं प्रतिमुच्य पूर्वस्मिन्नितरं मध्यमेन लक्षणेन दक्षिणमꣳसमायच्छेदुन्मुच्य पूर्वस्मादपरस्मिन्प्रतिमुच्य मध्यमेनैव लक्षणेन दक्षिणाꣳ श्रोणिमायच्छेदेवमुत्तरौ श्रोण्यꣳसौ ।

Take a cord of the length of the measure (of the side of the required square), and make ties at both its ends, a mark at its middle and at the middle points of its halves. Stretch the cord on the pṛishṭhyá line, and fix poles on the points marked by the two ties of the cord and by the three marks (five poles altogether). Fasten the ties at the second and fourth poles (reckoning from the east), stretch the cord towards the south having taken it by the middle mark, and make at the point, touched by the mark, a mark on the ground. Then fastening both ties at the middle pole, stretch the cord over the mark on the ground towards the south, having taken it by the middle mark, and fix a pole (at the spot reached by the stretched, doubled up, cord). Then fastening one tie at this pole and the other tie at the pole standing at the eastern end of the práchí, fix the south-east corner of the square by stretching the cord, having taken it by the middle mark. Then untying the rope from the eastern pole and fastening it at the western pole, fix the south-west corner, &c.; in the same way the north-east and north-west corner are found.

In this procedure the first step is to find the middle of the southern and of the northern sides of the required square by drawing a line at right angles through the middle point of the práchí. The method employed here for drawing a line at right angles on another is the simplest of all known to the S'ulvasútras, and essentially the same we make use of when describing intersecting arcs from two points equally distant to the right and left from some given point. In the later portions of the sútras this method is enjoined for the measurement of the agni (instead of cords canes of a certain length had to be employed there), and the followers of the White Yajur Veda had adopted it for the same purpose (see Indische Studien, XIII., p. 233, ff).

The second part of the procedure—to find the four corners of the square after having found the middle points of the sides—was of course easy and does not afford any special interest.

To Baudháyana the same method is known, but he restricts it in his paribhásha-sútras to the construction of oblongs; clearly without sufficient reason, since the method refers only to the construction of right angles, and the length of the sides is of no importance. A'pastamba gives no special rule at all for oblongs, and it is indeed not wanted.

I subjoin Baudháyana's rule:

दीर्घचतुरश्रं चिकीर्षन्यावच्चिकीर्षेत्तावत्यां भूमौ द्वौ शङ्कू निहन्यात्। द्वौ द्वावेकमे-केभितः समौ। यावती तिर्यङ्मानी तावतीꣳ रज्जुमुभयतः पाशां कृत्वा मध्ये लक्षणं करोति। पूर्वेषामन्त्ययोः पाशौ प्रतिमुच्य लक्षणेन दक्षिणायम्य लक्षणे लक्षणं करोति। मध्यमे पाशौ प्रतिमुच्य लक्षणस्योपरिष्टाद्दक्षिणापायम्य लक्षणे शङ्कुं निहन्यात्। सोऽꣳस एतेनो-त्तरोऽꣳसो व्याख्यातस्तथा श्रोणी।

He who wishes to make an oblong is to fix two poles on an area of the length which he intends to give to the oblong (*i. e.*, at the two ends of the práchí of that area). On both sides, *i. e.*, on the west and east sides of both these poles two other poles are to be fixed at equal distances. Then taking a cord of the length one intends to give to the side line (breadth) of the oblong, one makes ties at both its ends and a mark at its middle. Then one fastens the two ties at those two of the three eastern poles, which stand at the outside, stretches the cord towards the south holding it by the mark, and makes on this mark (*i. e.*, on the spot where the mark touches the ground after the cord has been stretched) a mark. Then fastening both ties at the middle pole one stretches the cord over the mark (on the ground) towards the south, and fixes a pole on the mark (*i. e.*, on the spot touched by the mark on the cord). That is the south-east corner of the oblong; thereby are explained likewise the north-east corner and the two western corners.

In the last place I give a method of chaturás'rakaraṇa, which is found in Baudháyana only, but there in the first place. It seems to be the most ancient of all the methods enumerated.

चतुरश्रं चिकीर्षन्यावच्चिकीर्षेत्तावतीꣳ रज्जुमुभयतः पाशौ कृत्वा मध्ये लक्षणं करोति । लेखामालिख्य तस्या मध्ये शङ्कुं निहन्यात्तस्मिन्पाशौ प्रतिमुच्य लक्षणेन मण्डलं परिलिखेत् । विष्कम्भान्तयोः शङ्कू निहन्यात् । पूर्वस्मिन्पाशं प्रतिमुच्य पाशेन मण्डलं परिलिखेत् । एव-मपरस्मिꣳस्ते यत्र समेयातां तेन द्वितीयं विष्कम्भमायच्छेत् । विष्कम्भान्तयोः शङ्कू निहन्यात् । पूर्वस्मिन्पाशौ प्रतिमुच्य लक्षणेन मण्डलं परिलिखेत् । एवं दक्षिणत एवं पश्चादेवमुत्तरत-स्तेषां येऽन्त्याः सꣳसर्गास्तच्चतुरश्रꣳ संपद्यते ।

If you wish to make a square, take a cord of the length which you desire to give to the side of the square, make a tie at both its ends and a mark at its middle; then having drawn the práchí line, fix a pole in its middle, and having fastened at that pole the two ties of the cord, describe with the mark a circle round it. Then fix poles at both ends of the diameter (formed by the práchí), and having fastened one tie at the eastern pole (the pole standing at the east end of the práchí), describe a circle with the other tie (*i. e.*, with the full length of the cord). In the same manner a circle is described round the pole at the west end of the práchí, and another diameter is drawn joining the points in which these two circles intersect (this diameter is the line pointing to the north and south points). A pole is fixed at both ends of this diameter. Having fastened both ties at the eastern pole, describe a circle round it with the mark. The same is to be done in the south, the west, and the north (*i. e.*, circles are to be described round the three other poles); the points of intersection of these four circles which (*i. e.*, the points) are situated in the four intermediate regions (north-east, north-west, &c.,) are the four corners of the required square.

Diagram 9.

Passing over some rules of less importance, I proceed to those which refer to the "squaring of the circle." It certainly is a matter of some interest to see the old áchâryas attempting this problem, which has since haunted so many unquiet minds. It is true the motives leading them to the investigation were vastly different from those of their followers in this arduous task. Theirs was not the disinterested love of research which distinguishes true science, nor the inordinate craving of undisciplined minds for the solution of riddles which reason tells us cannot be solved; theirs was simply the earnest desire to render their sacrifice in all its particulars acceptable to the gods, and to deserve the boons which the gods confer in return upon the faithful and conscientious worshipper.

It is true that they were not quite so successful in their endeavours as we might wish, and that their rules are primitive in the highest degree; but this tends at least to establish their high antiquity.

The rules are the following:

Baudháyana:

चतुरश्रं मण्डलं चिकीर्षन्नक्ष्णयार्धं मध्यात्प्राचीमभ्यापातयेद्यदतिशिष्यते तस्य सह तृतीयेन मण्डलं परिलिखेत्।

If you wish to turn a square into a circle, draw half of the cord stretched in the diagonal from the centre towards the práchí line (the line passing through the centre of the square and running exactly from the west towards the east); describe the circle together with the third part of that piece of the cord which will lie outside the square.

See diagram 10.

A cord is to be stretched from the centre e of the square a b c d towards the corner a; then the cord, being tied to a pole at e, is drawn towards the right hand side until it coincides in its position with the line e f; a piece of the cord, f h, will then of course lie outside the square. This piece is to be divided into three parts, and one of these three parts, f g, together with the piece e f, forms the radius of the circle, the area of which is to be equal to the area of the square a b c d.

