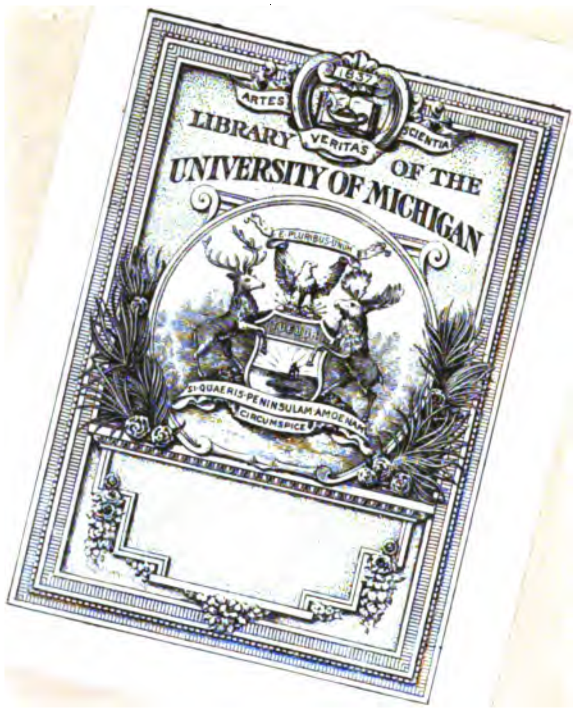
1837
ARTES
VERITAS
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E PLURIBUS UNUM
SI QUAERIS PENINSULAM AMOENAM
CIRCUMSPICE

THEORIA MOTUS LUNAE

EXHIBENS

OMNES EIUS INAEQUALITATES

IN

ADDITAMENTO

HOC IDEM ARGUMENTUM ALITER TRACTATUR

SIMULQUE OSTENDITUR

QUEMADMODUM MOTUS LUNAE CUM OMNIBUS

INAEQUALITATIBUS

INNUMERIS ALIIS MODIS

REPRAESENTARI

ATQUE AD CALCULUM REUOCARI POSSIT

AUCTORE

L. EULERO

IMPENSIS

ACADEMIAE IMPERIALIS SCIENTIARUM

PETROPOLITANAE

ANNO 1753.

Academiam Scientiarum Petropolitanam triennio abhinc omnes, qui ingenii viribus confisi, ad examinandam Neutonianam de motu Lunae Theoriam, animum applicare vellent, inuitasse, atque ei, qui in hac parte tenuisset primas, praemii loco proposuisse nummos aureos centum; postmodum Celeberrimum Clairautum huius certaminis exstitisse victorem, publico Academicorum, qui Petropoli agunt, iudicio diuulgatum.

Celeberrimus Eulerus, Academiae Petropolitanae membrum honorarium, officii ſui eſſe exiſtimauit, ferre vna cum ceteris de illa Clairauti diſſertatione iudicium. Transmiſit ergo huc ad nos una cum ſua ſententia ampliſſimam de eodem argumento diſſertationem; quae quo celerius innoteſceret, viſum eſt Academiae Praeſidi, minoris Ruſſiae Hetmano Illuſtriſſimo Cyrillo Gregoridae, Comiti Raſumouio, eam tradere Academicis in ſolenni conuentu examinandam, ea fini, vt ſi ſuffragio Academicorum comprobata, dignaque iudicata foret, quae orbis eruditi proponeretur theatro, ea praelo quam maturrime ſubiiceretur; quandoquidem ille mos iam inde a principio obtinuit, vt, quae in publico coetu praeleguntur diſſertationes, eae vel ante ſolennem actum, vel paruulo intermiſſo ſpatio typis diuulgentur poſtea.

Cete-

Ceterum dissertatio illa constat magnam partem calculis, veris quidem illis et omnibus numeris absolutis, sed propter nimiam sui molem atque difficultatem auditu molestissimis: quae si recitarentur publice, periculum erat, ne auditorum animi auscultandis iis deficerent, neue Academia Summorum patientia Virorum abvti, caeterosque enicare odio videretur. Praeuidit hoc incommodum, prouiditque Illustrissimi Praesidis sapientia. Mandavit Astronomiae Professori V. C. Nicetae Popouio, cuius id temporis dicendi erat prouincia, vt prolixam illam Euleri dissertationem, omissis calculis, redigeret in compendium, et quae inde excepisset, auditorum causa recitaret publice. Quo quidem facto et auribus hominum consuluit, et tamen rerum capitum participes eos facere aequabili temperamento instituit: atque diſſertationem, ne quid forte naeuorum obreperet, ipſo auctore coram typis excudi aequum cenſuit.

Iam qui illo tempore interfuerant conuentui Academicorum ſolenni auditores, aequis nimis auribusque auſcultarunt excerpta recitantem Euleriana Popouium: eſt ergo quod ſperare liceat, et iis, quorum intereſt, vt qui hoc genere ſtudiorum maxime delectantur, factum iri ſatis ipſa diſſertatione. Datum Petropoli Nov. 1752.

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Caput

INTRO-

INTRODUCTIO.

Cum nullum ſit dubium, quin ſummi Newtoni Theoria, qua motum Planetarum feliciſſimo cum ſucceſſu certis legibus adſtrinxit, plurimum ad motum Lunae accuratius determinandum contulerit, maximi ſane in Aſtronomia momenti eſt quaeſtio: vtrum haec Newtoni Theoria omnino ſit ſufficiens omnibus motus inaequalitatibus, quae in Luna obſeruantur, exactiſſime explicandis nec ne? Quanquam autem a Newtoni aſſeclis plerumque affirmari ſolet, nullam in motu Lunae obſeruari inaequalitatem, cuius ratio in iſta Theoria non contineatur; tamen tantum abeſt, vt hic conſenſus a quoquam perſpicue ſit monſtratus, vt potius applicatio huius Theoriae ad Lunam tantis implicetur calculi difficultatibus, quibus penitus euoluendis vires ingenii humani vix ſufficere videntur. Plurimae quidem adhuc prodierunt Tabulae Lunares, quae ex Theoria Newtoniana deductae perhibentur, ſed praeterquam quod ſaepius ultra 5′ ab obſeruationibus diſcrepent, earum conuenientia cum Theoria ipſa neutiquam eſt euicta; quin potius pleraeque Tabulae inaequalitatum non tam Theoriae quam obſeruationibus ſunt ſuperſtructae. Huiusmodi ergo tabularum ſiue conſenſus ſiue diſſenſus cum obſeruationibus neque ad Theoriam Newtonianam pleniſſime confirmandam, neque ad eam infringendam allegari

gari poteſt: nam quatenus iſtae tabulae obſeruationibus ſatisfaciunt, hoc non ſoli Theoriae eſt tribuendum; quatenus autem cum obſeruationibus minus conueniunt, hoc ne Theoriae quidem imputari poterit, propterea quod iſtae Tabulae non ſoli Theoriae innituntur.

Quaeſtio itaque, cujus mentionem feci, recte enodari nequit, niſi ante eiusmodi Tabulae exhibeantur, quae ex ſola Theoria, nullis obſeruationibus in ſubſidium vocatis, ſint formatae; tum enim demum ex huiusmodi Tabularum collatione cum ingenti obſeruationum ſummo ſtudio inſtitutarum copia diiudicare licebit, vtrum Theoria omnibus obſeruationibus reſpondeat, an vero correctione quapiam indigeat. Non difficile quidem eſt ex principiis mechanicis motum Lunae aequationibus analyticis complecti; quoniam autem hae aequationes plures variabiles inter ſe permixtas continent, atque adeo differentialibus ſecundi ordinis implicantur, earum reſolutio maximis difficultatibus eſt obnoxia; et quoniam alio modo niſi per approximationem ſuſcipi non poteſt, vtcunque inſtituatur, ſemper non leue dubium remanet, vtrum partes, quae in calculo ſunt neglectae et praetermiſſae, nihil, quod in motu Lunae eſſet notabile, efficere potuerint. Hoc modo explicatio motuum Lunae tota ad ſolam Analyſin transfertur, ac difficultates, quibus premitur, inde oriuntur, quod Analyſis nondum ſatis eſt exculta.

Cum igitur Theoria Newtoniana hoc principio latiſſime patente innitatur, quod omnia corpora coeleſtia ſe mutuo attrahant in ratione reciproca duplicata diſtantiarum,

rum, si motum Lunae secundum hanc Theoriam definire velimus, vires erunt spectandae omnes, quibus Luna sollicitatur. Atque inter has vires primaria est ea, qua ad terram vrgetur, quae si sola adesset, terraque quiesceret, Luna in ellipsi perfecta secundum regulas Keplerianas motum suum circa terram absolueret. At cum Luna praeterea aeque ac terra ipsa etiam ad solem trahatur, hac vi motus ille regularis non mediocriter perturbabitur: atque haec vis a sole profecta omnium difficultatum, quae in determinatione motus Lunae offenduntur, causa est existimanda. Reliquae enim vires, quibus forte Luna secundum Theoriam Newtoni ad reliquos planetas vrgeri deberet, tam sunt exiguae, vt effectus inde oriundus merito pro nihilo haberi queat.

Solas ergo vires solis ac terrae in computum duci oportet, si motum Lunae secundum Theoriam definire velimus, atque cum ex his viribus formulae analyticae fuerint erutae, quae motum Lunae complectantur, omne studium in his formulis ita euoluendis erit impendendum, vt inde ad quoduis tempus propositum locus Lunae assignari, ac more apud Astronomos solito secundum longitudinem et latitudinem definiri queat. Hinc porro Tabulae Astronomicae pro motu Lunae erunt condendae, quibus omnes inaequalitates tam in longitudine quam in latitudine exhibeantur, ex quibus si pro cujusuis obseruationis momento locus Lunae computetur, consensus vel dissensus calculi ab obseruationibus Theoriam vel confirmabit, vel ejus defectum declarabit. Neque tamen Theoria sola hujusmodi Tabulis construendis sufficit, sed quaedam elementa extrinſecus ab obſeruationibus aſſumi oportet, quae ſunt 1°. Excentricitas orbitae Lunaris, quae ſalua theoria vel major vel minor eſſe potuiſſet; pendet enim a motu Lunae primitus impreſſo, quem Theoria non determinat, ſed tanquam cognitum aſſumit. 2°. Locus Lunae medius pro quapiam Epocha propoſita, qui pariter ex obſeruationibus eſt concludendus. 3°. Locus Apogei orbitae Lunaris pro Epocha quadam data. 4°. Tempus periodicum Lunae ſecundum motum medium, quod pendet a diſtantia Lunae media a Terra, ideoque ex ſola Theoria definiri nequit. 5°. Locus nodorum Lunae pro Epocha quadam data: et 6to denique inclinatio media orbitae Lunaris ad planum Eclipticae.

His autem ſex elementis per obſeruationes definitis reliqua omnia, quibus ad locum Lunae pro quouis tempore aſſignandum opus eſt, ex ſola Theoria ſunt petenda; quae primo ad quoduis tempus locum Apogei eiusque ideo motum verum praebere debet, vt inde ex loco Lunae medio eius anomalia media colligi queat. Deinde Theoria quoque omnes correctiones ſeu Proſtaphaereſes, quae loco Lunae medio vel additae vel ſublatae eius locum verum exhibeant, ſuppeditare debet; atque iſtae correctiones, quae motus inaequalitates appellari ſolent, partim ab Anomalia media Lunae, partim ab eius Phaſi ſeu elongatione a ſole, partimque ab Anomalia ſolis media pendent, ex quo triplici fonte numerus inaequalitatum in immenſum augetur. Porro etiam Theoria motum nodi eiusque omnes inaequalitates indicare tenetur, ac denique etiam pro quouis tempore orbitae Lunaris veram inclinationem

da

ad eclipticam, vt inde eius latitudinem veram eruere liceat.

Cum autem, vt iam innui, nemo adhuc omnes inaequalitates, quae in motu Lunae reperiuntur, ex Theoria elicuerit, vt ex iis iudicium ferri possit, quantum haec Theoria cum obseruationibus conueniat; etsi nullum est dubium quin discrimen, si quod deprehenderetur, admodum paruum sit futurum: iam pridem haec quaestio ex solo motu Apogei dirimi est coepta, dum aliis motus Apogei ex obseruationibus cognitus magnopere a Theoria discrepare est visus, alii autem etiam hoc loco pulcerrimum consensum Theoriae et veritatis iactauerunt. Mirum autem est, ipsum Newtonum nihil circa motum Apogei ex Theoria statuisse, sed eum ex solis obseruationibus in calculum transtulisse, cum tamen motum nodorum summa sagacitate ex Theoria elicuisset, atque veritati consentaneum ostendisset. Cur igitur motum Apogei plane silentio praeterierit, nulla alia ratio subesse videtur, nisi quod animaduerterit hunc motum, prouti ex Theoria prodiret, obseruationibus parum fore conformem. Ex iis enim, quae Newtonus in suo immortali opere de motu absidum in genere tradidit, non admodum difficile videtur motum apogei Lunae definire: verum hic praeter expectationem euenit, vt motus apogei annuus vix 20° superans reperiatur, cum tamen ex obseruationibus constet, Apogeum Lunae interuallo unius anni vltra 40° promoueri.

Siue autem ista motus Apogei quantitas 20° legitime sit ex Theoria deriuata, siue minus; consideratio Apogei tutissimum praebet remedium quaestionem de sufficientia

Theoriae Newtonianae decidendi. Quamuis enim ex Theoria inaequalitas quaepiam in ipſo motu Lunae aliquot minutis ſecundis vel etiam primis maior minorue prodiret, quam experientia monſtraret, tamen tantilla differentia merito vel leui cuipiam errori in obſeruationibus, vel vitio in approximatione commiſſo tribueretur; quandoquidem aliunde certum eſt Theoriam Newtonianam non admodum a veritate recedere. At longe aliter eſt comparata ratio motus Apogei: quodſi enim vires Lunam ſollicitantes tantillum a Theoria Newtoniana diſcrepent, vt ex iis in ipſo motu Lunae vix perceptibile diſcrimen naſceretur, tamen inde in motu Apogei annuo differentia plurium graduum oriri poterit. Quae tanta differentia cum nulli errori vel obſeruationum vel ipſius calculi, ſiquidem omni cura inſtituatur, tribui queat, inueſtigatio motus apogei certiſſimum ſuppeditat criterium iudicandi, vtrum quaepiam Theoria veritati ſit conſentanea nec ne?

Quodſi ergo calculo rite adminiſtrato Theoria Newtoni reperiatur tantum Apogei Lunaris motum exhibere, quantus per obſeruationes deprehenditur, ſcilicet ultra 40° quotannis; fortius certe argumentum, quo veritas hujus Theoriae indubie demonſtretur, deſiderari nequit. Sin autem contra eueniat, vt progreſſio Apogei annua ex Theoria rite deriuata notabiliter a 40° deficiat, hinc certo erit concludendum Theoriam Newtonianam correctione quapiam indigere, neque vires, quibus Luna reuera ſollicitatur, exactiſſime huic Theoriae eſſe conformes.