A'pastamba gives the same rule in different words:

चतुरश्रं मण्डलं चिकीर्षन्मध्यात्कोट्यां निपातयेत् पार्श्वतः परिकृष्यातिशयतृतीयेन सह मण्डलं परिलिखेत्। सा नित्या मण्डलम्। यावद्धीयते तावदागन्तु।

If you wish to turn a square into a circle, stretch a cord from the centre towards one of the corners, draw it round the side and describe the circle together with the third part of the piece standing over; this line gives a circle exactly as large as the square; for as much as there is cut off from the square (*viz.* the corners of the square), quite as much is added to it (*viz.* the segments of the circle, lying outside the square).

I must remark that Kapardisvámin, A'pastamba's commentator, combines the two words "sá nityá" into sánityá (= sá anityá), and explains: this line gives a circle, which is not exactly equal to the square. But I am afraid we should not be justified in giving to A′pastamba the benefit of this explanation. The words ‘ yávad dhíyate, &c.’ seem to indicate that he was perfectly satisfied with the accuracy of his method and not superior, in this point, to so many circle-squarers of later times. The commentator who, with the mathematical knowledge of his time, knew that the rule was an imperfect one, preferred very naturally the interpretation which was more creditable to his author.

Kátyáyana's S′ulvaparis′ishṭa:

चतुरश्रं मण्डलं चिकीर्षन्मध्यादꣳसे निपात्य पार्श्वतः परिलिख्य तत्र यदतिरिक्तं भवति तस्य तृतीयेन सह मण्डलं परिलिखेत्।

Let us now see what the result of the above rule would be by making the side of the square equal to 2. a c = 2; a i = 1; a e = $\sqrt{2}$ = 1·414213...; $\frac{0·414213}{3}$ = 0·138071; radius of the circle = 1·138071.

Multiplying the square of 1·138071 by π = 3·141592..., we find as area of the circle: 4·069008........., while the area of the square = 4.

The next thing was to find a rule for turning a circle into a square. There we have at first a rule given by Baudháyana only:

मण्डलं चतुरश्रं चिकीर्षन्विष्कम्भमष्टौ भागान्कृत्वा भागमेकोनत्रिꣳशधा विभज्याष्टाविꣳशतिभागानुद्धरेद्भागस्य च षष्ठमष्टमभागोनम्।

If you wish to turn a circle into a square, divide the diameter into eight parts, and again one of these eight parts into twenty-nine parts; of these twenty-nine parts remove twenty-eight and moreover the sixth part (of the one left part) less the eighth part (of the sixth part).

The meaning is: $\frac{7}{8} + \frac{1}{8·29} - \frac{1}{8·29·6} + \frac{1}{8·29·6·8}$ of the diameter of a circle is the side of a square the area of which is equal to the area of the circle.

Considering this rule closer, we find that it is nothing but the reverse of the rule for turning a square into a circle.

It is clear, however, that the steps taken according to this latter rule could not be traced back by means of a geometrical construction; for if we have a circle given to us, nothing indicates what part of the diameter is to be taken as the “ atis′ayaṭritaya” (the piece f g in diagram 10).

It was therefore necessary to express the rule for turning a square into a circle in numbers. This was done by making use of the “ savis′esha”, which we have considered above. Baudháyana assumed a i as equal to 12 añgulis (= 408 tilas), and therefore a e = 16 añgulis, 33 tilas. Difference = 4 añg. 33 til. = 169 til.; the third part of this difference = 56⅓ til. Radius of the circle $= e\ f\ (= a\ i) + g\ f = 408$ til. $+ 56\frac{1}{3}$ til. $= 464\frac{1}{3}$ til. In other words: if half the side of a square is 408 til. long, the length of the radius of a circle, which is equal in area to the square, amounts to $464\frac{1}{3}$ til.; or, if the radius of a circle is $464\frac{1}{3}$ til., half the side of the corresponding square is 412 til. In order to avoid the fraction, both numbers were turned into thirds, and the radius made $= 1393$, half the side $= 1224$. Finally, the diameter was taken instead of the radius, and the whole side of the square instead of half the side.

To generalize this rule, it was requisite to express 1224 in terms of 1393. One eighth of $1393 = 174\frac{1}{8}$; this multiplied by $7 = 1218\frac{7}{8}$. Difference between $1218\frac{7}{8}$ and $1224 = 5\frac{1}{8}$. Dividing 174 (Baudháyana takes 174, instead of $174\frac{1}{8}$, neglecting the fraction as either insignificant or, more likely, as inconvenient) by 29 we get 6; subtracting from 6 its sixth part we get 5 and adding to this the eighth part of the sixth part of six, we get $5\frac{1}{8}$.

In other words: $$1224 = \frac{7}{8} + \frac{1}{8 \cdot 29} - \frac{1}{8 \cdot 29 \cdot 6} + \frac{1}{8 \cdot 29 \cdot 6 \cdot 8} \text{ of } 1393$$

(due allowance made for the neglected $\frac{1}{8}$.)

Another simpler and less accurate rule for squaring the circle is common to the three Sútrakárás.

Baudháyana:

अपि वा पञ्चदश भागान्कृत्वा द्वावुद्धरेदेषानित्या चतुरश्रकरणी ।

Or else divide (the diameter) into fifteen parts and remove two; that (the remaining thirteen parts) is the gross side of the square.

A'pastamba:

मण्डलं चतुरश्रं चिकीर्षन्विष्कम्भं पञ्चदश भागान्कृत्वा द्वावुद्धरेत्त्रयोदशावशिष्यन्ते सा नित्या चतुरश्रम्।

Kátyáyana:

मण्डलं चतुरश्रं चिकीर्षन्विष्कम्भं पञ्चदश भागान्कृत्वा द्वावुद्धरेच्छेषः करणी ।

If we assume a circle with 15 for diameter, the area of the corresponding square would, according to this rule, be 169, while the area of the circle is 176. 714.......

These are the most interesting of the paribhásha-sútras. In the following I shall extract the description of three kinds of the agnichayana, of the vakrapakshas'yenachiti, as given by A'pastamba; of the sárarathachakrachiti and of the s'mas'ánachiti. The two latter are described by Baudháyana only. I select these three chitis, because the first of them was, as it appears, most in use, and because some particular skill was required for the construction of the agnikshetra of the two latter chitis.

The vakrapaksha s'yena itself could be constructed in different forms. Two forms are described by Baudháyana, two by A'pastamba. And as two different prastáras were necessary for each chiti, we have altogether eight different prastáras for the vakrapaksha s'yena, each of them consisting of two hundred bricks. The following extract contains A'pastamba's rules for the first kind of the vakrapaksha s'yena.

(Description and diagrams of all the other kinds will be given in the 'Paṇḍit'. A sketch of one prastára of the second kind of the s'yenachit is to be found in Burnell's Catalogue; it is, as we are informed there, taken from an agni actually constructed and used. There is, however, an error in the reference to the sútra according to which it is said to be constructed, this sútra not being Baudháyana's, but A'pastamba's, paṭala VI.)

श्येनचितं चिन्वीत सुवर्गकाम इति विज्ञायते।

He who wishes for heaven, may construct the altar shaped like a falcon; this is the tradition.

वक्रपक्षो व्यस्तपुच्छो भवति।

His wings are bent and his tail spread out.

पश्चात्प्राङुदूहति पुरस्तात्प्रत्यङुदूहति।

On the west side the wings are to be drawn towards the east, on the east side towards the west.

एवमिव हि वयसां मध्ये पक्षनिर्णामो भवतीति विज्ञायते।

For such is the curvature of the wings in the middle of the birds, says the tradition.

यावानग्निः सारत्निप्रादेशः सप्तविधः संपद्यते प्रादेशं चतुर्थमात्मनश्चतुर्भागीयाश्चाष्टौ तासां तिस्रः शिर इतरत्पक्षयोर्विभजेत्।

Of the whole area covered by the sevenfold agni with aratni and prádes'a take the prádes'a, the fourth part of the átman (body without head, wings, and tail) and eight quarter bricks; of those latter, six form the head of the falcon; the remainder is to be divided between the two wings.

This sútra determines what portions of the legitimate area of the agni have to be allotted to the different parts of the falcon construction. The whole area of the saptavidha agni is seven purushas with the addition of the two aratnis on the wings and the prádes'a of the tail, altogether $7\frac{1}{2}$ purushas. Now the fourth part of the átman (of the primitive s'yenachiti) = one purusha and the prádes'a, *i. e.*, an oblong of 120 añgulis by 12 añgulis = $\frac{1}{10}$ square purusha and eight quarter bricks, (*i. e.*, square bricks the side of which is equal to the fourth part of a purusha = 30 añgulis, so that they cover together an area of $\frac{1}{2}$ square purusha) are given to the wings in addition to the area which they cover in the primitive agnî, only they have to cede in their turn three of the eight quarter bricks, which are employed for the formation of the head. The original area of both wings together being $2\frac{2}{5}$ purushas, their increased area amounts to $2\frac{2}{5} + 1\frac{3}{5} - \frac{3}{16} = 3\frac{13}{16}$ square purushas, for one wing to $1\frac{29}{32}$ square purushas.

अर्धदशमा अरत्नयोऽङ्गुलिश्च चतुर्भागोना पक्षायामः।

Nine and a half aratnis (= 238 añgulis) and three quarters of an añguli are the length of the wing.

The breadth of the wing is the same as in the primitive s'yena, *i. e.*, = one purusha = 120 añgulis. Dividing the area of the wing mentioned above by the breadth we get the length. Up to this, the wing has the shape of a regular oblong ; the following rules show how to produce the curvature.

द्विपुरुषाꣳ रज्जुमुभयतः पाशां करोति मध्ये लक्षणम्।

Make ties at both ends of a cord of two purushas length and a mark in its middle.

पक्षस्यापरयोः कोट्योरन्तौ नियम्य लक्षणेन प्राचीनमायच्छेदेवं पुरस्तात्स निर्णामः।

Having fastened the two ends of the cord at the two western corners of the oblong forming the wing, take it by the mark and stretch it towards the east ; the same is to be done on the eastern side (*i. e.*, the cord is fastened at the two east corners and stretched towards the east). This is the curvature of the wings.

By stretching the cord, fastened at the west corners, a triangle is formed by the west side of the oblong and the two halves of the cord, and this triangle has to be taken away from the area of the wing. In its stead the triangle formed, when the cord is stretched from the eastern corners, is added to the wing.

एतेनोत्तरः पक्षः व्याख्यातः।

Thereby the northern wing is explained.

The curvature is brought about in the same way.

आत्मा द्विपुरुषायामोऽध्यर्धपुरुषव्यासः।

The átman is two purushas long, one and a half purushas broad.

This is not the final area of the átman, as we shall see further on ; but an oblong of the stated dimensions has to be constructed and by cutting pieces from it we get the area we want.

पुच्छेऽर्धपुरुषव्यासं पुरुषं प्रतीचीनमायच्छेत्।

At the place of the tail stretch a purusha towards the west, with the breadth of half a purusha.

That means : construct an oblong, measuring one purusha from the east to the west, half a purusha from the north to the south.

तस्य दक्षिणतोऽन्यमुत्तरतश्च तावदक्ष्णया अवच्छिद्यार्धपुरुषोऽप्यये स्यात्।

To the south and to the north of this oblong, construct two other oblongs like it, and dividing them by their diagonals remove their halves, so that half a purusha remains as breadth at the jointure of átman and tail.

The result is the form of the tail which we see in the diagram.

शिरस्यर्धपुरुषेण चतुरश्रं कृत्वा पूर्वस्याः करण्या अर्धात्तावति दक्षिणोत्तरयोर्निपातयेत्

At the place of the head a square is to be made with half a purusha, and from the middle of its east side cords are to be stretched to the middle of the northern and the southern side.

The triangles cut off by these cords are to be taken away from the area of the head.

अप्ययान्प्रति श्रोण्यंसानपच्छिन्द्यात्।

Then the four corners of the átman are cut off in the direction towards the joining lines. This finishes the measurement of the s'yena. Its four corners are cut off by four cords connecting the ends of the lines in which the átman and the wings touch each other with the ends of the lines in which head and tail are joined to the átman.

A'pastamba now proceeds to the rules for the different sorts of bricks required for the construction of the agni on the agnikshetra.

करणं पुरुषस्य पञ्चमायामं षष्ठ्यासं यथायोगनतं तत्प्रथमम्।

One class of bricks has the length of the fifth of a purusha, the breadth of a sixth, bent in such a way as to fit (the place in which they are to be employed). This is the first class.

By "nata, bent" the sútrakára means to indicate that the sides of the brick do not form right angles. The shape of the brick is rhomboidical, the angles, which the sides form with each other, are the same which the wings of the s'yena form with the body. (See the diagrams of the two layers of this chiti 11 and 12, in which the bricks are marked with numbers.)

ते द्वे प्राचीसंहिते तद्द्वितीयम्।

Two of those bricks joined with their long side form the second class.

These are the bricks used in the second layer at the point where the curvature of the wings takes place.

प्रथमस्य षड्भागमष्टमभागेन वर्धयेद्यथायोगनतेन तत्तृतीयम्।

Increase that side of the first description which has the length of the sixth of a purusha, by the eighth part of a purusha which is bent in such a way as to fit in its proper place; this is the third class.

These are the bricks employed in the second layer, at the place where átman and wings join. They consist of two parts; the one part equal to a brick of the first class lies in the wing; the second part, an oblong of 24 añgulis by 15 añgulis, lies in the átman.

चतुर्भागीयाध्यर्धा तस्याश्चतुर्भागीयामात्रमक्ष्णया भिद्यात्तच्चतुर्थम् ।

From a brick of which the area exceeds by a half the area of that brick the side of which is the fourth part of a purusha (this latter would be 30 añg. by 30 añg., the increased brick is 45 añg. by 30 añg.), and divide that part of it which is equal to the brick, the side of which is equal to the fourth part of a purusha, by its diagonal (removing half of it). This is the fourth class.

We get a trapezium, the sides of which are equal to 15 añg., 30 añg., 45 añg. and, in the language of the sútras, to the savis'esha of 30 ($= \sqrt{1800}$); they would have put this last side equal to $42\frac{2 6}{6 8}$ añgulis and very likely have expressed the fraction as 14 tilas.

चतुर्भागीयार्धं पञ्चमम् ।

Bricks which are equal to the half of those of which the side is the fourth of a purusha, form the fifth class. Oblongs of 30 añg. by 15 añg.

तस्याक्ष्णयाभेदः षष्ठम् ।

The division of the above bricks by the diagonal produces bricks of the sixth class.

Rectangular triangles (the sides: 30 añg., 15 añg., $\sqrt{1125.}$)

पुरुषस्य पञ्चमभागं दशमभागव्यासं प्रतीचीनमायच्छेत्तस्य दक्षिणतोऽन्यमुत्तरतश्च ताव-
क्ष्णया दक्षिणापरयोः कोट्योरालिखेत् तत्सप्तमम् ।

Draw an oblong the length of which from the east to the west is the fifth part of a purusha (= 24 añgulis) and the breadth the tenth part (12 añg.); to the north and the south of this oblong draw two other oblongs, and divide those by the diagonals dividing their south-western corners. This is the seventh class.

We get the rhomboidical bricks employed in the second layer on both sides of the tail. Two of their sides are = 24 añg., the two others = $\sqrt{720}$.

एवमन्यदुत्तरमुत्तरस्याः कोट्या आलिखेत्तदष्टमम् ।

In the same way another description of bricks is formed; only this time the oblong on the north side has to be divided by the (other) diagonal which divides the northern (north-western) corner. This is the eighth class.

Result: the trapeziums employed in the middle of the tail in the second layer.

चतुर्भागीयाक्ष्णयोभयतोभेदो नवमम् ।

The ninth description of bricks is got by dividing a square brick the side of which is equal to the fourth part of a purusha, by both diagonals (into four triangles).