Verum haec ipſa quaeſtio, vtrum Theoria Newtoniana ad verum apogei Lunae motum perducat nec ne? eſt profunfundiſſimae indaginis, atque ſummam in calculo circumſpectionem ac ſollertiam requirit. Quanquam enim ex principiis generalibus, vnde vulgo motus abſidum definiri ſolet, ſatis luculenter ſemiſſis tantum pro motu Apogei Lunae elicitur; tamen quoniam in calculo plures termini, qui in determinationem motus Lunae ingrediuntur, ob paruitatem ſunt reiecti, merito dubitatio ſuboritur, num hi ipſi termini, ſi eorum ratio eſſet habita, non iſtum defectum compenſare valuerint? Quin etiam non defuere Geometrae, qui conſenſum huius Theoriae cum vero Apogei motu demonſtrare ſunt conati: verum plerumque non difficile erat paralogiſmum in ipſorum ratiociniis deprehendere. Maximam autem hoc loco attentionem meretur iudicium profundiſſimi Geometrae Clairaut, qui cum primum validiſſimis argumentis ſtatuiſſet, Newtoni Theoriam non vltra dimidium veri apogei Lunaris motus ſuppeditare, ſubito in contrariam abiit ſententiam ſtatuens hanc Theoriam elegantiſſime cum veritate conſpirare; neque certe tantae perſpicaciae Vir a priſtina ſententia, quam omni ſtudio propugnauerat, receſſiſſe eſt putandus, niſi firmiſſimis argumentis eo eſſet adactus.

Cum autem omnes rationes, quae Ipſum ad hanc retractationem impulerint, nondum publice expoſuerit, liceat mihi quidem, qui ſemper contrariae ſententiae fui addictus, tantiſper arduam hanc quaeſtionem tanquam nondum deciſam ſpectare, donec per propriam inueſtigationem inuenero, quid de ea ſit ſtatuendum. Poſtquam enim iam a longo temporis interuallo plurimum ſtudii in indagatione motus Lunae conſumſiſſem, ac variis methodis inſiſtens

ſemper

ſemper concluſionem Theoriae Newtonianae minus fauentem eſſem adeptus; quam tamen pro rite demonſtrata venditare non eram auſus, propterea quod approximatione eſſem vſus, ac ſemper ſuſpicio quaedam ratione terminorum praetermiſſorum remaneret: nuper in aliam incidi viam hanc inueſtigationem ſuſcipiendi, quae mihi multo certior videtur, ita vt per eam nulla dubitatione interiecta ad veritatem penetrare confidam. Ne autem ſi forte Theoriam Newtonianam minus ſufficientem inuenero, calculum ſecundum aliam Theoriam de nouo inſtituere cogar, ſtatim meam inueſtigationem in latiori ſenſu exordiar, viresque quibus Luna ad terram ſollicitatur, non exacte ſed proxime tantum quadratis diſtantiarum reciproce proportionales aſſumam: deinceps ſcilicet innoteſcet, vtrum haec aberratio a regula Newtoniana locum habeat nec ne? Calculum autem ita adornabo, vt quicquid euenerit, non ſolum verum apogei motum aſſequar, ſed etiam omnes Lunae inaequalitates inde elicere valeam, quas deinceps Aſtronomorum more tabulis complecti licebit.

Primum ergo problema in latiſſimo ſignificatu concipiam, vt corporis a viribus quibuscunque ſollicitati motum ſim inueſtigaturus: deinde vires, quibus Luna actu vrgeri cenſenda eſt, in calculum introducam, et aequationes Lunae motum determinantes exhibebo. Has porro aequationes variis modis in alias formas transmutabo, donec eas eo perduxero, vbi ad finem propoſitum maxime accommodatae videbuntur: quo cum peruenero, tandem tam motum Apogei, quam cunctas Lunae inaequalitates motus ex calculo deriuare ſtudebo.

CAPUT

CAPUT I.

DE MOTU CORPORIS A VIRIBUS QUIBUSCUNQUE SOLLICITATI.

§. 1.

Quoniam corpus a viribus quibuscunque sollicitari ponimus, fieri potest, vt eius motus non in eodem plano absoluatur. Hinc ad ejus motum per calculum ita repraesentandum, vt ad quoduis tempus verus locus, in quo corpus versabitur, assignari queat, conueniet corporis motum ad planum quoddam fixum pro lubitu assumtum referri. Exhibeat igitur Tabula hoc planum, atque corpus iam versetur extra hoc planum in puncto L, vnde ad planum demittatur perpendiculum L M; eritque punctum M locus corporis ad planum relatus. Quod si ergo ad quoduis tempus propositum hunc corporis locum relatum M, simulque eius a plano distantiam L M indicare valeamus, verus corporis locus L ad hoc tempus innotescet. Fig. 1.

§. 2. Ad locum autem puncti M commodius determinandum, assumamus in plano rectam quandam fixam CQ pro axe, ita vt ducta ex M ad hanc rectam perpendiculari M P, locus puncti M more apud Geometras recepto per coordinatas orthogonales definiatur. Assumto ergo porro in axe puncto quodam fixo C, vnde abscissae C P computentur, erit P M applicata puncto M respondens, & ipsum punctum L determinabitur per tres coordinatas inter se normales CP, PM & ML.

M L. Cum igitur praesenti temporis momento corpus in L versari ponatur, vocentur istae tres coordinatae eo spectantes:

$$CP = p; \quad PM = q \quad \text{et} \quad ML = r$$

elapso autem temporis elemento, quod per dt indicemus, coordinatae ternae tum locum corporis indicantes erunt:

$$p + dp; \quad q + dq; \quad \text{et} \quad r + dr.$$

§. 3. Quaecunque nunc fuerint vires, quibus corpus sollicitatur, eae semper per cognitam virium resolutionem reduci poterunt ad ternas vires secundum directiones ternarum coordinatarum vrgentes. Ponamus ergo vim $= P$, qua corpus in L secundum directionem ipsi P C parallelam trahitur: eam porro vim $= Q$, qua corpus secundum directionem ipsi M P parallelam trahitur: eamque denique vim $= R$, qua corpus secundum directionem L M sollicitatur. Has scilicet vires ita directas concipio, vt si corpus earum actioni libere obediret, valores coordinatarum p, q, r inde diminuerentur. His positis, ex principiis Mechanicae constat, si elementum temporis dt pro constanti assumatur, motum corporis his tribus formulis differentio-differentialibus determinari

$$\text{I. } ddp = -\tfrac{1}{2} P dt^2; \quad \text{II. } ddq = -\tfrac{1}{2} Q\, dt^2; \quad \text{III. } ddr = -\tfrac{1}{2} R dt^2.$$

§. 4. Verum hae coordinatae ad vsum astronomicum, ad quem hic potissimum respicimus, non satis sunt accommodatae. Nam si spectatorem in C constitutum assumimus, locus L, vbi corpus cernetur, commodissime me per quantitatem rectae C M, et angulum Q C M vna cum angulo M C L repraesentatur: atque si tabula planum eclipticae referat, rectaque C Q ad principium arietis sit directa, angulus Q C M in Astronomia vocari solet sideris longitudo, angulus M C L vero eius latitudo, et recta C M eius distantia curtata. Vocemus ergo porro:

I. Distantiam curtatam seu rectam C M $= x$

II. Longitudinem seu angulum Q C M $= \phi$

III. Latitudinem seu angulum M C L $= \psi$

ac posito constanter sinu toto $= 1$, erunt valores coordinatarum ante adhibitarum:

$CP = p = x \cos \phi$; $PM = q = x \sin \phi$ & $ML = r = x \operatorname{tang} \psi$ atque distantia corporis vera a puncto C erit $CL = x \sec. \psi = \frac{x}{\cos \psi}$.

§. 5. Sumtis nunc differentialibus more consueto obtinebimus:

$$dp = dx \cos \phi - x d\phi \sin \phi;\ dq = dx \sin \phi + x d\phi \cos \phi$$

$$\text{et } dr = dx \operatorname{tang} \psi + \frac{x d\psi}{\cos \psi^2}$$

atque hinc denuo differentialibus sumendis reperietur,

$$ddp = ddx \cos \phi - 2dx d\phi \sin \phi - x dd\phi \sin \phi - x d\phi^2 \cos \phi$$

$$ddq = ddx \sin \phi + 2dx d\phi \cos \phi + x dd\phi \cos \phi - x d\phi^2 \sin \phi$$

$$ddr = ddx \operatorname{tang} \psi + \frac{2dx d\psi}{\cos \psi^2} + \frac{x dd\psi}{\cos \psi^2} + \frac{2x d\psi^2 \sin \psi}{\cos \psi^3}$$

Binae priores formulae rite combinatae suppeditabunt sequentes multo concinniores

$$ddp \cos\phi + ddq \sin\phi = ddx - x d\phi^2$$

$$ddq \cos\phi - ddp \sin\phi = 2 dx d\phi + x dd\phi$$

sicque habebitur:

$$ddx - x d\phi^2 = -\tfrac{1}{2} dt^2 \; (P \cos\phi + Q \sin\phi)$$

$$2dx d\phi + x dd\phi = -\tfrac{1}{2} dt^2 \; (Q \cos\phi - P \sin\phi)$$

Tertiam vero aequationem deinceps magis tractabilem efficiemus.

§. 6 Manifestum autem est formulam $P \cos\phi + Q \sin\phi$ praebere vim ex viribus P et Q compositam, qua corpus in L secundum directionem rectae MC vrgetur, formulam vero alteram $Q \cos\phi - P \sin\phi$ exprimere vim ex eadem resolutione secundum directionem MQ ad MC normalem directam. Cum igitur hae duae vires assumtis binis P et Q aequiualeant, ponamus esse

I. Vim corpus L secundum MC trahentem $= V$

II. Vim corpus L secundum MQ trahentem $= T$

manente tertia vi corpus ad planum normaliter secundum LM vrgente $= R$. Atque sequentes habebimus aequationes:

I. $2 dx d\phi + x dd\phi = -\tfrac{1}{2} T dt^2$

II. $ddx - x d\phi^2 = -\tfrac{1}{2} V dt^2$

III. $ddx \tang\psi + \frac{2dx d\psi}{\cos\psi^2} + \frac{x dd\psi}{\cos\psi^2} + \frac{2x d\psi^2 \sin\psi}{\cos\psi^3} = -\tfrac{1}{2} R dt^2$

§. 7. Quo autem effectum tertiae vis R commodius ad calculum reuocemus, more apud Astronomos recepto contemplemur planum, in quo corpus durante elemento

elemento temporis dt mouetur, et quod ſimul per punctum C tranſeat. Hoc igitur planum cum plano aſſumto interſectionem alicubi formabit, quae ſit recta C☊, ac linea nodorum appellari ſolet; ſicque erit ☊CL planum orbitae, in qua corpus L praeſenti inſtanti mouetur, et angulus, quo hoc planum ☊CL ad planum fixum QCM inclinatur, vocatur inclinatio orbitae ad eclipticam pro tempore praeſenti. Cum igitur ex his duabus rebus latitudo ſideris definiri ſoleat, ponamus.

Longitudinem nodi aſcendentis ſeu angulum QC☊ $= \pi$ ac inclinationem orbitae ☊CL ad eclipticam $= \varrho$ atque loco latitudinis ψ has duas quantitates π et ϱ definire oportebit.

§. 8. Tertia ergo aequatio in duas diſpertietur, ad quas inueniendas ex M et L ad lineam nodorum C☊ ducantur normales MN et LN, eritque angulus LNM menſura inclinationis orbitae ad eclipticam; ideoque LNM $= \varrho$. Tum vero ob angulum ☊CM $= \Phi - \pi$ et CM $= x$ erit:

$$CN = x \operatorname{cof}(\Phi - \pi) \text{ et } MN = x \operatorname{fin}(\Phi - \pi)$$

hinc elicietur ML $= x \operatorname{tang} \varrho \operatorname{fin}(\Phi - \pi)$, vnde prodit tang MCL $= \operatorname{tang} \psi = \operatorname{tang} \varrho \operatorname{fin}(\Phi - \pi)$, quae formula inſeruit latitudini ψ ex cognita inclinatione ϱ et loco nodi eiusue longitudine π inueniendae, ſi quidem iam cognita fuerit longitudo ſideris Φ. Quoniam autem ſidus elemento temporis dt in eodem plano manet, in differentiatione formulae $\operatorname{tang} \psi = \operatorname{tang} \varrho \operatorname{fin}(\Phi - \pi)$,

 quanti-

quantitates π et ϱ tanquam constantes spectari poterunt, eritque idcirco.

$$\frac{d\,\psi}{\cos\psi^2} = d\,\Phi \text{ tang } \varrho \cos(\Phi - \pi)$$

§. 9. Interim tamen nihil impedit, quominus in eadem differentiatione quantitates π et ϱ tanquam variabiles tractemus, quales reuera esse possunt successu temporis; vnde orietur haec aequatio:

$$\frac{d\,\psi}{\cos\psi^2} = \frac{d\,\varrho}{\cos\varrho^2} \sin(\Phi - \pi) + (d\Phi - d\pi) \text{ tang } \varrho \cos(\Phi - \pi)$$

hicque valor ipsius $\frac{d\,\psi}{\cos\psi^2}$ collatus cum praecedente praebebit hanc aequalitatem:

$$\frac{d\,\varrho}{\cos\varrho^2} \sin(\Phi - \pi) = d\pi \text{ tang } \varrho \cos(\Phi - \pi)$$

vnde obtinemus $\frac{d\,\varrho}{\sin\varrho\cos\varrho} = \frac{d\pi\cos(\Phi - \pi)}{\sin(\Phi - \pi)} = \frac{d\,\pi}{\text{tang}(\Phi - \pi)}$.

Cum iam sit $\frac{d\,\varrho}{\sin\varrho\cos\varrho} = \frac{d.\text{ tang }\varrho}{\text{tang }\varrho} = d.\,l\text{ tang }\varrho$, erit

$$d.\,l\text{ tang }\varrho = \frac{d\,\pi}{\text{tang }(\Phi - \pi)}$$

ex quo, si longitudo nodi iam fuerit reperta, sine labore inclinatio ad eclipticam ϱ investigari poterit.