Therewith the dimensions of all required bricks are detailed; it remains to show how the area of the s'yena is to be covered with them.

उपधाने षष्टिः षष्टिः पक्षयोः प्रथमा उदीचीनपद्ध्यात् ।

When placing the bricks we have to put down sixty of the first kind in each wing, turned towards the north.

पुच्छपार्श्वयोरष्टावष्टौ षष्ठ्यः ।

On both sides of the tail eight of the sixth description.

तिस्रोऽग्रे तत एकां ततस्तिस्र तत एकाम् ।

Three of them in the top (*i. e.*, in each of the two western corners of the tail), then one (to the east of the three), then again three, then again one.

पुच्छाप्यये चतुर्थ्यौ विमध्ये ।

At the place where the tail is joined to the body, two bricks of the fourth description are placed, so as to lie partly in the body, partly in the tail. (They are composed of a triangle and an oblong; the triangle belongs to the body, the oblong to the tail).

तयोः पश्चात्पञ्चम्यावनीकसंहिते

To the west of these two, bricks of the fifth kind are placed touching each other with their faces (their short sides).

They touch each other, says one of the commentators, with their faces, like two fighting rams.

शेषे दश चतुर्थ्यः ।

Ten bricks of the fourth kind cover the remainder of the tail.

श्रोण्यंसेषु चाष्टौ प्राचीः प्रतीचीश्च ।

In the four corners of the átman eight bricks of the fourth description are placed, turned towards the east and towards the west.

शेषे च षड्विंशतिरष्टौ षष्ठ्यश्चतस्रः पञ्चम्यः ।

In the remainder of the átman are to be placed twenty-six of the fourth class, eight of the sixth, four of the fifth.

शिरसि चतुर्थ्यौ विमध्ये ।

In the head two bricks of the fourth kind, situated partly in the átman.

तयोश्च पुरस्तात्प्राच्यावेव द्विशतः प्रस्तारः ।

To the east of those, two of the fourth kind turned towards the east. These altogether form one layer of two hundred bricks.

The rules for the second layer follow.

अपरस्मिन्प्रस्तारे पञ्च पञ्च निर्णामयोर्द्वितीयाः ।

In the second layer place five bricks of the second kind in both wings on the place of curvature.

अप्ययोश्च तृतीया आत्मानमष्टमभागोपेताः ।

And bricks of the third kind stretching into the átman with that part, one side of which is an eighth purusha, are to be placed on the two lines in which the wings are joined to the átman.

शेषे पञ्चचत्वारिंशत्प्रथमाः प्राचीः ।

In the remaining part of each wing forty-five bricks of the first class are to be placed, turned towards the east.

Twenty-five in the southern half of the southern wing, twenty in its northern half; twenty-five in the northern half of the northern wing, twenty in its southern half.

पुच्छस्य पार्श्वयोः पञ्च सप्तम्यः ।

Five bricks of the seventh class are to be placed on the northern side of the tail and five on its southern side.

द्वितीयाचतुर्थ्योश्चान्यतरतः प्रतिसंहितामेकैकाम् ।

At the side of the second (of the above mentioned bricks) on one side (of the tail), and at the side of the fourth on the other side, one brick of the seventh class is to be placed.

शेषे त्रयोदशाष्टम्यः ।

In the remaining part of the tail thirteen bricks of the eighth class are to be placed.

श्रोण्यंसेषु चाष्टौ चतुर्थ्यो दक्षिणा उदीचीश्च ।

In the four corners of the átman place eight bricks of the fourth kind, turned towards the south and the north.

शेषे च विंशतिस्त्रिंशत्षष्ठ्य एका पञ्चमी ।

In the remaining part of the átman twenty bricks of the fourth kind, thirty of the sixth and one of the fifth, are to be placed.

शिरसि चतुर्थ्यौ तयोश्च पुरस्ताच्चतस्रो नवम्यः ।

Two of the fourth kind are to be placed in the head, and to the east of those four of the ninth kind.

एष द्विशतः प्रस्तारः ।

This gives again a layer of two hundred bricks.

व्यत्यासं चिनुयाद्यावतः प्रस्तारांश्चिकीर्षेत् ।

By turns the layers are to be constructed as many as we may wish to make.

The third layer is equal to the first, the fourth to the second, the fifth again to the first, and so on.

Next I extract from the third paṭala of Baudháyana's S'ulva-sútra the rules for the construction of the sárarathachakrachit, the altar shaped like a wheel with spokes. *Vide* Diagrams 13, 14, 15.

पुरुषार्धीयस्याद्दशेनेष्टकाः समचतुरश्राः कारयेन्मानार्थाः ।

With the fifteenth part of half a purusha square bricks are made; they are used for measuring (only for the measurement of the area of the sárarathachakrachit, not for the construction of the agni).

A square is made equal to half a square purusha and its fifteenth part taken; then bricks are made, equal to this fifteenth part.

तासां द्वे शते पञ्चविंशतिश्च सारत्निप्रादेशः सप्तविधः सम्पद्यते ।

Two hundred and twenty-five of these bricks constitute the sevenfold agni together with aratni and prádes'a.

The sevenfold agni with aratni and prádes'a means, as mentioned above, the agni the area of which is equal to seven and a half square purushas. As fifteen of the bricks mentioned in the first sútra make half a square purusha, seven and a half purushas require two hundred and twenty-five.

तास्वन्याश्चतुःषष्टिमावपेत् ।

To these (two hundred and twenty-five bricks) sixty-four more are to be added.

We get thereby altogether two hundred and eighty-nine bricks.

ताभिः चतुरश्रं करोति ।

With these bricks a square is to be formed.

तस्य षोडशेष्टका पार्श्वमानी भवति ।

The side of the square comprises sixteen bricks.

त्रयस्त्रिंशदतिशिष्यन्ते ।

Thirty-three bricks still remain.

ताभिरन्तात्सर्वतः परिचिनुयात् ।

These are to be placed on all sides round the borders (of the square; *i. e.*, according to the commentary, on the north side and east side of the square).

Thereby all 289 bricks are arranged in a square, the side of which is formed by seventeen bricks. It is strange that we are not directed to construct the whole square at once, but are told to form at first a square out of 256 bricks and then to place the remaining 33 bricks around it. I have to propose only the following explanation. The commentator describing the whole procedure tells us to form at first in the middle of the agnikshetra a small square with four bricks, then to increase this square into a larger one, of nine bricks, by adding five bricks, to increase this square in its turn into a larger one of sixteen, and so on. While we place the additional bricks by turns on the north and east side and on the south and west side of the initial square of four bricks, the growing square loses and regains by turns its situation right in the centre of the agnikshetra; it loses it when it is increased for the first time, regains it when increased for the second time, loses it again when increased for the third time, and so on. When it is increased for the fourteenth time or, to put it in another way, when 256 bricks have been laid down, the centre of the square coincides again with the centre of the agnikshetra, and it is again displaced from there when thirty-three bricks more are added on the north and east side, and the whole square is composed of 289 bricks. The whole agni was therefore slightly displaced, and for this reason perhaps Baudháyana preferred not to call it a real chaturas'ra, but a figure made out of a chaturasra of 256 bricks with the addition of 33 bricks. There is reason for wonder that the displacement of the agni was not remedied in some way; it would have been a very easy matter.

नाभिः षोडश मध्याः ।

The sixteen middle bricks form the nave of the wheel.

We must remember that the bricks mentioned here are only used for measuring out the agnikshetra, and consequently understand by the sixteen middle bricks the area covered by them. In order to cut a square of the required size out of the centre of the large square, the commentator directs us to fix poles in the centre of the four bricks forming the corners of the square of twenty-five bricks situated in the middle of the large square and to join these four poles by cords; the area included by these cords is equal to that of sixteen bricks.

चतुःषष्टिरराश्चतुःषष्टिर्वेदिः ।

Sixty-four bricks form the spokes of the wheel, sixty-four the vedi.