§. 10. Differentiemus formulam primo inuentam

$$\frac{d\,\psi}{\cos\psi^2} = d\,\Phi \text{ tang } \varrho \cos(\Phi - \pi)$$

iterum, et cum sit $d.\text{ tang }\varrho = \frac{d\pi \text{ tang }\varrho \cos(\Phi - \pi)}{\sin(\Phi - \pi)}$

erit:

erit:

$$\frac{dd\psi}{\cos\psi^2} + \frac{2d\psi^2 \sin\psi}{\cos\psi^3} = dd\Phi \tang \varrho \cos(\Phi - \pi)$$

$$+ \frac{d\Phi d\pi \tang\varrho \cos(\Phi-\pi)^2}{\sin(\Phi-\pi)} - d\Phi(d\Phi - d\pi)\tang\varrho \sin(\Phi-\pi)$$

seu $\frac{dd\psi}{\cos\psi^2} + \frac{2d\psi^2 \sin\psi}{\cos\psi^3} = dd\Phi \tang\varrho \cos(\Phi-\pi)$

$$+ \frac{d\Phi d\pi \tang\varrho}{\sin(\Phi-\pi)} - d\Phi^2 \tang\varrho \sin(\Phi-\pi)$$

qui valores pro ψ in tertia aequatione superiori substituti suppeditabunt:

$$ddx \tang\varrho \sin(\Phi-\pi) + 2dxd\Phi \tang\varrho \cos(\Phi-\pi) + xdd\Phi \tang\varrho \cos(\Phi-\pi)$$

$$+ \frac{xd\Phi d\pi \tang\varrho}{\sin(\Phi-\pi)} - xd\Phi^2 \tang\varrho \sin(\Phi-\pi) = -\tfrac{1}{2} R dt^2$$

quae transmutatur in hanc:

$$(ddx - xd\Phi^2)\tang\varrho \sin(\Phi-\pi) + (2dxd\Phi + xdd\Phi)\tang\varrho \cos(\Phi-\pi)$$

$$+ \frac{xd\Phi d\pi \tang\varrho}{\sin(\Phi-\pi)} = -\tfrac{1}{2} R dt^2$$

§. 11. Commode, hic euenit vt in ista formula illae ipsae expressiones differentio-differentiales $ddx - xd\Phi^2$ et $2dxd\Phi + xdd\Phi$ occurrant, quae ex actione duarum reliquarum virium sunt enatae: vnde si formularum harum valores aequiualentes $-\tfrac{1}{2} V dt^2$ et $-\tfrac{1}{2} T dt^2$ substituamus, impetrabimus

$$-\tfrac{1}{2} V dt^2 \tang\varrho \sin(\Phi-\pi) - \tfrac{1}{2} T dt^2 \tang\varrho \cos(\Phi-\pi) + \frac{xd\Phi d\pi \tang\varrho}{\sin(\Phi-\pi)} = -\tfrac{1}{2} R dt^2$$

qua differentiale $d\pi$, quo promotio elementaris lineae nodorum indicatur, ita determinabitur, vt sit

$$d\pi = \tfrac{1}{2} dt^2 . \frac{\sin(\Phi-\pi)}{xd\Phi} \left(V \sin(\Phi-\pi) + T \cos(\Phi-\pi) - \frac{R}{\tang\varrho}\right)$$

Deinde

Deinde cum ſit $d.\,l\,\mathrm{tang}\,\varrho = \frac{d\pi}{\mathrm{tang}\,(\phi - \pi)}$, erit

$$d.l\,\mathrm{tang}\,\varrho = \tfrac{1}{2}dt^2.\,\frac{\mathrm{cof}(\phi-\pi)}{x\,d\phi}\left(V\,\mathrm{fin}(\phi-\pi)+T\,\mathrm{cof}(\phi-\pi)-\frac{R}{\mathrm{tang}\,\varrho}\right)$$

Duas ergo has aequationes loco ſuperioris tertiae, ex qua latitudo ψ inueniri debebat, in calculum introduci conueniet; inuentis enim π et ϱ erit $\mathrm{tang}\,\psi = \mathrm{tang}\,\varrho\ \mathrm{fin}\,(\phi - \pi)$.

§. 12. Hinc patet lineam nodorum nunquam eſſe mobilem, quin ſimul inclinatio ϱ variationi ſit obnoxia. Eadem enim vis $V\,\mathrm{fin}\,(\phi-\pi) + T\,\mathrm{cof}\,(\phi-\pi) - \frac{R}{\mathrm{tang}\,\varrho}$, quae lineae nodorum motum imprimit eius longitudinem π immutando, ſimul in inclinatione ϱ variationem generat. Nulli autem plane immutationi tam linea nodorum, quam inclinatio erunt ſubjectae, ſi vis illa euaneſcat, quod euenit ſi media directio omnium virium corpus L ſollicitantium in ipſum planum ☊CL, in quo corpus ſemel moueri coepit, perpetuo incidat; hicque eſt caſus, quo corpus continuo in eodem plano moueri pergit. Generatim ergo corporis a tribus viribus V, T, R ſollicitati motus quatuor ſequentibus aequationibus determinatur.

I. $2\,dx\,d\phi + x\,dd\phi = -\tfrac{1}{2}\,T\,dt^2$

II. $ddx - x\,d\phi^2 = -\tfrac{1}{2}\,V\,dt^2$

III. $d\pi = \tfrac{1}{2}dt^2.\,\frac{\mathrm{fin}\,(\phi-\pi)}{x\,d\phi}\left(V\,\mathrm{fin}(\phi-\pi)+T\,\mathrm{cof}(\phi-\pi)-\frac{R}{\mathrm{tang}\,\varrho}\right)$

IV. $d.\,l\,\mathrm{tang}\,\varrho = \frac{d\pi}{\mathrm{tang}\,(\phi - \pi)}$

quas ergo quouis caſu oblato reſolui oportet.

CAPUT

CAPUT II.

INUESTIGATIO VIRIUM LUNAM SOLLICITANTIUM.

§. 13.

Cum Lunae motus, qualis ex centro terrae ſpectaretur, definiri debeat, ſit C terrae centrum, ad quod etiam praecipua vis, qua Luna vrgetur, directa concipitur; atque tabula exhibeat planum eclipticae, in quo nunc quidem Sol exiſtat in S, Luna vero ſupra hoc planum verſetur in L latitudinem habens borealem, vnde ad planum eclipticae perpendiculum demittatur LM. Hinc ductis rectis CL, CM, CS, itemque CQ initium arietis verſus, vnde longitudines computari ſolent, fiant ſequentes denominationes. Fig. 2.

1. Longitudo Solis ſeu angulus ACS $= \theta$
2. Longitudo Lunae ſeu angulus ACM $= \phi$
3. Latitudo Lunae ſeu angulus MCL $= \psi$
4. Diſtantia Solis a Terra CS $= y$
5. Diſtantia Lunae a Terra curtata CM $= x$

§. 14. Sit iam AM proiectio orbitae lunaris in planum eclipticae; ac planum, in quo Luna nunc mouetur, per centrum terrae ductum, planum eclipticae interſecet ſecundum rectam C☊, quae lineam nodorum pro tempore praeſenti exhibebit: ac terminus ☊ quidem nodum aſcendentem referet, ſiquidem lunam ſecundum regionem AM promoueri ponamus. Quod ſi ergo porro vocemus

6. Longitudinem nodi asc: $AC☊ = \pi$

7. Incl. orbitae Lunae ad eclipticam $= \rho$

hinc latitudo Lunae geocentrica ita definietur, vt sit $\tang \psi = \tang. \sin(\phi - \pi)$. Vnde incrementum latitudinis $d\psi$ commode assignabitur, cum sit, vt supra vidimus $d. \tang \psi = \frac{d\psi}{\cos \psi^2} = d\phi \tang \rho \cos(\phi - \pi)$: ac praeterea ex motu nodi cognito variatio inclinationis ita definietur, vt sit $d. \tang \rho = \frac{d\pi \tang \rho}{\tang(\phi - \pi)}$, seu $d\rho = \frac{d\pi \sin \rho \cos \rho}{\tang(\phi - \pi)}$.

§. 15. Cum nunc primum Luna ad centrum terrae C secundum directionem LC attrahatur, sit haec vis $= M$. Deinde sit vis, qua Luna ad solem S vrgetur secundum $LS = N$; atque his duabus viribus Luna proprie vrgeri censenda est. Praeterea vero cum Terra ipsa, ad quam motum Lunae referimus, in motu versetur, ut eam tanquam quiescentem considerare queamus, non solum motum Terrae, sed etiam vires, quibus Terra vrgetur, toti mundo, secundum plagas oppositas imprimi concipiamus. Sit igitur vis qua Terra ad Solem vrgetur $= S$, & vis qua ad Lunam trahitur $= L$, his viribus contrario modo in lunam translatis, Luna sequentibus viribus sollicitata habebitur

1. Secundum directionem LC vi $= M + L$

2. Secundum directionem LS vi $= N$

3. Secundum directionem MR ipsi SC parallelam vi $= S$.

§. 16. Hunc

§. 16. Nunc primo has vires ad ternas directiones supra assumtas MC, MQ et LM reducamus; ac primo quidem vis $M + L$ dabit

pro directione MC vim $= (M + L) \cos \psi$

pro directione LM vim $= (M + L) \sin \psi$

Secunda vis N vero dabit

pro directione LM vim $= \frac{LM}{LS} . N$

pro directione MS vim $= \frac{MS}{LS} . N$

at haec vlterius resoluta ducta Mr ipsi CS parallela dabit:

pro directione MC vim $= \frac{MC}{LS} . N$

pro directione Mr vim $= \frac{CS}{LS} . N$

Haec postrema a vi tertia S subtracta relinquet vim $S - \frac{CS}{LS} . N$, qua Luna secundum directionem MR sollicitatur, quae ob angulum $CMR = SCM = \phi - \theta$ dabit

pro directione MC vim $= (S - \frac{CS}{LS} . N) \cos (\phi - \theta)$

pro directione Mq vim $= (S - \frac{CS}{LS} . N) \sin (\phi - \theta)$

vbi directio Mq contraria est directione MQ.

§. 17. His iam viribus cum ternis initio assumtis V, T et R comparandis, inueniemus pro his viribus sequentes valores:

 1. pro-

1. pro directione MC vim V =

$$(M+L)\cos\psi + \frac{MC}{LS}.N + (S - \frac{CS}{LS}.N)\cos(\varphi-\theta)$$

2. pro directione MQ vim $T = -(S - \frac{CS}{LS}.N)\sin(\varphi-\theta)$

3. pro directione LM vim $R = (M+L)\sin\psi + \frac{LM}{LS}.N.$

Cum nunc sit $CM = x$, $CS = y$, et angulus $SCM = \varphi - \theta$; erit $MS = \sqrt{(xx - 2xy\cos(\varphi-\theta) + yy)}$, et ob $LM = x\tang\psi$ erit $LS = \sqrt{(yy - 2xy\cos(\varphi-\theta) + xx\sec.\psi^2)}$, quae distantia Solis a Luna LS breuitatis gratia ponatur $= u$, vt sit $u = \sqrt{(yy - 2xy\cos(\varphi-\theta) + xx\sec.\psi^2)}$. His ergo valoribus introductis erunt vires nostrae:

1. $V = (M+L)\cos\psi + \frac{Nx}{u} + S\cos(\varphi-\theta) - \frac{Ny}{u}\cos(\varphi-\theta)$

2. $T = -S\sin(\varphi-\theta) + \frac{Ny}{u}\sin(\varphi-\theta)$

3. $R = -(M+L)\sin\psi + \frac{Nx\tang\psi}{u}$

§. 18. Quia nunc est $\tang\psi = \tang\varrho\sin(\varphi-\pi)$ et $\sin\psi = \tang\psi\cos\psi$ erit:

$$\frac{R}{\tang\varrho} = (M+L)\cos\psi\sin(\varphi-\pi) + \frac{Nx\sin(\varphi-\pi)}{u}$$

tum vero habebitur

$$V\sin(\varphi-\pi) + T\cos(\varphi-\pi) = (M+L)\cos\psi\sin(\varphi-\pi) + \frac{Nx\sin(\varphi-\pi)}{u}$$
$$+ S\cos(\varphi-\theta)\sin(\varphi-\pi) - \frac{Ny\cos(\varphi-\theta)\sin(\varphi-\pi)}{u}$$
$$- S\sin(\varphi-\theta)\cos(\varphi-\pi) + \frac{Ny\sin(\varphi-\theta)\cos(\varphi-\pi)}{u}$$

quae ob $\cos(\varphi-\theta)\sin(\varphi-\pi) - \sin(\varphi-\theta)\cos(\varphi-\pi) = \sin(\varphi-\pi)$, dat

V sin

$$V \sin(\varphi-\pi) + T\cos(\varphi-\pi) - \frac{R}{\tang \rho} = S\sin(\theta-\pi) - \frac{Ny}{u}\sin(\theta-\pi)$$

ex quo aequationes motum Lunae continentes erunt:

$$\text{I. } 2dxd\varphi + xdd\varphi = -\tfrac{1}{2}dt^2 \left(\frac{Ny}{u} - S\right)\sin(\varphi-\theta)$$

$$\text{II. } ddx - xd\varphi^2 = -\tfrac{1}{2}dt^2\left((M+L)\cos\psi + \frac{Nx}{u} - \left(\frac{Ny}{u} - S\right)\cos(\varphi-\theta)\right)$$

$$\text{III. } d\pi = -\tfrac{1}{2}dt^2 \cdot \frac{\sin(\varphi-\pi)\sin(\theta-\pi)}{x\,d\varphi}\left(\frac{Ny}{u} - S\right)$$

$$\text{IV. } d.\,l\tang\rho = \frac{d\pi}{\tang(\varphi-\pi)}$$

vbi $\theta - \pi$ exprimit angulum ☊ CS seu distantiam Solis a nodo ascendente.

§. 19. Jam secundum Theoriam Newtoni, si massam Terrae ponamus $=$ ♁ ac Lunae $=$ ☾, ob distantiam $CL = \frac{x}{\cos\psi}$, foret vis $M = \frac{♁\cos\psi^2}{xx}$ et vis $L = \frac{☾\cos\psi^2}{xx}$, sicque vis tota $M+L = (♁ + ☾) \cdot \frac{\cos\psi^2}{xx}$. Quo autem, si forte haec Theoria insufficiens deprehendatur, rem generalius complectamur, ponamus hanc vim:

$$M + L = (♁ + ☾)\cos\psi^2 \left(\frac{1}{xx} - \frac{1}{bb}\right)$$

vbi terminus $\frac{1}{bb}$ defectum huius vis a Theoria Newtoniana exhibeat; qui cum sit minimus, pro constanti haberi poterit saltem pro exigua variabilitate, quam distantia x subit. Vim autem Solis exacte Theoriae Newtonianae conformem assumere poterimus; quoniam etiamsi inde recederet, differentia non solum foret quam minima, sed quia pro Luna aeque discreparet ac pro Terra, in nostris formulis nullius plane esset momenti.

§. 20. Posita ergo Solis massa $= \odot$, erit vis, qua Terram ad se attrahit $S = \frac{\odot}{yy}$, vis autem qua Lunam ad se trahit $N = \frac{\odot}{uu}$. His ergo valoribus virium in calculum inductis, motus lunae ex quatuor sequentibus aequationibus determinari debet:

$$\text{I. } 2dx d\varphi + x dd\varphi = -\tfrac{1}{2} dt^2 \left(\frac{\odot y}{u^3} - \frac{\odot}{yy}\right) \sin(\varphi - \theta)$$

$$\text{II. } ddx - x d\varphi^2 = -\tfrac{1}{2} dt^2 (\text{♁} + \text{☾}) \cos\psi^2 \left(\frac{1}{xx} - \frac{1}{bb}\right)$$
$$- \tfrac{1}{2} dt^2 \left(\frac{\odot x}{u^3} - \frac{\odot y}{u^3}\cos(\varphi - \theta) + \frac{\odot}{yy}\cos(\varphi - \theta)\right)$$

$$\text{III. } d\pi = -\tfrac{1}{2} dt^2 . \frac{\sin(\varphi - \pi)\sin(\theta - \pi)}{x d\varphi} \left(\frac{\odot y}{u^3} - \frac{\odot}{yy}\right)$$

$$\text{IV. } d.\, l \tan \rho = \frac{-d\pi}{\tan(\varphi - \pi)}$$

Hic iam primum curandum est, vt elementum temporis, quod est quantitas heterogenea, ex calculo eliminemus; id quod commodissime fiet, si motum medium solis vtpote tempori proportionalem, loco temporis in calculum introducemus.