Out of the entire square of 289 bricks another square has to be cut out, containing the area for the spokes and for the void spaces between the spokes. This square would be equal to the area occupied by 144 bricks, but we have to deduct from that the 16 bricks in the centre which constitute the nave. Thus 128 bricks are divided equally between spokes and interstices. The required square is cut out by poles being fixed in the centre of the four bricks which form the corners of the square of 13 × 13 bricks and by joining the four poles with cords.

नेमिः शेषाः ।

The remaining bricks form the felloe of the wheel.—One hundred and forty-four bricks having been employed for nave and spokes, one hundred and forty-five remain for the felloe. The measurement of the agnikshetra being finished therewith, the bricks used for measuring are no longer wanted. As result of the described proceeding we have three squares, the largest of which encloses the two smaller ones. The smallest, situated in the centre, is meant for the nave; the two larger ones mark the interior and exterior edges of the felloe. It remains to turn these three squares into circles.

नाभिमन्ततः परिलिखेत्

The nave is to be circumscribed at its borders with a circle, *i. e.* the square forming the nave is to be turned into a circle. This was of course executed according to the general rule which has been discussed above.

नेमिमन्ततश्चान्तरतश्च परिलिखेत् ।

After having likewise turned into circles the squares, marking the outer and inner edge of the felloe—

नेमिनाभ्योरन्तरालं द्वात्रिंशधा विभज्य विपर्यासं भागानुद्धरेत् ।

One divides the area lying between felloe and nave into thirty-two parts, and takes out the second, fourth, sixth, &c., parts.

That means: the second, &c., parts are excluded from the agnikshetra and not to be covered with bricks.

एवमावाप उद्धृतो भवति ।

In this manner the added part (*i. e.*, the sixty-four bricks by which the square of 289 bricks exceeded the legitimate area of the saptavidha agni) is removed again.

By following all the preceding directions we get indeed a wheel, the area of which (with exclusion of the interstices between the spokes) is equal to that of the saptavidha agni; of course, we have to make the necessary allowance for the inevitable error introduced by the square having to be turned into a circle. It remains to retrace the steps by which Baudháyana succeeded in rendering the area of the sárarathachakra pretty well equal to that of the chaturasra s′yena.

A look at the diagram of the sárarathachakrachit shows at once that one preliminary question must first be settled, the question what the relative size of the wheel's different parts was to be. As far as we can see, there was no fixed rule regarding this matter, and wheels of various shapes might therefore have been adopted. Baudháyana does not state at the outset what the shape of his wheel will be, but from the result of his rules we may conclude his intention. The entire square—or the entire circle into which the square is turned—comprises 289 bricks, or simpler 289 parts, of which 145 form the felloe, the remaining 144 the spokes, interstices, and the nave. It appears therefore probable that Baudháyana's intention was to allot to the felloe an area equal to that of spokes, &c., together. The reason why the two parts were not made exactly equal will appear from the following.

The task was, in the first place, to draw two squares—representing the outer and the inner edge of the felloe—the area of one of which was the double of the area of the other. For this purpose Baudháyana made use of his "savis′esha," *i. e.*, of the rule teaching that the square of 16 $\frac{33}{34}$ is almost equal to double the square of 12; only he substituted here, in order to facilitate the operation, 17 to 16 $\frac{33}{34}$. Accordingly, he began by drawing a square the area of which amounted to seven and a half square purushas, divided it into 289 parts, by dividing its side into 17 parts, and drew in the centre of this square another one comprising 144 such parts (by the method described above). To these two squares representing the outer and inner edges of the felloe a third one, marking the area of the nave, had to be added. For this purpose from the square of 144 parts a small square of 16 parts, amounting to the eighth part of the whole, was cut out. Lastly, of the 128 parts left for the space between nave and felloe, 64 were removed, so that 64 were left for the sixteen spokes.

Now by removing 64 parts, the agnikshetra was unduly reduced; it had to contain 289 parts, and it only contained 225. This deficiency had of course to be made up in some way, and the way how to do that was not very difficult to find. Sixty-four of two hundred and eighty-nine parts were lost in the act of cutting out the interstices of the spokes, therefore the area of the initial square had to be such that it would be equal to $7\frac{1}{2}$ square purushas after having been diminished by $\frac{64}{289}$. Accordingly, the square equal to $7\frac{1}{2}$ purushas had not to be divided into 289 parts, but into 225 parts, and 64 parts had to be added moreover, so that the loss of these 64 parts reduced the agnikshetra just to the right size.

Hence Baudháyana's rules to make bricks equal to the two hundred and twenty-fifth part of the agni, to add sixty-four such bricks, &c.

The rules now following teach how to cover the kshetra of the sára-rathachakra with two hundred bricks.

नेमिं चतुःषष्टिं कृत्वा व्यवलिख्य मध्ये परिलिखेत्।

Having divided the felloe into sixty-four parts and having drawn the separating lines, a circle is to be described in the middle (of the felloe).

ता अष्टाविंशतिशतं भवन्ति।

Thus we get one hundred and twenty-eight (bricks placed in the felloe).

अरांश्चतुर्धा विभजेत्।

Every spoke is to be divided into four parts. We get therefore sixty-four bricks in all spokes together.

नाभिमष्टधा विभजेत्।

The nave is to be divided into eight parts (by radii).

एष प्रथमः प्रस्तारः।

This is the first layer.

Again, in order to avoid the "bheda", a different division of the agnikshetra had to be adopted for the second layer.

अपरस्मिन्प्रस्तारे नाभिमन्ततश्चतुर्थवेलायां परिलिखेत्।

In the second layer a circle is to be described in the nave at the distance of a quarter from the edge.

नेमिमन्तरतः।

In the same manner a circle is to be described in the felloe at the distance of a quarter from its inner edge.

नेमिमन्तरतश्चतुःषष्टिं कृत्वा व्यवलिखेत् ।

After having divided the felloe at its inner edge into sixty-four parts, draw the dividing lines.

अराणां पञ्चधा विभाग आपरिकर्षण्योः ।

The spokes are divided into five parts, each up to the two circles (in nemi and nábhi). That means: the area of a spoke is considered to extend into the felloe and the nave up to the two circles which had been drawn in them at the distance of a quarter from the edge, and this whole area is divided into five parts.

नेम्यामन्तरालेषु द्वे द्वे ।

Two bricks are placed in each of the interstices in the nemi (the interstices between the spokes).

नाभ्यन्तरालेष्वेकैकाम् ।

And one brick in the interstices in the nave.

यच्छेषं नाभेस्तदष्टधा विभजेत् ।

The remainder of the nave is to be divided into eight parts.

स एष षोडशप्रकरणः सारो रथचक्रचित् ।

This is the construction in the shape of a wheel with spokes, which requires altogether sixteen different kinds of bricks.

As remarked above, the third and fifth layers are to be made equal to the first, the fourth to the second.

I lastly extract the chapter treating of the s'mas'ánachit. It is not easy to say what would be the correct definition of a s'mas'ána in the sense in which it is used in the s'ulvasútra; it seems to be a construction on which the dead body was placed, perhaps the pile on which it was burnt. There is, however, no doubt about the form of the chiti, which will appear clear enough from the diagram. *Vide* Diagrams 16, 17, 18.

श्मशानचितं चिन्वीतेति विज्ञायते । सर्वमग्निं चतुरश्रान्पञ्चदशभागान्कृत्वा ॥

" He may construct the s'mas'ánachiti", such is the tradition. Having divided the whole agni into fifteen squares.

The area of the agni, $7\frac{1}{2} = \frac{15}{2}$ square purushas, divided in this manner, yields fifteen squares, of one half square purusha each.

तेषामाख्यातमुपधानम् ।

The arrangement of these fifteen squares has already been taught.

As the commentator explains, the subject has been treated in a previous portion of Baudháyana's kalpasútra, from which he quotes the following:

श्मशानचितं चिन्वीत यः कामयेत पितृलोक ऋध्नुयामिति षट् प्राच्यः पुरुषास्त्रयः पुरस्तात्तिर्यञ्चौ तौ द्वौ स आत्मा ।

He who wishes for prosperity in the world of the fathers, may construct the s'mas'ánachiti. Six purushas are the length of the práchí line, three the length of the eastern side, two the length of the western side.

Purusha means here not the ordinary purusha, but the measure of the side of one of the fifteen squares into which the agni has been divided. The form of the chiti is that of a trapezium (as the sútras would call it: an oblong shorter on one side), the east side of which is equal to three reduced purushas, &c.

The area of this trapezium is consequently equal to $7\frac{1}{2}$ square purushas.

This area has now to be divided into two hundred parts.

त्रिभिर्भागैर्भागार्धव्यासं दीर्घचतुरश्रं विहृत्य पूर्वस्याः करण्या अर्धाच्छ्रोणी प्रत्यालिख्यान्त्यावुद्धरेत्।

With three of these parts construct an oblong of the breadth of one part (an oblong of which one side is equal to three times the side of one of the fifteen squares, and the other equal to one time the side), draw from the middle of the east side of this oblong lines to the two west corners, and cut off the two side pieces.

After the removal of these two pieces, there remains a praüga, an acutangular equilateral triangle.

तस्य दशधा विभागः।

This triangle is divided into ten parts.

For the details of this division, we must consult the commentator:

तस्य प्रउगस्य प्रउगाकारा उभयतःप्रउगाकारा इष्टका यथा भवन्ति तथा दशधा विभागः। अन्यथाविभागे करणबहुत्वं स्यात्। तत्रैवं विभागः। प्रउगपृथ्वनीके समान्तराणि त्रीणि चिह्नानि कृत्वा चतुरो विभागान्कृत्वा प्रउगपार्श्वयोरपि तथा कृत्वा पृथ्वनीकप्रथमचिह्नादारभ्येतरपार्श्वप्रथमचिह्नं प्रत्यालिखेत्। एवं द्वितीयचिह्नादारभ्य पार्श्वद्वितीयम्। एवं तृतीयचिह्नादारभ्य तथा तृतीयम्। तथा तृतीयचिह्नादारभ्य इतरपार्श्वप्रथमचिह्नं प्रत्यालिखेत्। एवमितरयोः। एवं विभक्ते पृथ्वनीकस्थाः प्रउगाकाराश्चतस्र इष्टकाः। ततस्तिस्र उभयतःप्रउगाकृतयः। ततो द्वे। तत एका चुबुकान्ता। एवं षडुभयतःप्रउगाश्चतस्रः प्रउगाः। एवं दशेष्टकैकस्मिन्प्रउगे भवन्ति।

The division of this triangle is to be made in such a way as to produce bricks of the shape of triangles and double triangles (two triangles joined with their bases). If we adopted another division, we should get different classes of bricks. (The sútras always study the greatest shortness in their expressions and say in this case only: the division is into ten parts. Now, the commentator remarks, this can only mean: into ten triangles and double triangles; for if we divide the large triangle in any other manner, the eight parts would be of different shape, and then the sútrakára would have been bound to give rules for manufacturing bricks of these different shapes). The division of the triangle is effected in the following manner. We make on the "broad face", *i. e.*, the base of the triangle (the sútrakáras compare the triangle with a face, the base—we have to imagine the triangle turned round, so that the base is uppermost—representing the broad *i. e.*, upper part and the top the chin, chubuka) three marks at equal distances from each other (thus dividing it into four parts). Having divided the two other sides of the triangle in the same way, we begin by drawing a line from the first mark on the base to the first mark on the nearer of the two other sides. Then a line is drawn joining the second mark on the base with the second mark on the side, and a third line joining the third mark on the base with the third mark on the side. After that, a line is drawn joining the third mark on the base with the first mark on the third side of the triangle. The same is done with the other marks. By this division we get four triangular bricks standing on the base of the large triangle; over these we have three double-triangular bricks; then two double-triangles; then one double triangle in the 'chin' of the large triangle. Altogether six double triangles and four triangles. Thus we have ten bricks in one of the large triangles.

तानि विंशतिः सर्वोऽग्निः संपद्यते ।

Twenty such (large triangles as described in the last sútra but one) form the whole agni.

One of these triangles is the half of an oblong, the area of which is equal to the tenth part of the whole agni.

The arrangement of these twenty large triangles, every one of which is subdivided into ten praügas and ubhayatahpraügas, may be seen in the sketch of the first layer of the s'mas'ánachiti, and I omit therefore the detailed description given by the commentator.

Baudháyana proceeds to the rules for the second layer.

अपरस्मिन्प्रस्तारे प्रउगं मध्येऽनूचीनं विभजेत् ।

For the second layer we divide one triangle lengthways (bisecting the base by a perpendicalar from the top).

Here again we depend on the commentary for explanation.

अग्निक्षेत्रे भागप्रमाणव्यासानि षड्भागप्रमाणायतानि पञ्च महाप्रउगाणि शेरते । तत्र प्रत्यगग्राणि त्रीणि प्रागग्रे द्वे । तेषां महाप्रउगानां प्रउगद्वयमिह विवक्षितम् । अनूचीनमिति प्रउगविशेषणम् । षड्भागायतमित्यर्थः । दक्षिणतः प्रत्यगपस्थितप्रउगमध्ये भागप्रमाणप्रत्यनीकमध्यादारभ्यापश्चिमस्कन्धाग्रादालिखेत् । एवमुत्तरस्मिन्नपि पार्श्वे स्थितं विभजेत् ।

In the whole agnikshetra (of the s'mas'ánachiti) there are five triangles, the height of which is equal to the measure of six parts (to six times the side of the fifteenth part of the agnikshetra), and the base of which is equal to one such part (the area of one such triangle is $\frac{3}{15}$ of the agnikshetra, therefore all five = the whole agnikshetra, $7\frac{1}{2}$ square purushas). (If we divide the agni into these five triangles), the top of three among them is turned towards the west, that of two towards the east. Two of these five triangles are meant in the sútra (only two come really into question, as we shall see further on). By "lengthways" a modification of the triangle is to be understood; the meaning is a triangle of six parts' height. (And this triangle is to be got in the following way). On the south side of the agni a line is to be drawn through the middle of the triangle situated there, the top of which is turned towards the west; this line reaches from the middle of the base the measure of which is one part to the top of the triangle. In the same way the triangle on the north side of the agni is to be divided.

The result is the two long rectangular triangles on the north and south sides of the second layer of the s'mas'ánachiti.

तस्य षड्धा विभागः ।

This triangle is divided into six parts.

Commentary: प्रत्यगग्रं षड्भागायतं महाप्रउगार्धं तिर्यक् त्रिधा विभजेत् । तत्र पूर्वखण्डस्य पूर्वतिर्यङ्मान्यां समान्तराले द्वे चिह्ने कृत्वा प्रथमचिह्नादारभ्यार्जवेनापरतिर्यङ्मानीं प्रत्यालिखेत् । एवं द्वितीयचिह्नादारभ्य । एवं मध्यमखण्डस्य पूर्वान्तमध्यादारभ्यार्जवेनापरान्तमालिखेत् । एवं विभक्ते प्रथमे खण्डे बाह्यपार्श्वत अर्धप्रउगाकारा एकेष्टका । मध्यतो द्वे दीर्घचतुरश्रे । मध्यमखण्डस्य बाह्यत एका प्रउगार्धा अन्तरत एका दीर्घचतुरश्रा । अपरो भागः प्रउगार्धरूप एव । एवं षड्धा विभागः । एवमुत्तरतः ।

The diagram of the second layer, in which the two triangles are divided in the manner described above, renders a translation of the commentator's words unnecessary.

ते द्वे पार्श्वयोरुपदध्यात् ।

These two (large triangles, divided into six parts each) are to be placed on both sides (of the second layer).

In the following sútras those bricks are described which fill the space between the two triangles.