§. 21. Cum igitur etiam motus Solis in his aequationibus sit ratio habenda, eum prius inuestigemus: et quoniam pro terra quiescente Sol a sola vi $\frac{\odot}{yy}$ ad terram

ram sollicitari concipiendus est, si formulas pro luna inuentas ad solem accommodemus, obtinebimus:

$$2\,dy\,d\theta + y\,dd\theta = 0$$

$$ddy - y\,d\theta^2 = -\tfrac{1}{4}\,dt^2 . \frac{\odot}{yy}$$

si iam distantiam Solis a terra mediam ponamus $= b$ ejusque anomaliam mediam $= q$; casu quo excentricitas orbitae solaris esset nulla, foret semper $y = b$ & $d\theta = dq$: vnde altera aequatio dabit $-b\,dq^2 = -\tfrac{1}{4}\,dt^2 . \frac{\odot}{bb}$. Quare loco elementi temporis dt elementum anomaliae mediae solis ita in calculum introduci debet, vt vbique loco $\tfrac{1}{4}\,dt^2$ scribatur $\frac{b^3\,dq^2}{\odot}$, id quod tam in his formulis pro Sole, quam in superioribus pro Luna fieri poterit.

§. 22. Cum iam b denotet distantiam solis a terra mediam, sit eius vera distantia $y = b\,\omega$, et anomalia eius vera $= s$, erit $d\theta = ds$, quandoquidem a motu apogei solis animum abstrahimus. Hinc itaque erit

$$2\,d\omega\,ds + \omega\,dds = 0$$

$$dd\omega - \omega\,ds^2 = -\frac{dq^2}{\omega\omega},$$

quarum prior integrata dat $\omega\omega\,ds = C\,dq$ ob dq constans, ideoque $\omega\,ds^2 = \frac{CC\,dq^2}{\omega^3}$; qui valor in altera aequatione substitutus praebet,

$$dd\omega = \frac{CC\,dq^2}{\omega^3} - \frac{dq^2}{\omega\omega}.$$

quae

quae per $2\,d\omega$ multiplicata et integrata dat:

$$\frac{d\omega^2}{dq^2} = D - \frac{CC}{\omega\omega} + \frac{2}{\omega}$$

vnde fit $dq = \frac{\omega\, d\omega}{\sqrt{(D\omega\omega + 2\omega - CC)}}$

ac proinde $ds = \frac{C\, d\omega}{\omega \cdot \sqrt{(D\omega\omega + 2\omega - CC)}}$

§. 23. Quanquam autem hinc valores finiti haud difficulter deduci possent, tamen alia vtar methodo, quae in motu Lunae maiorem praestabit vtilitatem. Inuento autem $\omega\omega\, ds = C\, dq$, alteram aequationem ita transformo, vt elementi constantis dq ratio non amplius habeatur:

$$dq.\, d.\frac{d\omega}{dq} - \omega\, ds^2 = - \frac{dq^2}{\omega\omega}$$

Sit nunc $\omega = \frac{1}{u}$, vt habeat $ds = C\, uu\, dq$, et $d\omega = - \frac{du}{uu}$, et ob $dq = \frac{ds}{Cuu}$ erit $\frac{d\omega}{dq} = - \frac{C\,du}{ds}$; hinc sumto iam elemento ds constante, erit

$$\frac{-d's}{Cuu} \cdot \frac{C\,ddu}{ds} - \frac{ds^2}{u} = - \frac{ds^2}{CCuu} \text{ seu}$$

$$ddu + u\,ds^2 = \frac{ds^2}{CC}$$

vnde statim elicitur $u = \frac{1 - e \cos s}{CC}$, vbi e excentricitatem orbitae solaris indicabit.

§. 24. Hinc porro habebitur $\omega = \frac{CC}{1 - e\cos s}$, et $y = \frac{CCb}{1 - e\cos s}$, anomalia vera s ab apogeo computata; vnde distantia apogei

apogei a terra posito $s = 0$ erit $= \frac{CCb}{1-e}$, et distantia perigei posito $s = 180°$ prodit $= \frac{CCb}{1+e}$; sicque distantia media media fiet $= \frac{CCb}{1-ee}$, quae cum per hypothesin aequalis esse debeat ipsi b, statui oportet $CC = 1-ee$: hincque erit

$$y = \frac{b(1-ee)}{1-e\cos s} \quad \text{et} \quad \omega = \frac{1-ee}{1-e\cos s}$$

Porro autem aequatio $\omega\omega\, ds = C\, dq = dq\sqrt{(1-ee)}$ abibit in hanc:

$$dq = \frac{(1-ee)^{\frac{3}{2}}\, ds}{(1-e\cos s)^2} \quad \text{et} \quad q = \int \frac{(1-ee)^{\frac{3}{2}}\, ds}{(1-e\cos s)^2}$$

ex qua, vti satis constat, vel data anomalia vera s inueniri potest anomalia media q, vel vicissim. His itaque formulis motum Solis continentibus in determinatione motus Lunae vtamur.

§. 25. Primo ergo loco $\frac{1}{2}dt^2$ vbique scribamus $\frac{b^3\, dq^2}{☉}$ et $b\omega$ loco y, quo facto nostrae aequationes fient

$$\text{I. } 2\,dx\,d\Phi + x\,dd\Phi = -b^3 dq^2 \left(\frac{b\omega}{u^3} - \frac{1}{bb\omega\omega}\right) \sin(\varphi-\theta)$$

$$\text{II. } ddx - x\,d\Phi^2 = -\frac{(♁+☾)\, b^3 dq^2}{☉} \cos\psi^3 \left(\frac{1}{xx} - \frac{1}{bb}\right)$$

$$-b^3 dq^2 \left(\frac{x}{u^3} - \frac{b\omega\cos(\Phi-\theta)}{u^3} + \frac{\cos(\varphi-\theta)}{bb\omega\omega}\right)$$

$$\text{III. } d\pi = -b^3 dq^2 . \frac{\sin(\Phi-\pi)\sin(\theta-\pi)}{x\,d\Phi} \left(\frac{b\omega}{u^3} - \frac{1}{bb\omega\omega}\right)$$

Ponatur porro $u = bv$, adque in calculum quoque introducatur distantia media lunae a terra, quae sit $= a$, positoque $x = az$, prodibit:

$$\text{I. } 2dzd\varphi + zdd\varphi = -\frac{bdq^2}{a}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\sin(\varphi - \theta)$$

$$\text{II. } ddz - zd\varphi^2 = -\frac{(\text{♁} + \text{☾})\,b^3}{\odot\, a^3}\,dq^2 \cos\psi^3\left(\frac{1}{zz} - \frac{aa}{bb}\right)$$
$$- \frac{zdq^2}{v^3} + \frac{b\omega dq^2 \cos(\varphi - \theta)}{av^3} - \frac{bdq^2 \cos(\varphi - \theta)}{a\omega\omega}$$

$$\text{III. } d\pi = -\frac{bdq^2}{azd\varphi}\sin(\varphi - \pi)\sin(\theta - \pi)\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)$$

§. 26. Ponamus nunc ad abbreuiandum:

$\frac{(\text{♁} + \text{☾})\,b^3}{\odot\, a^3} = m$; $\frac{(\text{♁} + \text{☾})\,b^3}{\odot\, abb} = \mu$, seu $\mu = \frac{maa}{bb}$ quarum litterarum valores m et μ per obseruationes definiri debent; tum vero sit $\frac{a}{b} = \nu$, quae est quantitas valde parua a parallaxi solis pendens. Hisque valoribus introductis, aequationes nostrae sequentes induent formas:

$$\text{I. } 2dzd\varphi + zdd\varphi = -\frac{1}{\nu}dq^2\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)(\sin\varphi - \theta)$$

$$\text{II. } ddz - zd\varphi^2 = -\frac{mdq^2\cos\psi^3}{zz} + \mu dq^2\cos\psi^3$$
$$- \frac{zdq^2}{v^3} + \frac{1}{\nu}dq^2\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\cos(\varphi - \theta)$$

$$\text{III. } d\pi = -\frac{dq^2}{\nu zd\varphi}\sin(\varphi - \pi)\sin(\theta - \pi)\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)$$

$$\text{IV: } d.\,l\tan\rho = \frac{d\pi}{\tan(\varphi - \pi)}$$

§. 27.

§. 27. Cum iam posuerimus:

$$x = az;\quad y = b\omega;\quad v = \frac{u}{b} \quad \text{et} \quad \nu = \frac{a}{b}$$

erit $u = \sqrt{(bb\omega\omega - 2ab\omega z \cos(\phi - \theta) + aazz \sec. \psi^2)}$
atque $v = \sqrt{(\omega\omega - 2\nu\omega z \cos(\phi - \theta) + \nu\nu zz \sec. \psi^2)}$
vbi notandum est quantitates ω, q et s ex motu solis ita inter se pendere, vt sit

$$\omega = \frac{1 - ee}{1 - e\cos s} \quad \text{et} \quad dq = \frac{(1 - ee)^{\frac{3}{2}} ds}{(1 - e\cos s)^2} = \frac{\omega\omega ds}{\sqrt{(1 - ee)}}$$

ita vt huic ad datum quoduis tempus tam valor ipsius ω quam anomaliae verae s definiri possit. Modum autem has formulas ad calculum reuocandi hic non trado, quia eum alias fusius iam exposui: hoc solum hic notari conveniet, excentricitatis orbitae solaris valorem ex obseruationibus colligi $e = 0,01680$.

§. 28. Nunc antequam vlterius progredi queamus, valorem irrationalem ipsius v tolli conueniet, quod facile per seriem praestabitur maxime conuergentem, ob ν fractionem valde paruam; sumta enim parallaxi solis $= 12''$, quia parallaxis lunae media est $= 3380''$, erit $\frac{a}{b} = \nu = \frac{12}{3380} = \frac{1}{282}$. Hinc sufficit seriei illius conuergentis, quam reperiemus, aliquot tantum terminos ab initio assumsisse; quia reliqui ob paruitatem continuo magis crescentem tuto omitti poterunt. Cum autem angulus $\phi - \theta$, qui distantiam solis a luna secundum longitudinem denotat, in hac resolutione frequentissime occurret, breuitatis gratia ponamus $\phi - \theta = \eta$
ita vt pro v sequentem habeamus valorem irrationalem

$$v = \sqrt{(\omega\omega - 2\nu\omega z \cos\eta + \nu\nu zz \sec. \psi^2)}.$$

§. 29. Quoniam ergo in nostris formulis occurrit $\frac{1}{v^3}$ ob $\frac{1}{v^3} = (\omega\omega - 2v\omega z \cos\eta + vvzz \sec\psi^2)^{-\frac{3}{2}}$, nanciscemur

$$\frac{1}{v^3} = \frac{1}{\omega^3} + \frac{3vz\cos\eta}{\omega^4} - \frac{3vvzz\sec.\psi^2}{2\omega^5} + \frac{15vvzz}{2\omega^5}\cos\eta^2$$

vbi terminos altiores ipsius v potestates inuoluentes sine haesitatione reiicere possumus; in ipsis aequationibus autem tantum in prima ipsius v potestate subsistemus. Habebimus ergo:

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right) = \frac{3z\cos\eta}{\omega^3} + \frac{3vzz}{2\omega^4}(5\cos\eta^2 - \sec.\psi^2) \text{ seu}$$

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right) = \frac{3z\cos\eta}{\omega^3} + \frac{3vzz}{4\omega^4}(5 + 5\cos 2\eta - 2\sec.\psi^2)$$

hincque porro:

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\sin(\varphi - \theta) = \frac{3z\sin 2\eta}{2\omega^3} + \frac{3vzz}{8\omega^4}(5\sin\eta + 5\sin 3\eta - 4\sin\eta\sec.\psi^2)$$

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\cos(\varphi - \theta) = \frac{3z}{2\omega^3}(1 + \cos 2\eta) + \frac{3vzz}{8\omega^4}(15\cos\eta + 5\cos 3\eta - 4\cos\eta\sec.\psi^2)$$

$$\text{atque } \frac{z}{v^3} = \frac{z}{\omega^3} + \frac{3vzz}{\omega^4}\cos\eta$$

§. 30. Substituamus hos valores in nostris aequationibus atque obtinebimus:

$$\text{I. } 2dz\,d\varphi + z\,dd\varphi = -dq^2\left(\frac{3z\sin 2\eta}{2\omega^3} + \frac{3vzz}{8\omega^4}(5\sin\eta + 5\sin 3\eta - 4\sin\eta\sec.\psi^2)\right)$$

$$\text{II. } ddz - z\,d\varphi^2 = -\frac{m\,dq^2\cos\psi^3}{zz} + \mu\,dq^2\cos\psi^3 + \frac{z\,dq^2}{2\omega^3} + \frac{3z\,dq^2}{2\omega^3}\cos 2\eta$$
$$+ \frac{3vzz\,dq^2}{8\omega^4}(7\cos\eta + 5\cos 3\eta - 4\cos\eta.\sec.\psi^2)$$

III. dx

III. $d\pi = -\frac{dq^2}{2d\Phi}\sin(\varphi-\pi)\sin(\theta-\pi)\left(\frac{3z\cos\eta}{\omega^3} + \frac{3vzz}{4\omega^4}(5 + 5\cos 2\eta - 2\,\mathrm{sec.}\,\psi^2)\right)$

IV. $d.\,l\,\mathrm{tang}\,\varrho = \frac{d\pi}{\mathrm{tang}\,(\Phi-\pi)}$.

Hic iam obſervare licet, cum angulus ψ nunquam fere 5° ſuperet, eiusque ſecans nonniſi in terminis iam per v multiplicatis, ac propterea reſpectu reliquorum valde parvis occurrat, ſine vllius erroris ſenſibilis metu in his terminis poni poſſe ſec. $\psi = 1$.

§. 31. Deinde vt etiam ex maioribus terminis coſ ψ eliminemus; conſideremus formulam $\mathrm{tang}\,\psi = \mathrm{tang}\,\varrho\,\sin(\Phi-\pi)$, eritque $\mathrm{sec.}\,\psi = \frac{1}{\cos\psi} = \sqrt{(1+\mathrm{tang}\,\varrho^2\sin(\Phi-\pi)^2)}$

Hinc ergo habebimus:

$$\cos\psi^3 = (1 + \mathrm{tang}\,\varrho^2 . \sin(\Phi-\pi)^2)^{-\frac{3}{2}}$$

et cum tang ϱ^2 nunquam fere fractionem $\frac{1}{114}$ ſuperet erit ſatis exacte:

$$\cos\psi^3 = 1 - \tfrac{3}{2}\,\mathrm{tang}\,\varrho^2 . \sin(\Phi-\pi)^2 \text{ — vel etiam}$$
$$\cos\psi^3 = 1 - \tfrac{3}{4}\,\mathrm{tang}\,\varrho^2 + \tfrac{3}{4}\,\mathrm{tang}\,\varrho^2\cos 2(\Phi-\pi)$$

qui valor pro coſ ψ^3 in termino maiore $\frac{m\,dq^2\cos\psi^3}{zz}$ ſubſtitui poteſt: in altero autem termino $\mu\,dq^2\cos\psi^3$ quia per ſe eſt valde paruus, atque adeo ſecundum Theoriam Newtoni euaneſceret, nihil impedit, quo minus loco coſ ψ^3 ſcribamus vnitatem.