भागतृतीयायामाश्चतुर्थव्यासाः कारयेत ।

Bricks are to be made as long as the third part (of the side of one of the fifteen squares which compose the agnikshetra), and as broad as the fourth part.

तासामर्ध्यास्तिर्यग्भेदाः ।

And other bricks equal to one half of the bricks of the first class, produced by dividing the latter by a horizontal line.

ता अन्तयोरुपधाय शेषमग्निं बृहतीभिः प्राचीभिः प्रच्छादयेत् ।

Having put bricks of the second class on the east and west end of the agni, the remaining space is to be covered with the large bricks of the first description.

Covering the agni as directed, we place at first eight ardhya bricks on the east end and eight on the west end. The space left empty between these two rows requires 17 × 8 = 136 bṛihatí bricks. Now, summing up all bricks employed we get (1) 136 bṛihatyas (2) 16 ardhyás (3) twelve bricks in the two triangles on the north and south side together. Sum: 164 bricks.

But we want, according to the general rule, 200 bricks, and therefore the following sútra.

अर्धेष्टकाभिः सङ्ख्यां पूरयेत् ।

Finally the number is to be made full with ardhya-bricks.

That means: thirty-six bṛihatyas are taken out, and seventy-two ardhyás put in their places. The sketch of the layer in question shows where this had to be done.

So far the rules for the s'mas'ánachiti resemble those for the other chitis, but the following sútras refer to an interesting peculiarity. I give at first a passage from a previous part of Baudháyana's Kalpasútra, quoted by the commentator.

तस्य मात्रा यदि ग्रीवदघ्नं पुरस्तान्नाभिदघ्नं पश्चात् । यदि नाभिदघ्नं पुरस्ताज्जानु—दघ्नं पश्चात् । यदि जानुदघ्नं पुरस्ताद्गुल्फदघ्नं पश्चात् । यदि गुल्फदघ्नं पुरस्तात्समं भूमेः पश्चात् । स एष श्मशानचित्पितृलोककामस्येति ।

When its measure is such as to reach up to the neck on the east side, it reaches up to the navel on the west side; when it reaches up to the navel on the east side, it reaches up to the knee on the west side; when it reaches up to the knee on the east side, it reaches up to the ankle on the west side; when it reaches up to the ankle on the east side, it is on a level with the ground on the west side. Such is the s'mas'ánachiti of him who desires the world of the fathers.

We see from these words that, contrary to the general rule which prescribed a perfectly horizontal surface for the chitis, the s'mas'ánachit had to be higher at its east end than at its west end. The commentator adds: hastipṛishṭhavach chinvíteti: the chiti is to be constructed so as to resemble the back of an elephant which is sloping down towards a person viewing the animal from behind. This peculiar shape of the s'mas'ánachiti required consequently a set of rules for preserving, notwithstanding the different height, the same cubic content of the whole mass of bricks.

ऊर्ध्वप्रमाणमग्नेः पञ्चमेन वर्धयेत् ।

The height of the agni is to be increased by one fifth.

The height of the agni, when constructed for the first time and in five layers, is—as mentioned above—one jánu = 32 añgulis; when constructed for the second time and in ten layers, it is the double, and it is three times as much when, in the third construction, the number of layers amounts to fifteen. A fifth of the usual height has to be added to the height of the s'mas'ánachiti.

तत्सर्वं त्रेधा विभज्य द्वयोर्भागयोश्चतुर्थेन नवमेन वा चतुर्दशेन वेष्टकाः कारयेत्।

Divide all this—the height inclusive the added fifth part—into three parts, and make bricks with the fourth or the ninth or the fourteenth part of two of these three parts.

With the fourth for the agni of five layers, with the ninth for the agni das'achitíka, with the fifteenth for the panchadas'achitíka.

ताभिश्चतस्रो वा नव वा चतुर्दश वा चितीरुपधाय शेषमवाच्यमक्ष्णयापच्छिन्द्यादर्धमुद्धरेत्।

Having constructed with these bricks either four or nine or fifteen layers, the remaining part of the height (amounting to one third) is to be divided in a downward direction by the diagonal and half of it to be removed.

That means: the fifth layer is to be constructed with bricks the height of which is equal to the third part of the whole height; and then half of the whole layer is to be cut off following the direction of the diagonal of the northern and southern side. In this way the cubic content of the whole chiti comes out right. Increasing the height of the agni of five layers by its fifth part, we get $32 + 6\frac{2}{5} = 38\frac{2}{5}$ añgulis. This divided by three and the quotient multiplied by two, gives $25\frac{3}{5}$. The fourth part of this, $6\frac{2}{5}$ añgulis is the height of the bricks of each of the four first layers. The fifth layer, before being cut in two, is $12\frac{4}{5}$ añgulis high; after the removal of its half, it has this height only on its east side, the height on the west side being equal to 0. Thus its middle height is $6\frac{2}{5}$, and consequently the middle height of the whole chiti $= 32$ añgulis. In the same way we get as height of the agni of ten layers $76\frac{4}{5}$ añgulis on the east side, $51\frac{1}{5}$ on the west side, 64 añgulis as middle height. The corresponding numbers for the pan̆chadas'achitíka agni are $115\frac{1}{5}$, $76\frac{4}{5}$, 96.

Regarding the time in which the S'ulvasútras may have been composed, it is impossible to give more accurate information than we are able to give about the date of the Kalpasútras. But whatever the period may have been during which Kalpasútras and S'ulvasútras were composed in the form we have now before us, we must keep in view that they only give a systematically arranged description of sacrificial rites, which had been practised during long preceding ages. The rules for the size of the various vedis, for the primitive shape and the variations of the agni, &c., are given by the bráhmaṇas, although we cannot expect from this class of writings explanations of the manner in which the manifold measurements and transformations had to be managed. Many of the rules, which we find now in Baudháyana, A'pastamba, and Katyáyana, expressed in the same or almost the same words, must have formed the common property of all adhvaryus long before they were embodied in the Kalpasútras which have come down to us. Besides, the quaint and clumsy terminology often employed for the expression of very simple operations—for instance in the rules for the addition and subtraction of squares—is another proof for the high antiquity of these rules of the cord, and separates them by a wide gulf from the products of later Indian science with their abstract and refined terms.

This leads to another consideration. Clumsy and ungainly as these old sútras undoubtedly are, they have at least the advantage of dealing with geometrical operations in really geometrical terms, and are in this point superior to the treatment of geometrical questions which we find in the Lílávatí and similar works. They tell us that the diagonal of a square or of an oblong produces an area equal to double the area of the square or to the squares of the sides of the oblong—not that the square of the number of units into which the diagonal is divided is equal to double the square of the number expressing the side of the square or to the sum of the squares of the two numbers which represent the sides of the oblong.

Let us see how Bháskara words the proposition about the rectangular triangle (instead of which the sútras speak of the square and the oblong). We read in the chapter on kshetravyavahára in the Lílávatí the following:

— तत्कृत्योर्योगपदं कर्णः ।

The square root of the sum of the squares of these (of the two shorter sides of a rectangular triangle) is the diagonal.

दोःकर्णवर्गयोर्विवरान्मूलं कोटिः ।

The square root of the difference of the squares of the diagonal and one of the short sides (called "doh") is the other short side (koṭih), etc.

It is apparent that these rules are expressed with a view to calculation, and we find indeed that Bháskara immediately proceeds to examples which are exercises in arithmetic, not in geometry.

कोटिश्चतुष्टयं यत्र दोस्त्रयं यत्र का श्रुतिः ।
कोटिं दोः कर्णतः कोटिश्रुतिभ्यां च भुजं वद ॥

A geometrical truth interests the later Indian mathematicians but in so far as it furnishes them with convenient examples for their arithmetical and algebraic rules; purely geometrical constructions, as the samása and nirhára of squares, described in the S'ulvasútras, find no place in their writings.

It is true that the exclusively practical purpose of the S'ulvasútras necessitated in some way the employment of practical, that means in this case, geometrical terms, and it might be said that the later mathematicians would have employed the same methods when they had had to deal with the same questions.

But a striking proof of the contrary is given by the commentators of the S'ulvasútras who represent the later development of Indian mathematics. Trustworthy guides as they are in the greater number of cases, their tendency of sacrificing geometrical construction to numerical calculation, their excessive fondness, as it might be styled, of doing sums renders them sometimes entirely misleading. I shall illustrate this by some examples.

As mentioned above, the area of the saptavidha agni had, at each repetition of the construction of the altar, to be increased by one square purusha. In order to effect this increase, without changing the proportion of the single parts of the agni, Baudháyana gives the following rule:

That which is different from the original form of the agni (*i. e.*, that area which has to be added to the 7½ square purushas of the primitive agni) is to be divided into fifteen parts, and two of these parts are to be added to every one of the seven square purushas of the primitive agni (the one remaining part is consequently added to the remaining half purusha); with seven and a half of these increased purushas, the agni has to be constructed.

According to the commentator, we have to apply this rule in the following fashion. The one square purusha, which has to be added to the saptavidha agni, contains 14400 square añgulis. We divide 14400 by fifteen, multiply the quotient by two, and add the product to 14400: result = 16320. These 16320 añgulis are the square content of the new increased square purusha, and we have therefore, in order to get the required measure of length, to extract the square root of 16320. This root indicates the length which had to be given to the cane used for measuring out the ashṭavidha agni.

Such a proceeding is of course not countenanced by the rules of the S'ulvasútras themselves. Baudháyana's method was undoubtedly the following. The square purusha which had to be added was divided into fifteen parts, either into fifteen small oblongs, by dividing one side of the square into three, the other into five parts or into fifteen small squares; in the latter case, the panchadas'amakaraṇí had to be found according to the paribhásha rules. Two of these fifteenth parts were then combined into one; if squares, by taking the dvikaraṇí of one of them; if oblongs, by turning one of them into a square and then taking the dvikaraṇí. Lastly—following the rules for chaturasra-samása—the square containing the two fifteenth parts was added to a square purusha, and the side of the resulting square furnished the measure of the purusha which had to be employed for the ashṭavidha agni.

Another example is furnished by the rules for the paitrikí vedi, the altar used at the piṭriyajna, the area of which had to be equal to the ninth part of the vedi used at the soma sacrifices. The measures of the sides of this vedi have been mentioned above; its area amounts to 972 square padas.

Now for constructing the paitrikí vedi from the saumikí vedi, Baudháyana gives the following short rule:

महावेदेस्तृतीयेन समचतुरश्रकृतायास्तृतीयकरणीति नवमस्तु भूमेर्भागो भवति ।

The commentator, supplying several words, explains this sútra in the following way: If we make a square, the area of which is equal to 972 square padas, its side will be equal to 31 padas, 2 añgulis, and 26 tilas. The third part of this (= 10 padas, 5 añgulis, and 31 tilas) is to be taken for the side of a square, the area of which will be equal to the ninth part of the mahávedi.

For a proof we are directed to turn the 972 square padas into square tilas by multiplying 972 by 225 and then by 1056, to extract the square-root of the result, to turn the tilas again into padas by dividing the square-root by 34 and then by fifteen, and finally to divide the result by three.

In accordance with this process, the commentator translates the above sútra in the following manner:

The side (" karaṇí" to be supplied) of that area (" bhúmeh" to be supplied) which is made a square with the third part of the mahávedi (which has been itself turned into a square previously) is the ṭritíyakaraṇí; the ninth part (of the mahávedi) is produced (by making a square with this ṭritíyakaraṇí).—This translation is certainly wrong. In the first place, the word 'karaṇí', which the commentator supplies, could not be missed in the text of the sútra. In the second place, the commentator ascribes to the word 'tritíyakaraṇí' a meaning which it cannot possibly have. He interprets it as the line which is the third part (of the side of the mahávedi); but that line is called the navamakaraṇí, as its square is equal to the ninth part of the area of the mahávedi, and tritíyakaraṇí can only mean the line which produces, or the square of which is the third part (of some area).

To arrive at the right understanding of the sútra, we must consider by what method the task of constructing the paitrikí vedi could be accomplished in the shortest way. The thing was to construct a square, the area of which would be equal to the ninth part of another area which contained 972 square padas, *i. e.*, to 108 square padas. If 108 would yield an integral square-root, the matter would have been easy enough; but this not being the case, another method had to be devised. The commentator, as we have seen, proposes to construct a square of 972 padas, and to take the third part of its side; but this method besides, as shown above, not agreeing with the words of the sútra, required several tedious preparatory constructions. The same remark applies to the direct construction of a square of 108 padas, and a shorter process could therefore not but be highly welcome. Now the third part of 972 is 324, and the square-root of 324 is exactly 18; in other words, the side of a square of 324 square padas is eighteen padas. Accordingly, instead of the navamakaraṇí of 972, the tritíyakaraṇi of 324 was sought for, and we know from the paribhásha rules that this could be easily managed. Accordingly, Baudháyana's rule has to be translated as follows: The ṭritíyakaraṇí of that area which is made a square with the third part of the mahávedi (*i. e.*, of a square of 324 padas) is it (*viz.* the side of a square of 108 padas); the result is the ninth part of the area (of the mahávedi).

Thus we see that the pre-conceived opinion of the commentator about the method to be employed for the solution of the problem leads him to a perfectly mistaken interpretation of the sútra.

On the other hand, it is interesting to find some terms indicating a connexion between the first rudiments of science as contained in the S′ulvasútras and its later development. So for instance the term 'varga'. It is true that we should be able to account for the meaning in which it is used by later mathematicians—*viz.* that of the square of a number—without finding earlier indications of the manner how it came to be used in that sense. The origin of the term is clearly to be sought for in the graphical representation of a square, which was divided in as many 'vargas', or troops of small squares, as the side contained units of some measure. So the square drawn with a side of five padas' length could be divided into five vargas, each consisting of five small squares, the side of which was one pada long.

Nevertheless it is interesting to find this explanation of varga confirmed by a passage in A′pastamba.

यावत्प्रमाणा रज्जुस्तावतस्तावतो वर्गान्करोति ।

As many measures (units of some measure) a cord contains, so many troops or rows (of small squares) it produces (when a square is drawn on it).

But another case is more interesting still. The word 'karaṇí' is one of the most frequent mathematical terms in treatises as the Lílávatí, Víjagaṇita, &c., and there it is invariably used to denote a surd or irrational number; as the commentators explain it, that of which when the square-root is to be taken, the root does not come out exact. The square-roots of two, three, five, &c., are karaṇís. How the word came by that meaning, we are not told, but we are now able to explain it from the S′ulvasútras. As we have seen above, in these it always means the side of a square.

The connexion between the original and the derived meaning is clear enough. Karaṇí meant at first the side of any square, after that possibly the square-root of any number. Possibly I say, for in reality the mathematical meaning of karaṇí was restricted. It was not used to denote the square-roots of those numbers, the root of which can be exactly obtained, but only of those the root of which does not come out exact, of those in fact the root of which can be represented exactly only in a graphical way. It was not possible to find the exact square-root of eight for instance, but it was possible to draw a square, the area of which was equal to eight—let us say—square padas, and the side of which was therefore a graphical representation of the square-root of eight.

But we have to go still a step further back. 'Karaṇí' meant originally not the side of a square, but the rajjuh karaṇí, the cord used for the measuring of a square. And thus we see that the same word which expressed in later times the highly abstract idea of the surd number, originally denoted a cord made of reeds which the adhvaryu stretched out between two wooden poles when he wanted to please the Immortals by the perfectly symmetrical shape of their altar.

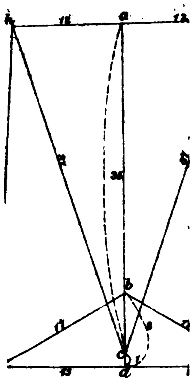


Fig. 4

he cord of 54 padas length;

ná sroní, d uttará sroní,

N

squares have been turned