§. 32. Hoc ergo modo ſi aequationes noſtras a conſideratione latitudinis Lunae ψ liberemus ad ſequentes perueniemus aequationes:

$$\text{I. } 2dzd\Phi + zdd\Phi = -dq^2\left(\frac{3x\sin 2\eta}{2\omega^3} + \frac{3yzz}{8\omega^4}(\sin\eta + 5\sin 3\eta)\right)$$

$$\text{II. } ddz - zd\Phi^2 = -\frac{mdq^2}{zz}\left(1 - \tfrac{3}{2}\operatorname{tang}\rho^2 + \tfrac{3}{2}\operatorname{tang}\rho^2\cos 2(\Phi-\pi)\right) + \mu dq^2$$
$$+ \frac{zdq^2}{2\omega^3} + \frac{3zdq^2}{2\omega^3}\cos 2\eta + \frac{3yzzdq^2}{8\omega^4}(3\cos\eta + 5\cos 3\eta)$$

$$\text{III. } d\pi = -\frac{dq^2}{zd\Phi}\sin(\Phi-\pi)\sin(\theta-\pi)\left(\frac{3x\cos\eta}{\omega^3} + \frac{3yzz}{4\omega^4}(3+5\cos 2\eta)\right)$$

$$\text{IV. } d.\,l\operatorname{tang}\rho = \frac{d\pi}{\operatorname{tang}(\Phi-\pi)}.$$

Nunc igitur in hoc erit incumbendum, vt ex his quatuor aequationibus omnia motus phaenomena, quae in Luna ſecundum Theoriam adeſſe debent, ſollicite eruantur, atque tum cum obſeruationibus conferantur.

CAPUT

CAPUT III.

INTRODUCTIO ANOMALIAE VERAE LUNAE IN PRAECEDENTES AEQUATIONES.

§. 33.

Quoniam noſtra quaeſtio circa Lunam verſatur, loco anomaliae mediae ſolis, quam pro tempore in calculum introduximus, magis conueniet motu Lunae medio vti, qui itidem tempori eſt proportionalis. Verum ex ſequentibus patebit calculum commodiorem reddi, ſi loco motus medii adhibeamus anomaliam Lunae mediam, cuius incrementa itidem tempori ſunt proportionalia. Sit itaque ad datum tempus anomalia media Lunae $= p$; et cum eius incrementum dp ad incrementum anomaliae mediae ſolis eodem tempuſculo acceptum datum ac per obſeruationes cognitam teneat rationem, ponamus $dp = ndq$. Tabulae autem Aſtronomicae pro interuallo 365 dierum praebent:
Motum anomaliae mediae Solis $11^s, 29^\circ, 44', 39'' = 1295079''$
Motum anomaliae mediae Lunae

$13^{Rev.}\ 2^s, 28^\circ, 43'\ 13'' = 17167393''$

vnde fit $n = \frac{17167393}{1295079} = 13,25586$

§. 34. Poſito ergo $\frac{dp}{n}$ loco dq, aequationes noſtrae erunt

$$\text{I. } 2dzd\Phi + zdd\Phi = -\frac{dp^2}{nn}\left(\frac{3x\sin 2\eta}{2\omega^3} + \frac{3vzz}{8\omega^4}(\sin\eta + 5\sin 3\eta)\right)$$

II. ddz

$$\text{II. } ddz - zd\Phi^2 = -\frac{mdp^2}{nnzz}(1 - \tfrac{3}{4}\operatorname{tang}\varrho^2 + \tfrac{3}{4}\operatorname{tang}\varrho^2 \cos 2(\Phi - \pi)) + \frac{\mu dp^2}{nn}$$

$$+ \frac{zdp^2}{2n^2\omega^3} + \frac{3zdp^2}{2n^2\omega^3}\cos 2\eta + \frac{3vzzdp^2}{8nn\omega^4}(3\cos\eta + 5\cos 3\eta)$$

$$\text{III. } d\pi = \frac{-dp^2}{nnzd\Phi}\sin(\Phi - \pi)\sin(\theta - \pi)\left(\frac{3z\cos\eta}{\omega^3} + \frac{3vzz}{4\omega^4}(3 + 5\cos 2\eta)\right)$$

$$\text{IV. } d.\, l\operatorname{tang}\varrho = \frac{d\pi}{\operatorname{tang}(\Phi - \pi)}$$

atque hic elementum dp aſſumtum eſt conſtans: ſimul autem patet terminos, qui per nn ſunt diuiſi, prae ceteris ſatis eſſe paruos, cum ſit $nn = 175, 71795$. Quae circumſtantia ſequentes approximationes non mediocriter adiuuabit.

§. 35. Nunc antequam vlterius progrediamur, aequationem primam per z multiplicemus, atque integratione in priori parte inſtituta obtinebimus

$$zzd\Phi = Cdp - \frac{dp}{nn}\int dp\left(\frac{3z^2\sin 2\eta}{2\,\omega^3} + \frac{3vz^3}{8\omega^4}(\sin\eta + 5\sin 3\eta)\right)$$

ponamus breuitatis gratia hoc membrum integrale

$$\int dp\left(\frac{3zz}{2\omega^4}\sin 2\eta + \frac{3vz^3}{8\omega^4}(\sin\eta + 5\sin 3\eta)\right) = S$$

quod integrale, ne introductio conſtantis incertitudinem pariat, ita capi aſſumo, vt nullum terminum mere conſtantem contineat, quippe qui iam in C eſſet comprehenſus. Hoc ergo circa determinationem integrationis probe obſeruato, erit $zzd\Phi = dp\left(C - \frac{S}{nn}\right)$: vbi terminus S aequabilem arearum deſcriptionem, quam Regula Kepleri in planetis primariis infert, perturbat; eſt enim

enim $\frac{1}{2} zz d\Phi$ elementum areae descriptae, quod si ipsi $C\,dp$ esset aequale, tempori exacte esset proportionale.

§. 36. Cum igitur sit $d\Phi = \frac{dp}{zz}\left(C - \frac{S}{nn}\right)$, erit $z d\Phi^2 = \frac{dp^2}{z^3}\left(CC - \frac{2}{nn} CS + \frac{1}{n^4} SS\right)$, quo valore substituto reliquae nostrae aequationes sequentes induent formas:

$$\text{II. } ddz = \frac{dp^2}{z^3}(CC - \frac{2}{nn} CS + \frac{1}{n^4} SS)$$
$$- \frac{m\,dp^2}{nnzz}(1 - \tfrac{1}{2}\operatorname{tang}\rho^2 + \tfrac{3}{2}\operatorname{tang}\rho^2 \cos 2(\Phi - \pi)) + \frac{\mu dp^2}{nn}$$
$$+ \frac{z\,dp^2}{2nn\omega^3} + \frac{3z\,dp^2}{2nn\omega^3}\cos 2\eta + \frac{3\nu zz\,dp^2}{8nn\omega^4}(3\cos\eta + 5\cos 3\eta)$$

$$\text{III. } d\pi = -\frac{z\,dp}{Cnn - S}\sin(\Phi - \pi)\sin(\theta - \pi)\left(\frac{3z\cos\eta}{\omega^3} + \frac{3\nu zz}{4\omega^4}(3 + 5\cos 2\eta)\right)$$

et quarta manet $d.\,l\operatorname{tang}\rho = \frac{d\pi}{\operatorname{tang}(\Phi - \pi)}$ vt ante.

Eo igitur pertigimus, vt inuestigari oporteat quantitates z, π et ρ, quibus inuentis obtinebitur Φ ex formula primum eruta. Cum autem sit $d\eta = d\Phi - d\theta$, ob $d\theta = ds = \frac{dq\sqrt{(1-ee)}}{\omega\omega} = \frac{dp\sqrt{(1-ee)}}{n\omega\omega}$, erit $d\eta = \frac{dp}{zz}\left(C - \frac{S}{nn}\right) - \frac{dp\sqrt{(1-ee)}}{n\omega\omega}$. Tum vero est vti vidimus $\omega = \frac{1-ee}{1-e\cos s}$, vnde et huius differentiale ad dp reduci poterit.

§. 37. Si hunc calculum prosequi vellemus, tota inuestigatio tandem eo rediret, vt definiretur quantum

 longi-

longitudo Lunae vera ab eius longitudine media, quae ex anomalia media p haberetur, discreparet: hoc autem discrimen nonnunquam vltra 8 gradus exsurgere posset, ideoque correctiones admodum notabiles requireret. Vt igitur nobis quam minimae correctiones inuestigandae relinquantur, expediet differentiam inter locum Lunae verum, et locum corporis quod secundum regulas Kepleri in ellipsi circa Terram reuolueretur, ita tamen mobili, vt eius motus absidum cum motu apogei Lunae per obseruationes cognito conueniret. Seu quod eodem redit, quaeramus primo ex anomalia Lunae media p secundum regulas Kepleri anomaliam eius veram quae sit $= r$, vnde si longitudo apogei fuerit $= \varpi$, quantita, $\varpi + r$ nunquam multum vltra gradum a longitudine Lunae vera differet: vnde discrimen multo facilius inueniri poterit, si quidem debita orbitae lunaris excentricitas in calculum inducatur. Hinc loco anomaliae Lunae mediae p eius anomaliam veram, quae scilicet mediae pro excentricitate rite assumta conueniat, in aequationes nostras inferamus.

§. 38. Tabulae quidem astronomicae excentricitatem orbitae lunaris plerumque variabilem statuunt; sed cum hic non de vera huius orbitae excentricitate quaestio sit, quam de excentricitate illius orbitae ellipticae mobilis, in qua corpus motum proxime motum Lunae referat; huius excentricitas media erit statuenda inter maximam ac minimam, quae vulgo orbitae lunari tribuuntur: vnde ista excentricitas media colligitur $= 0,05445$. Ne autem huic conclusioni nimium fidamus genera-

generatim hanc excentricitatem ponamus $= k$; atque anomalia vera per mediam ita determinabitur, vt sit $dp = \frac{(1-kk)^{\frac{3}{2}} dr}{(1-k\cos r)^2}$, vel sit brevitatis gratia $\frac{1-kk}{1-k\cos r} = t$, vt sit $dp = \frac{tt\,dr}{\sqrt{(1-kk)}}$. Porro autem reliqua differentialia ita ad elementum dr reuocabuntur, vt sit:

$$ds = \frac{ttdr\sqrt{(1-ee)}}{nn\omega\sqrt{(1-kk)}} = d\theta, \text{ et } d\eta = \frac{tt\,dr}{zz\sqrt{(1-kk)}}(C - \frac{S}{nn}) - \frac{tt\,dr\sqrt{(1-ee)}}{nn\omega\sqrt{(1-kk)}}.$$

§. 39. Si motus Lunae cum motu huius corporis, quod imaginamur, perfecte conueniret, tum vbique foret $z = \frac{1-kk}{1-k\cos r}$ seu $z = t$: quoniam autem hi duo motus inter se non conueniunt, non erit $z = t$. Ponamus ergo esse:

$$z = tu = \frac{(1-kk)u}{1-k\cos r} \text{ seu } x = \frac{(1-kk)au}{1-k\cos r}$$

vbi primum obseruo, quantitatem u valde parum ab vnitate recedere. Erit autem quantitas variabilis, quae alium terminum constantem praeter vnitatem non inuoluet: nam si alium terminum constantem contineret, is in a posset comprehendi, idque indicio esset distantiam mediam a non recte esse assumtam. Habebit ergo u huiusmodi formam $1 + Z$, vbi Z ex terminis nonnisi variabilibus constabit. Praeterea autem animaduerto, hanc quantitatem Z nullum terminum huius formae $\alpha \cos r$ complecti debere; quoniam hoc indicio esset excentricitatem k non recte esse assumtam, sed eam vel maiorem vel minorem accipi oportuisse.

 §. 40. His

§. 40. His igitur notatis, quod quantitas u primo terminum constantem $= 1$ contineat, tum vero nullum terminum formae $\alpha \cos r$ inuoluat, statuamus $z = tu$ seu $z = \frac{(1-kk)u}{1-k\cos r}$ posito breuitatis gratia $t = \frac{1-kk}{1-k\cos r}$. Atque cum supra elementum dp constans posuissemus, hac conditione exuenda erit $ddz = dp\, d.\frac{dz}{dp}$, et $\frac{ddz}{dp^2} = \frac{1}{dp}\, d.\frac{dz}{dp}$ Diuisa ergo secunda aequatione per dp^2, erit:

$$\text{II.}\quad \frac{1}{dp}\, d.\frac{dz}{dp} = \frac{CC}{t^3u^3} - \frac{2CS}{nnt^3u^3} + \frac{SS}{n^4t^3u^3}$$

$$-\frac{m}{nnttuu}(1-\tfrac{3}{4}\tan\rho^2 + \tfrac{3}{4}\tan\rho^2\cos 2(\phi-\pi)) + \frac{\mu}{nn} + \frac{tu}{2nn\omega^3}$$

$$+\frac{3tu\cos 2\eta}{2nn\omega^3} + \frac{3\nu ttuu}{8nn\omega^4}(3\cos\eta + 5\cos 3\eta)$$

$$\text{III.}\ d\pi = \frac{-tudp}{Cnn-S}\sin(\phi-\pi)\sin(\theta-\pi)\left(\frac{3tu\cos\eta}{\omega^3} + \frac{3\nu ttuu}{4\omega^4}(3 \mp 5\cos 2\eta)\right)$$

vbi nunc nullum differentiale assumtum est constans, sed iam pro lubitu quoduis differentiale constans assumi poterit.

§. 41. Posito autem $z = tu$ et $dp = \frac{ttdr}{\sqrt{(1-kk)}}$ existente $t = \frac{1-kk}{1-k\cos r}$ erit primo:

$$S = \int \frac{ttdr}{\sqrt{(1-kk)}}\left(\frac{3ttuu}{2\omega^3}\sin 2\eta + \frac{3\nu t^3u^3}{8\omega^4}(\sin\eta + 5\sin 3\eta)\right) \text{ seu}$$

$$S = \int \frac{dr}{\sqrt{(1-kk)}}\left(\frac{3t^4uu}{2\omega^3}\sin 2\eta + \frac{3\nu t^5u^3}{8\omega^4}(\sin\eta + 5\sin 3\eta)\right)$$

Hinc

Hinc fiet $d\Phi = \frac{dr}{uu\sqrt{(1-kk)}}\left(C - \frac{S}{nn}\right)$ atque

$$d\eta = \frac{dr}{uu\sqrt{(1-kk)}}\left(C - \frac{S}{nn}\right) - \frac{ttdr\sqrt{(1-ee)}}{n\omega\omega\sqrt{(1-kk)}}$$

Porro autem ob $dz = tdu + udt$, erit $\frac{dz}{dp} = \frac{tdu+udt}{ttdr}\sqrt{(1-kk)}$; at est $dt = -\frac{(1-kk)kdr \sin r}{(1-k\cos r)^2} = -\frac{kttdr \sin r}{1-kk}$; sicque fiet $\frac{dz}{dp} = \frac{du\sqrt{(1-kk)}}{tdr} - \frac{ku \sin r}{\sqrt{(1-kk)}}$; ac posito elemento dr constante erit $d.\frac{dz}{dp} = \frac{ddu\sqrt{(1-kk)}}{tdr} - \frac{dudt\sqrt{(1-kk)}}{ttdr} - \frac{kdu\sin r}{\sqrt{(1-kk)}} - \frac{kudr\cos r}{\sqrt{(1-kk)}}$ hincque ob $\frac{dt}{tt} = -\frac{kdr\sin r}{1-kk}$ habebitur:

$$d.\frac{dz}{dp} = \frac{ddu\sqrt{(1-kk)}}{tdr} - \frac{kudr\cos r}{\sqrt{(1-kk)}}.$$

§. 42. Hinc iam porro obtinemus pro secunda aequatione

$$\frac{1}{dp}d.\frac{dz}{dp} = \frac{(1-kk)ddu}{t^3dr^2} - \frac{ku\cos r}{tt}$$

qui valor substitutus in aequatione per $\frac{t^3}{1-kk}$ multiplicata orietur haec aequatio:

$$\text{II.}\ \frac{ddu}{dr^2} - \frac{ktu\cos r}{1-kk} = \frac{CC}{(1-kk)u^3} - \frac{2CS}{(1-kk)nnu^3} + \frac{SS}{n^4(1-kk)u^3}$$
$$- \frac{mt}{nn(1-kk)uu}(1 - \tfrac{3}{2}\text{tang}\rho^2 + \tfrac{3}{2}\text{tang}\rho^2\cos 2(\Phi-\pi)) + \frac{\mu t^3}{nn(1-kk)} + \frac{t^4 u}{2nn(1-kk)\omega^3}$$
$$+ \frac{3t^4u\cos 2\eta}{2nn(1-kk)\omega^3} + \frac{3\nu t^5 uu}{8nn\omega^4(1-kk)}(3\cos\eta + 5\cos 3\eta)$$

$$\text{III.}\ d\pi = -\frac{uudr\sin(\Phi-\pi)\sin(\theta-\pi)}{(Cnn-S)\sqrt{(1-kk)}}\left(\frac{3t^4\cos\eta}{\omega^3} + \frac{3\nu t^5u}{4\omega^4}(3+5\cos 3\eta)\right)$$

 Quartam

Quartam aequationem $d.\, l\,\mathrm{tang}\,\varrho = \frac{d\,\pi}{\mathrm{tang}\,(\varphi - \pi)}$, cum nullam mutationem subeat, superfluum foret continuo repetere.

§. 43. Conueniet autem quantitates constantes C et m, quarum valores nondum nouimus, saltem vero proxime indagare, quo facilius deinceps ipsam aequationum resolutionem dirigere queamus. Perspicuum autem est, si omnes quantitates a situ solis pendentes ex calculo deleantur, tum vtique fieri debere $u = 1$. Cum igitur primum S ab angulo η pendeat, terminos tam S quam η inuoluentes omittamus, ac pro ω quidem scribamus 1; quia tantum determinationem ad verum accedentem requirimus, quem in finem quoque inclinationem orbitae negligamus. Hinc aequatio secunda dabit:

$$-\frac{kt\cos r}{1-kk} = \frac{CC}{1-kk} - \frac{mt}{nn(1-kk)} + \frac{\mu t^3}{n^2(1-kk)} + \frac{t^4}{2nn(1-kk)} \text{ siue}$$

$$CC = \frac{mt}{nn} - \frac{\mu t^3}{nn} - kt\cos r - \frac{t^4}{2nn}$$

Cum autem sit $t = \frac{1-kk}{1-k\cos r} = 1 + k\cos r$ proxime, ob k valde paruum habebitur.

$$CC = \frac{m}{nn} - \frac{\mu}{nn} - \frac{1}{2nn}$$
$$+ \frac{m}{nn}k\cos r - \frac{3\mu k}{nn}\cos r - k\cos r - \frac{2k}{nn}\cos r$$

vnde perspicuum esse oportere.

$$\frac{m}{nn} = 1 + \frac{2+3\mu}{nn} \text{ et } CC = 1 + \frac{3+4\mu}{2nn}$$

§. 44. His

§. 44. His igitur constantium $\frac{m}{nn}$ et CC valoribus proximis inuentis ponamus esse reuera:

$$\frac{m}{nn} = 1 + \frac{2+3\mu+\gamma}{nn} \text{ et } CC = 1 + \frac{3+4\mu+\delta}{2nn} = \lambda\lambda$$

scribamus enim λ pro C, quia litteris maiusculis A, B, C, D etc. deinceps in operationibus sequentibus vtemur: sicque fiet

$$S = \frac{1}{\sqrt{(1-kk)}} \int dr \left(\frac{3t^4uu}{2\omega^3}\sin 2\eta + \frac{3\nu t^5u^3}{8\omega^4}(\sin\eta + 5\sin 3\eta)\right)$$

$$d\Phi = \frac{dr}{uu\sqrt{(1-kk)}}\left(\lambda - \frac{S}{nn}\right)$$

$$d\eta = \frac{dr}{uu\sqrt{(1-kk)}}\left(\lambda - \frac{S}{nn}\right) - \frac{ttdr\sqrt{(1-ee)}}{n\omega\omega\sqrt{(1-kk)}}$$

$$\text{II. } \frac{(1-kk)\,ddu}{dr^2} = ktu\cos r + \frac{\lambda\lambda}{u^3} - \frac{2\lambda S}{nnu^3} + \frac{SS}{n^4u^3} + \frac{\mu t^3}{nn} + \frac{t^4u}{2nn\omega^3}$$

$$- \frac{mt}{nnuu}\left(1 - \tfrac{3}{2}\operatorname{tang}\rho^2 + \tfrac{3}{4}\operatorname{tang}\rho^2\cos 2(\Phi-\pi)\right)$$

$$+ \frac{3t^4u\cos 2\eta}{2nn\omega^3} + \frac{3\nu t^5uu}{8nn\omega^4}(3\cos\eta + 5\cos 3\eta)$$

$$\text{III. } d\pi = -\frac{uudr\sin(\Phi-\pi)\sin(\theta-\pi)}{(\lambda nn - S)\sqrt{(1-kk)}}\left(\frac{3t^4}{\omega^3}\cos\eta + \frac{3\nu t^5u}{4\omega^4}(3+5\cos 2\eta)\right)$$

§. 45. Ponatur $\lambda = \kappa\sqrt{(1-kk)}$, vt sit $\kappa\kappa = 1 + \frac{3+4\mu+\delta}{2nn}$ defectum enim in termino indefinito δ complecti licet, existente $m = nn + 2 + 3\mu + \gamma$; tum vero ponatur $S = (1-kk)^{\frac{1}{2}}\int R dr$, vt sit $R = \frac{dS}{dr\sqrt{(1-kk)}}$; ac si pro t et ω valores restituamus, qui erant,

$t = \frac{1-kk}{1-k\cos r}$ et $\omega = \frac{1-ee}{1-e\cos s}$ habebimus:

R =

$$R = \frac{3\,(1-kk)^3\,(1-e\cos s)^3}{2\,(1-ee)^3\,(1-k\cos r)^4}\,uu\,\sin 2\eta$$

$$+\frac{3\nu(1-kk)^4\,(1-e\cos s)^4}{8(1-ee)^4\,(1-k\cos r)^5}\,u^3\,(\sin\eta + 5\sin 3\eta)$$

$$d\Phi = \frac{dr}{uu}\left(u - \frac{1}{nn}\int R\,dr\right);\; d\theta = ds = \frac{(1-kk)^{\frac{3}{2}}(1-e\cos s)^2}{n(1-ee)^{\frac{3}{2}}(1-k\cos r)^2}\,dr$$

$$\frac{d\eta}{dr} = \frac{u}{uu} - \frac{\int R\,dr}{nnuu} - \frac{(1-kk)^{\frac{3}{2}}(1-e\cos s)^2}{n(1-ee)^{\frac{3}{2}}(1-k\cos r)^2}$$

§. 46. Aequatio autem secunda facta hac substitutione, si per $1-kk$ diuidatur, abibit in sequentem:

$$\text{II.}\;\frac{ddu}{dr^2} = \frac{ku\cos r}{1-k\cos r} + \frac{uu}{u^3} - \frac{2u\int R\,dr}{nnu^3} + \frac{(\int R\,dr)^2}{n^4u^3} + \frac{\mu(1-kk)^2}{nn(1-k\cos r)^3}$$

$$-\frac{m}{nn(1-k\cos r)\,uu}\left(1-\tfrac{3}{4}\operatorname{tang}\varrho^2 + \tfrac{3}{4}\operatorname{tang}\varrho^2\cos 2(\Phi-\pi)\right)$$

$$+\frac{(1-kk)^3(1-e\cos s)^3}{2nn(1-ee)^3(1-k\cos r)^4}\,u\,(1+3\cos 2\eta)$$

$$+\frac{3\nu(1-kk)^4(1-e\cos s)^4}{8nn(1-ee)^4(1-k\cos r)^5}\,u^2\,(3\cos\eta + 5\cos 3\eta)$$

$$\text{III.}\; d\pi = -\frac{dr\sin(\Phi-\pi)\sin(\Phi-\pi)}{(uuu - R\int dr)}\left(\frac{3(1-kk)^3(1-e\cos s)^3}{(1-ee)^3(1-k\cos r)^4}\,uu\cos\eta\right.$$

$$\left.+\frac{3\nu(1-kk)^4(1-e\cos s)^4}{4(1-ee)^4(1-k\cos r)^5}\,u^3\,(3+5\cos 2\eta)\right)$$

Ac si ε denotet inclinationem mediam orbitae lunaris, quantitas $1-\frac{3}{4}\operatorname{tang}\varrho^2 + \frac{3}{4}\operatorname{tang}\varrho^2\cos 2(\varphi-\pi)$ in has duas partes discespi poterit:

$$\left(1-\tfrac{3}{4}\operatorname{tang}\varepsilon^2\right) + \tfrac{3}{4}\left(\operatorname{tang}\varepsilon^2 - \operatorname{tang}\varrho^2 + \operatorname{tang}\varrho^2\cos 2(\varphi-\pi)\right)$$

quarum illa est constans, haec vero proprie a nodo et inclinatione pendet.

§. 47.

§. 47. Euoluamus autem producta illa ex μ et ν orta, et quoniam excentricitates k et e sunt valde paruae, sufficit ad eos vsque terminos tantum progredi, qui coefficientes habeant kk, ek et ee, eosque qui per altiores potestates sint multiplicati omittere. Hinc erit:

$$\frac{1}{1-k\cos r} = 1 + \tfrac{1}{2}kk + k\cos r + \tfrac{1}{2}k^2\cos 2r$$

$$\frac{k\cos r}{1-k\cos r} = \tfrac{1}{2}kk + k\cos r + \tfrac{1}{2}k^2\cos 2r$$

$$\frac{(1-kk)^2}{(1-k\cos r)^3} = 1 + 3k\cos r,$$ quia hic terminus per μ multiplicatur.

$$\frac{(1-kk)^{\frac{3}{2}}}{(1-k\cos r)^2} = 1 + 2k\cos r + \tfrac{3}{2}kk\cos 2r$$

$$\frac{(1-kk)^3}{(1-k\cos r)^4} = 1 + 2kk + 4k\cos r + 5kk\cos 2r$$

$$\frac{(1-kk)^4}{(1-k\cos r)^5} = 1 + 5k\cos r,$$ quia hic terminus iam per ν est multiplicatus.

§. 48. Porro vero pro terminis ex ν enatis est:

$$\frac{(1-e\cos s)^2}{(1-ee)^{\frac{3}{2}}} = 1 + 2ee - 2e\cos s + \tfrac{1}{2}ee\cos 2s$$

$$\frac{(1-e\cos s)^3}{(1-ee)^3} = 1 + \tfrac{9}{2}ee - 3e\cos s + \tfrac{3}{2}ee\cos 2s$$

$$\frac{(1-e\cos s)^4}{(1-ee)^4} = 1 - 4e\cos s,$$ quia hic factor tantum in minimis terminis occurrit.

Hinc ergo colligimus:

$$\frac{(1-kk)^{\frac{3}{2}}(1-e\cos s)^2}{(1-ee)^{\frac{3}{2}}(1-k\cos r)^2} = 1 + 2ee + 2k\cos r + \tfrac{3}{2}kk\cos 2r - 2e\cos s - 2ek\cos(r+s) - 2ek\cos(r-s) + \tfrac{1}{2}ee\cos 2s$$

$$\frac{(1-kk)^3(1-e\cos s)^3}{(1-ee)^3(1-k\cos r)^4} = 1 + 2kk + \tfrac{3}{2}ee + 4k\cos r + 5kk\cos 2r - 3e\cos s - 6ek\cos(r-s) - 6ek\cos(r+s) + \tfrac{3}{2}ee\cos 2s$$

$$\frac{(1-kk)^4(1-e\cos s)^4}{(1-ee)^4(1-k\cos r)^5} = 1 + 5k\cos r - 4e\cos s,$$

atque hinc fiet: $d\Phi = \frac{dr}{uu}(u - \frac{1}{nn}\int R\,dr)$ atque

$$\frac{ds}{dr} = \frac{1+2ee}{n} + \frac{2k}{n}\cos r - \frac{2e}{n}\cos s - \frac{2ek}{n}(\cos r - s) + \frac{3kk}{2n}\cos 2r + \frac{ee}{2n}\cos 2s - \frac{2ek}{n}\cos(r+s)$$

$$\frac{d\eta}{dr} = \frac{u}{uu} - \frac{\int R\,dr}{nnuu} - \frac{1-2ee}{n} - \frac{2k}{n}\cos + r\frac{2e}{n}\cos s + \frac{2ek}{n}\cos(r-s) - \frac{3kk}{2n}\cos 2r - \frac{ee}{2n}\cos 2s + \frac{2ek}{n}\cos(r+s)$$

§. 49. Introductis nunc his valoribus euolutis in formulas nostras, iisque, qui per sinum cosinumue alterius anguli sunt multiplicati, pariter secundum simplices angulos explicatis, obtinebimus primum valorem ipsius R, qui erit:

$$R = \tfrac{3}{2}u^2\left\{\begin{array}{l}(1+2kk+\tfrac{5}{2}ee)\sin 2\eta + 2k\sin(2\eta-r) + 2k\sin(2\eta+r)\\ + \tfrac{5}{2}kk\sin(2\eta-2r) + \tfrac{5}{2}kk\sin(2\eta+2r)\\ - \tfrac{3}{2}e\sin(2\eta-s) - \tfrac{3}{2}e\sin(2\eta+s)\\ + \tfrac{3}{4}ee\sin(2\eta-2s) + \tfrac{3}{4}ee\sin(2\eta+2s)\\ - 3ek\sin(2\eta-r+s) - 3ek\sin(2\eta+r-s)\\ - 3ek\sin(2\eta-r-s) - 3ek\sin(2\eta+r+s)\end{array}\right.$$

$$+ \tfrac{3}{8}\nu u^3\left\{\begin{array}{l}\sin\eta + \tfrac{5}{2}k\sin(\eta-r) - 2e\sin(\eta-s)\\ 5\sin 3\eta + \tfrac{5}{2}k\sin(\eta+r) - 2e\sin(\eta+s)\\ + \tfrac{25}{2}k\sin(3\eta-r) - 10e\cos(3\eta-s)\\ + \tfrac{25}{2}k\sin(3\eta+r) - 10e\cos(3\eta+s)\end{array}\right.$$

§. 50.

§. 50. Aequatio autem secunda principalis sequentem induet formam:

$$\text{II. } \frac{ddu}{dr^2} = \frac{uu}{u^3} - \frac{2u\int R\,dr}{nn\,u^3} + \frac{(\int R\,dr)^2}{n^4 u^3}$$

$$+ \frac{3m\,\text{tang}\,\rho^2}{4n\,nuu}\,(1-\text{cos}\,2(\phi-\pi))\,(1+k\,\text{cos}\,r)$$

$$- \frac{m}{nnuu}(1+\tfrac{1}{2}kk+k\,\text{cos}\,r+\tfrac{1}{2}k^2\,\text{cos}\,2r) + \frac{\mu}{nn}(1+3k\,\text{cos}\,r)$$

$$+ u\,(\tfrac{1}{2}kk + k\,\text{cos}\,r + \tfrac{1}{2}kk\,\text{cos}\,2r)$$

$$+ \frac{u}{2nn}\left\{\begin{array}{l} 1+2kk+\tfrac{9}{2}ee+4k\,\text{cos}\,r-3e\,\text{cos}\,s-6ek\,\text{cos}\,(r-s) \\ \qquad +5kk\,\text{cos}\,2r+\tfrac{9}{2}ee\,\text{cos}\,2s-6ek\,\text{cos}\,(r+s) \end{array}\right.$$

$$+ \frac{3u}{2nn}\left\{\begin{array}{l} (1+2kk+\tfrac{9}{2}ee)\,\text{cos}\,2\eta+2k\,\text{cos}\,(2\eta-r)+2k\,\text{cos}\,(2\eta+r) \\ +\tfrac{5}{2}kk\,\text{cos}\,(2\eta-2r)+\tfrac{5}{2}kk\,\text{cos}\,(2\eta+2r) \\ -\tfrac{3}{2}e\,\text{cos}\,2\,(2\eta-s)-\tfrac{3}{2}e\,\text{cos}\,(2\eta+s) \\ +\tfrac{3}{4}ee\,\text{cos}\,(2\eta-2s)+\tfrac{3}{4}ee\,\text{cos}\,(2\eta+2s) \\ -3ek\,\text{cos}\,(2\eta-r+s)-3ek\,\text{cos}\,(2\eta+r-s) \\ -3ek\,\text{cos}\,(2\eta-r-s)-3ek\,\text{cos}\,(2\eta+r+s) \end{array}\right.$$

$$+ \frac{3\nu uu}{8nn}\left\{\begin{array}{l} 3\,\text{cos}\,\eta+5\,\text{cos}\,3\eta+\tfrac{15}{2}k\,\text{cos}\,(\eta-r)+\tfrac{15}{2}k\,\text{cos}\,(\eta+r) \\ -6e\,\text{cs}\,(\eta-s)-6e\,\text{cs}\,(\eta+s)+\tfrac{25}{2}k\,\text{cs}\,(\eta-r)+\tfrac{25}{2}k\,\text{cs}\,(\eta+r) \\ -10e\,\text{cos}\,(3\eta-s)-10e\,\text{cos}\,(3\eta+s) \end{array}\right.$$

vbi terminos, qui adhuc vlteriori euolutione indigent, primo loco posui, et cum terminus tang ρ^2 implicans iam sit valde paruus, in eius multiplicatore secundam ipsius k potestatem omisi: sin autem alicuius momenti videantur, loco $1 + k\,\text{cos}\,r$ scribi poterit $1 + \tfrac{1}{2}kk + k\,\text{cos}\,r + \tfrac{1}{2}kk\,\text{cos}\,2r$.

§. 51. Pro longitudine vero nodi inuenienda aequatio sequens prodibit resoluenda:

$$d\pi = -\frac{3uudr\sin(\phi-\pi)\sin(\theta-\pi)}{unn-\int Rdr}\cos\eta\,(1+2kk+\tfrac{3}{2}ee+4k\cos r$$
$$+5kk\cos 2r-3e\cos s+\tfrac{3}{2}ee\cos 2s)$$
$$-\frac{3vu^3dr\sin(\phi-\pi)\sin(\theta-\pi)}{4(unn-\int Rdr)}(3+5\cos 2\eta)(1+5k\cos r)$$

At est $\sin(\phi-\pi)\sin(\theta-\pi)=\frac{1}{2}\cos\eta-\frac{1}{2}(\phi+\theta-2\pi)$; vnde $\sin(\phi-\pi)\sin(\theta-\pi)\,c\!f\eta=\frac{1}{4}+\frac{1}{4}c\!f 2\eta-\frac{1}{4}c\!f2(\phi-\pi)-\frac{1}{4}c\!f2(\theta-\pi)$ et $\sin(\phi-\pi)\sin(\theta-\pi)\cos 2\eta=\frac{1}{4}\cos\eta+\frac{1}{4}\cos 3\eta$
$-\frac{1}{4}\cos(3\phi-\theta-2\pi)-\frac{1}{4}\cos(3\theta-\phi-2\pi)$

Tum vero ob $\int Rdr$ valde paruum prae unn, erit satis exacte

$$\frac{1}{unn-\int Rdr}=\frac{1}{unn}+\frac{\int Rdr}{uun^4}+\frac{(\int Rdr)^2}{u^3n^6}$$

vbi quidem postremus terminus tuto omitti potest.

§. 52. Praeterea vero ponatur $u=1+\frac{v}{nn}$, vt sit $ddu=\frac{ddv}{nn}$, et reiectis terminis per n^4 diuisis, qui iam per exiguam quantitatem sunt multiplicati, erit

$$\frac{d\phi}{dr}=u-\frac{2uv}{nn}+\frac{3uv^2}{n^4}-\frac{\int Rdr}{nn}+\frac{2v\int Rdr}{n^4}$$

$$\frac{d\eta}{dr}=u-\frac{1-2ee}{n}-\frac{2k}{n}\cos r+\frac{2e}{n}\cos s-\frac{3kk}{2n}\cos 2r-\frac{ee}{2n}\cos 2s$$
$$+\frac{2ek}{n}\cos(r-s)+\frac{2ek}{n}\cos(r+s)$$
$$-\frac{2uv-\int Rdr}{nn}+\frac{3uv^2+2v\int Rdr}{n^4}\qquad\text{atque}$$

$$R=\tfrac{3}{2}(1+2kk\mp\tfrac{5}{2}ee)\sin 2\eta+3k\sin(2\eta-r)-\tfrac{3}{2}e\sin(2\eta-s)$$
$$+3k\sin(2\eta+r)-\tfrac{3}{2}e\sin(2\eta+s)$$
$$+\frac{3v}{nn}\sin 2\eta\quad+\tfrac{15}{4}kk\sin(2\eta-2r)+\tfrac{3}{8}ee\sin(2\eta-2s)$$
$$+\tfrac{15}{4}kk\sin(2\eta+2r)+\tfrac{3}{8}ee\sin(2\eta+2s)$$
$$+\frac{3vv}{2n^4}\sin 2\eta\quad-\tfrac{3}{2}ek\sin(2\eta-r+s)-\tfrac{3}{2}ek\sin(2\eta+r-s)$$
$$-\tfrac{3}{2}ek\sin(2\eta-r-s)-\tfrac{3}{2}ek\sin(2\eta+r+s)$$
$$+$$

$$+\frac{6kv}{nn}\sin(2\eta-r)+\frac{6kv}{nn}\sin(2\eta+r)$$

$$-\frac{9ev}{2nn}\sin(2\eta-s)-\frac{9ev}{2nn}\sin(2\eta+s)$$

$$+\tfrac{3}{8}v\sin\eta+\tfrac{15}{16}vk\sin(\eta-r)-\tfrac{3}{4}ve\sin(\eta-s)$$

$$+\tfrac{75}{16}vk\sin(3\eta-r)-\tfrac{15}{4}ve\sin(3\eta-s)$$

$$+\tfrac{15}{8}v\sin 3\eta+\tfrac{15}{16}vk\sin(\eta+r)-\tfrac{3}{4}ve\sin(\eta+s)$$

$$+\tfrac{75}{16}vk\sin(3\eta+s)-\tfrac{15}{4}ve\sin(3\eta+s)$$

§. 53. Ipsa vero aequatio secunda per hanc substitutionem, postquam per nn fuerit multiplicata, in formam sequentem abibit.

II. $$\frac{ddv}{dr^2}=nnnn-3nnv+\frac{6nnvv}{nn}-2n\int R\,dr$$

$$+\frac{6nv}{nn}\int R\,dr+\frac{1}{nn}\left(\int R\,dr\right)^2$$

$$+\frac{3m\tan g\,\rho^2}{4}(1-\cos 2(\varphi-\pi))(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)$$

$$-\frac{3mv\tan g\,\rho^2}{2nn}(1-\cos 2(\varphi-\pi))\quad(1+k\cos r)$$

$$-\frac{3mvv}{n^4}(1+k\cos r)-m(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)$$

$$+\frac{2mv}{nn}(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)+nn(\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)$$

$$+v(\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)+\mu(1+3k\cos r)$$

$$+\tfrac{1}{2}+kk+\tfrac{9}{4}ee+2k\cos r-\tfrac{3}{2}e\cos s-3ek\cos(r-s)$$

$$+\tfrac{5}{2}kk\cos r+\tfrac{3}{4}ee\cos 2s-3ek\cos(r+s)$$

$$+\frac{v}{2nn}(1+4k\cos r-3e\cos s)$$

$$+\tfrac{3}{2}(1+2kk+\tfrac{9}{2}ee)\cos 2\eta+3k\cos(2\eta-r)-\tfrac{9}{4}e\cos(2\eta-s)$$

$$+3\cos(2\eta+r)-\tfrac{9}{4}e\cos(2\eta+s)$$

$$+\tfrac{15}{4}kk\cos(2\eta-2r)+\tfrac{9}{8}ee\cos(2\eta-2s)$$
$$+\tfrac{15}{4}kk\cos(2\eta+2r)+\tfrac{9}{8}ee\cos(2\eta+2s)$$
$$-\tfrac{9}{2}ek\cos(2\eta-r+s)-\tfrac{9}{2}ek\cos(2\eta+r-s)$$
$$-\tfrac{9}{2}ek\cos(2\eta-r-s)-\tfrac{9}{2}ek\cos(2\eta+r+s)$$

$$+\frac{3v}{2nn}\begin{cases}\cos 2\eta+2k\cos(2\eta-r)+2k\cos(2\eta+r)\\ -\tfrac{3}{2}e\cos(2\eta-s)-\tfrac{3}{2}e\cos(2\eta+s)\end{cases}$$

$$+\tfrac{3}{8}v\begin{cases}+3\cos\eta+\tfrac{15}{2}k\cos(\eta-r)-6e\cos(\eta-s)\\ +5\cos 3\eta+\tfrac{15}{2}k\cos(\eta+r)-6e\cos(\eta+s)\\ +\tfrac{25}{2}k\cos(3\eta-r)-10e\cos(3\eta-s)\\ +\tfrac{25}{2}k\cos(3\eta+r)-10e\cos(3\eta+s)\end{cases}$$

$$+\frac{3vv}{4nn}(3\cos\eta+5\cos 3\eta)$$

§. 54. Cum autem sit $m = nn + 2 + 3\mu + \gamma$ et $\varkappa\varkappa = 1 + \frac{3+4\mu+\delta}{2nn}$, si hi valores substituantur, plures termini se mutuo destruent, aequatioque prodibit sequenti forma concinniori contenta: vbi quidem in terminis per se minimis loco m scribi licebit nn, et 1 loco $\mu\varkappa$ vel $\varkappa$.

II. Aequatio.

$$\frac{ddv}{dr^2} = \tfrac{1}{2}\delta - \gamma + \tfrac{9}{4}ee - \gamma k\cos r + \tfrac{3}{2}kk\cos 2r$$
$$-2\left(1+\frac{3+4\mu+\delta}{4nn}\right)\int R\,dr + \frac{1}{nn}\left(\int R\,dr\right)^2$$
$$-v\left(1-\tfrac{3}{2}kk-3k\cos r-\tfrac{3}{2}kk\cos 2r\right)$$
$$+\frac{vv}{nn}(3-3k\cos r)-\tfrac{3}{2}e\cos s+\tfrac{3}{4}ee\cos 2s-3ek\cos(r-s)$$
$$-3ek\cos(r+s)+\tfrac{3}{2}\left(1+2kk+\tfrac{9}{2}ee\right)\cos 2\eta$$

+

$$+\,3k\cos(2\eta-r)+\tfrac{15}{4}kk\cos(2\eta-2r)-\tfrac{9}{4}e\cos(2\eta-s)$$
$$+\,3k\cos(2\eta+r)+\tfrac{15}{4}kk\cos(2\eta+2r)-\tfrac{9}{4}e\cos(2\eta-s)$$
$$+\,\tfrac{9}{8}ee\cos(2\eta-2s)-\tfrac{9}{2}ek\cos(2\eta-r+s)-\tfrac{9}{2}ek\cos(2\eta-r-s)$$
$$+\,\tfrac{9}{8}ee\cos(2\eta+2s)-\tfrac{9}{2}ek\cos(2\eta+r-s)-\tfrac{9}{2}ek\cos(2\eta+r+s)$$

$$+\,\frac{v}{nn}\left\{\begin{array}{l}2\gamma-\tfrac{3}{2}\delta+6k\cos r+2(3\mu+\gamma)k\cos r+6\int R\,dr\\ -\tfrac{3}{2}e\cos s+\tfrac{3}{2}\cos 2\eta+3k\cos(2\eta-r)+3k\cos(2\eta+r)\\ -\tfrac{9}{4}e\cos(2\eta-s)-\tfrac{9}{4}e\cos(2\eta+s)\end{array}\right.$$

$$+\,\tfrac{3}{8}\nu\left\{\begin{array}{l}3\cos\eta+\tfrac{15}{2}k\cos(\eta-r)-6e\cos(\eta-s)\\ +5\cos 3\eta+\tfrac{15}{2}k\cos(\eta+r)-6e\cos(\eta+s)\\ +\tfrac{25}{2}k\cos(3\eta-r)-10e\cos(3\eta-s)\\ +\tfrac{25}{2}k\cos(3\eta+r)-10e\cos(3\eta+s)\end{array}\right.$$

$$+\,\frac{3\nu v}{4nn}(3\cos\eta+5\cos 3\eta)$$

$$+\,\tfrac{3}{4}(nn+2+3\mu+\gamma)\frac{(1-2v)}{nn}\,\mathrm{tang}\,\rho^2(1-\cos 2(\varphi-\pi))$$
$$(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)$$

§. 55. Pro loco nodi autem inveniendo prodibit ſequens aequatio.

$$\frac{d\pi}{dr}=\frac{-3}{\varkappa nn}\left(1+\frac{2\varkappa v+\int R\,dr}{\varkappa nn}\right)(1+2kk+\tfrac{9}{2}ee)$$

$$\left\{\begin{array}{l}\tfrac{1}{4}+\tfrac{1}{4}\cos 2\eta-\tfrac{1}{4}\cos 2(\varphi-\pi)-\tfrac{1}{4}\cos 2(\theta-\pi)\\ +k\cos r+\tfrac{1}{2}k\cos(2\eta-r)-\tfrac{1}{2}k\cos(2\varphi-2\pi-r)\\ -\tfrac{3}{4}e\cos s+\tfrac{1}{2}k\cos(2\eta+r)-\tfrac{1}{2}k\cos(2\varphi-2\pi+r)\\ \qquad-\tfrac{3}{8}e\cos(2\eta-s)-\tfrac{1}{2}k\cos(2\theta-2\pi-r)\\ \qquad-\tfrac{3}{8}e\cos(2\eta+r)-\tfrac{1}{2}k\cos(2\theta-2\pi+r)\end{array}\right.$$

$$-\,\frac{3\nu}{4\varkappa nn}\left\{\begin{array}{l}\tfrac{11}{4}\cos\eta+\tfrac{5}{4}\cos 3\eta-\tfrac{3}{2}\cos(3\varphi+\theta-2\pi)\\ -\tfrac{5}{4}\cos(3\varphi-\theta-2\pi)-\tfrac{5}{4}\cos(3\theta-\varphi-2\pi)\end{array}\right.$$

At

At pro inclinatione orbitae habebitur:

$$\frac{d.l\,\mathrm{tang}\,\rho}{dr} = \frac{-3}{xnn}\left(1 + \frac{2xv + \int R\,dr}{xxn}\right)\left(1 + 2kk + \tfrac{3}{2}ee\right)$$

$$\left\{\begin{array}{l} \tfrac{1}{4}\sin 2(\varphi - \pi) + \tfrac{1}{4}\sin 2(\theta - \pi) - \tfrac{1}{4}\sin 2\eta \\ - \tfrac{1}{2}k\sin(2\eta - r) + \tfrac{1}{2}k\sin(2\varphi - 2\pi - r) \\ - \tfrac{1}{2}k\sin(2\eta + r) + \tfrac{1}{2}k\sin(2\varphi - 2\pi + r) \\ + \tfrac{3}{8}e\sin(2\eta - s) + \tfrac{1}{2}k\sin(2\theta - 2\pi - r) \\ + \tfrac{3}{8}e\sin(2\eta + s) + \tfrac{1}{2}k\sin(2\theta - 2\pi + r) \end{array}\right.$$

$$- \frac{3\nu}{4xnn}\left(-\tfrac{1}{4}\sin\eta - \tfrac{5}{4}\sin 3\eta + \tfrac{3}{2}\sin(\varphi + \theta - 2\pi)\right.$$
$$\left. + \tfrac{5}{4}\sin(3\varphi - \theta - 2\pi) + \tfrac{5}{4}\sin(3\theta - \varphi - 2\pi)\right)$$

Quomodo igitur his aequationibus ad motum Lunae cognoſcendum vti conueniat, in ſequentibus capitibus videamus.

CAPUT

CAPUT IV.

INUESTIGATIO INAEQUALITATIS LUNAE ABSOLUTAE, QUAE VARIATIO DICITUR.

§. 33.

Ex his aequationibus perſpicitur in determinationem motus Lunae plurimorum angulorum vel ſinus vel coſinus ingredi, qui anguli formantur per variam combinationem ſequentium 4 angulorum:
1. ex diſtantia Solis a Luna, quem angulum poſuimus $= \eta$
2. ex anomalia Lunae vera $= r$
3. ex anomalia Solis vera $= s$
4. ex diſtantia Lunae a nodo aſcendente $= \phi - \pi$.
Ne igitur a tanta angulorum multitudine obruamur, a caſibus ſimplicioribus ordiamur: ac primo quidem in eas tantum motus inaequalitates inquiramus, quae a ſolo angulo η pendeant, neque idcirco excentricitatem vel Solis vel Lunae implicent, neque ab orbitae lunaris inclinatione ad eclipticam afficiantur.

§. 57. Has igitur inaequalitates, quae a ſolo ſitu Solis reſpectu Lunae naſcuntur, atque ab Aſtronomis ſub nomine variationis comprehendi ſolent, ex praecedentibus aequationibus eliciemus, ſi tam excentricitatem Lunae k quam ſolis e pro nihilo habeamus, atque inclinationem orbitae lunaris ad eclipticam evaneſcentem ſtatuamus, ita vt ſit $k = 0$, $e = 0$ et tang $\rho = 0$. Sic enim obtinebimus eas inaequalitates Lunae, quae ab his elementis non pendent, ideoque tantum per angu-

lum

lum η determinantur; quae cum vnica tabula comprehendi queant, haec tabula variationem Lunae indicare dicitur. Interim tamen hic animaduerti oportet, partem quandam exiguam variationis quoque ab excentricitate orbitae Lunae k pendere, quam partem deinceps supplebimus, cum huius excentricitatis rationem sumus habituri.

§. 58. Reiectis ergo terminis k, e, et tang ρ continentibus, habebimus:

$$\frac{d\Phi}{dr} = \varkappa - \frac{2\varkappa v - \int R\,dr}{nn} + \frac{3\varkappa v^2 + 2v\int R\,dr}{n^4}$$

$$\frac{d\eta}{dr} = \varkappa - \frac{1}{n} - \frac{2\varkappa v - \int R\,dr}{nn} + \frac{3\varkappa v^2 + 2v\int R\,dr}{n^4}$$

$R = \frac{3}{4}\sin 2\eta + \frac{3v}{nn}\sin 2\eta + \frac{3}{8}\nu\sin\eta + \frac{15}{8}\nu\sin 3\eta$, ac denique

$$\frac{ddv}{dr^2} = \tfrac{1}{2}\delta - \gamma - 2\left(1 + \frac{3+4\mu+\delta}{4nn}\right)\int R\,dr + \frac{1}{nn}\left(\int R\,dr\right)^2 - v + \frac{3vv}{nn}$$
$$+ \tfrac{3}{4}\cos 2\eta + \frac{v}{nn}(2\gamma - \tfrac{3}{2}\delta) + \frac{3v\cos 2\eta}{2nn} + \frac{6v}{nn}\int R\,dr$$
$$+ \tfrac{9}{8}\cos\eta + \tfrac{15}{8}\nu\cos 3\eta$$

Hic autem notandum est esse $\varkappa = \sqrt{\left(1 + \frac{3+4\mu+\delta}{2nn}\right)}$; quoniam vero valores litterarum μ et δ demum cum per consensum obseruationum, tum per indolem calculi definire instituimus, hic ex obseruationibus petamus valores ipsius $\varkappa$; cum enim sit $\varkappa : 1 = d\Phi : dr$, hoc est vt motus Lunae medius ad motum anomaliae, erit $\varkappa = 1,0085272$. Fieri quidem potest, vt hic valor aliquantulum a vero differat, sed errorem si quis lateat infra detegemus, facillimeque emendabimus.

§. 59.

§. 59. Cum igitur iam supra inuenerimus esse $x = 13,25586$ ac proinde $xx = 175,71795$ erit $\frac{1}{x} = 0,075438$, ideoque $x - \frac{1}{x} = 0,933089$ Hic autem numerus, qui iam quasi medium valorem rationis $\frac{d\eta}{dr}$ exprimit, in omnibus operationibus, quae sequuntur, frequentissime occurret, hincque breuitatis gratia ponamus

$$x - \frac{1}{x} = a, \quad \text{vt sit } a + \frac{1}{x} = V\left(+\frac{3+4\mu+\delta}{2xx}\right)$$

eritque ergo $a = 0,933089$, qui valor quam minime a vero discrepat, vti mox patebit. Quod autem verus ipsius a valor aliquantulum diuersus esse possit, inde primo patet, quod minutias, quae ex terminis $\frac{3xv^2 + 2v\int R dr}{x^4}$ quantitati constanti accrescere potuissent, hic neglex-imus; tum vero fieri potest, vt ratio media differentialium $d\eta$ ad dr alia sit atque quantitatum finitarum η et r.

§. 60. Si has formulas attente contemplemur, mox deprehendemus valorem integralis $\int R dr$ constare ex cosinibus angulorum 2η, η, 3η, et 4η. Quanquam enim altiora quoque multipla huius anguli ingredientur, tamen facile patet, coefficientes eorum continuo fieri minores, ita vt in quadruplo tuto subsistere possimus: similis autem erit ratio valoris ipsius v. Hinc ponamus:

$$\int R dr = \mathfrak{A} \cos 2\eta + \mathfrak{B} \cos 4\eta + \mathfrak{a} v \cos \eta + \mathfrak{b} v \cos 3\eta$$
$$v = A \cos 2\eta + B \cos 4\eta + a v \cos \eta + b v \cos 3\eta$$

atque hos valores fictitios in formulis nostris substitua-

 mus:

mus, vt inde valores istorum coefficientium assumtorum determinare possimus: quippe qui modus aptissimus videtur ad cognitionem integralium perueniendi. Quia autem est circiter $\nu = \frac{1}{282}$, patet terminos per ν multiplicatos prae reliquis tam esse exiguos, vt eos qui multo fuerint minores, sine haesitatione praetermittere possimus.

§. 61. Per hos ergo valores assumtos consequemur:

$$\frac{d\Phi}{dr} = \varkappa - \frac{(2\varkappa A + \mathfrak{A})}{nn} \cos 2\eta - \frac{(2\varkappa B + \mathfrak{B})}{nn} \cos 4\eta$$
$$+ \frac{A(3\varkappa A + 2\mathfrak{A})}{2n^4} \cos 2\eta + \frac{A(3\varkappa A + 2\mathfrak{A})}{2n^4} \cos 4\eta$$
$$- \frac{(2\varkappa a + \mathfrak{a})}{nn} \nu \cos \eta - \frac{(2\varkappa b + \mathfrak{b})}{nn} \nu \cos 3\eta$$

atque ob $\varkappa - \frac{1}{n} = \alpha$ erit minimis terminis omissis, quia hi in operatione multo magis diminuerentur:

$$\frac{d\eta}{dr} = \alpha - \frac{(2\varkappa A + \mathfrak{A})}{nn} \cos 2\eta - \frac{(2\varkappa B + \mathfrak{B})}{nn} \cos 4\eta$$
$$- \frac{(2\varkappa a + \mathfrak{a})}{nn} \nu \cos \eta - \frac{(2\varkappa b + \mathfrak{b})}{nn} \nu \cos 3\eta.$$

His positis erit:

$$\frac{d.\cos 2\eta}{dr} = -\sin 2\eta . \frac{2d\eta}{dr} = -2\alpha \sin 2\eta - \frac{(2\varkappa B + \mathfrak{B})}{nn} \sin 2\eta + \frac{(2\varkappa A + \mathfrak{A})}{nn} \sin 4\eta$$
$$+ \frac{(2\varkappa a + \mathfrak{a})}{nn} \nu \sin \eta + \frac{(2\varkappa a + \mathfrak{a})}{nn} \nu \sin 3\eta$$
$$- \frac{(2\varkappa b + \mathfrak{b})}{nn} \nu \sin \eta$$

$$\frac{d.\cos 4\eta}{dr} = -\sin 4\eta.\frac{4d\eta}{dr} = -4\alpha\sin 4\eta + \frac{2(2\kappa A+\mathfrak{A})}{nn}\sin 2\eta$$

$$\frac{d.\cos\eta}{dr} = -\sin\eta.\frac{d\eta}{dr} = -\alpha\sin\eta$$

$$\frac{d.\cos 3\eta}{dr} = -\sin 3\eta.\frac{3d\eta}{dr} = -3\alpha\sin 3\eta$$

§. 62. Quod si iam secundum has formulas quantitas integralis $\int R\,dr$ differentietur, obtinebitur:

$$\begin{aligned} R = {} & \left(-2\alpha\mathfrak{A} - \frac{\mathfrak{A}(2\kappa B+\mathfrak{B})}{nn} + \frac{2\mathfrak{B}(2\kappa A+\mathfrak{A})}{nn}\right)\sin 2\eta \\ & + \left(\frac{\mathfrak{A}(2\kappa A+\mathfrak{A})}{nn} - 4\alpha\mathfrak{B}\right)\sin 4\eta \\ & + \left(\frac{\mathfrak{A}(2\kappa a+\mathfrak{a})}{nn} + \frac{\mathfrak{A}(2\kappa b+\mathfrak{b})}{nn} - \alpha\mathfrak{a}\right)\nu\sin\eta \\ & + \left(\frac{\mathfrak{A}(2\kappa a+\mathfrak{a})}{nn} - 3\alpha\mathfrak{b}\right)\nu\sin 3\eta \end{aligned}$$

Cum iam sit per hypothesin

$$R = \begin{array}{llll} \frac{3}{2}\sin 2\eta & +\frac{3A}{2nn}\sin 4\eta & +\frac{3}{8}\nu\sin\eta & +\frac{15}{8}\nu\sin 3\eta \\ -\frac{3B}{2nn} & & +\frac{3a}{2nn}\nu & +\frac{3a}{2nn}\nu \\ & & -\frac{3b}{2nn}\nu & \end{array}$$

prodibit terminis homogeneis comparandis:

$$2\alpha\mathfrak{A} = -\frac{3}{2} - \frac{\mathfrak{A}(2\kappa B+\mathfrak{B}) + 2\mathfrak{B}(2\kappa A+\mathfrak{A})}{nn} + \frac{3B}{2nn}$$

$$4\alpha\mathfrak{B} = -\frac{3A}{2nn} + \frac{\mathfrak{A}(2\kappa A+\mathfrak{A})}{nn}$$

$$\alpha\mathfrak{a} = -\frac{3}{8} - \frac{3(a-b)}{2nn} + \frac{\mathfrak{A}(2\kappa a+\mathfrak{a}) - \mathfrak{A}(2\kappa b+\mathfrak{b})}{nn}$$

$$3\alpha\mathfrak{b} = -\frac{15}{8} - \frac{3a}{2nn} + \frac{\mathfrak{A}(2\kappa a+\mathfrak{a})}{nn}$$

§. 63.

§. 63. Aequatio autem noſtra differentio-differentialis, ſi pro $\int R\,dr$ et v valores aſſumti ſubſtituantur, ſequentem induet formam:

$$\frac{ddv}{dr^2} = \begin{array}{lllll}
(\tfrac{1}{2}\delta-\gamma) & -2\kappa\mathfrak{A}\cos 2\eta & -2\kappa\mathfrak{B}\cos 4\eta & -2\kappa\mathfrak{a}v\cos\eta & -2\kappa\mathfrak{b}v\cos 3\eta \\
+\frac{\mathfrak{A}\mathfrak{A}}{2nn} & & +\frac{\mathfrak{A}\mathfrak{A}}{2nn} & & \\
 & -A & -B & -av & -bv \\
+\frac{3AA}{2nn} & & +\frac{3AA}{2nn} & & \\
 & +\frac{1}{4} & & & \\
 & +\frac{(4\gamma-3\delta)}{2nn}A & +\frac{(4\gamma-3\delta)}{2nn}B & +\frac{3b}{4nn}v & \\
+\frac{3A}{4nn} & +\frac{3B}{4nn} & +\frac{3A}{4nn} & +\frac{3a}{4nn}v & +\frac{3a}{4nn}v \\
+\frac{3A\mathfrak{A}}{nn} & & +\frac{3A\mathfrak{A}}{nn} & +\frac{3}{8}v & +\frac{15}{8}v
\end{array}$$

vbi quidem perſpicuum eſt, quinam termini reſpectu reliquorum tam ſint parui, vt ſine errore deleri queant.

§. 64. Quaeramus ergo primum differentiale $\frac{dv}{dr}$ ac reperietur:

$$\frac{dv}{dr} = \begin{array}{l}
\left(-2A\alpha - \frac{A(2\kappa B+\mathfrak{B})}{nn} + \frac{2B(2\kappa A+\mathfrak{A})}{nn}\right)\sin 2\eta \\
\left(-4\alpha B + \frac{A(2\kappa A\mp\mathfrak{A})}{nn}\right)\sin 4\eta \\
\left(-\alpha a + \frac{A(2\kappa a+\mathfrak{a})}{nn} - \frac{A(2\kappa b+\mathfrak{b})}{nn}\right)v\sin\eta \\
\left(-3\alpha b + \frac{A(2\kappa a+a)}{nn}\right)v\sin 3\eta \quad \ldots
\end{array}$$

pona-

ponatur autem breuitatis ergo:

$$\frac{dv}{dr} = -A' \sin 2\eta - B' \sin 4\eta - a' v \sin \eta - b' v \sin 3\eta$$

vt sit:

$$A' = 2\alpha A + \frac{A(2\kappa B + \mathfrak{B})}{nn} - \frac{2B(2\kappa A + \mathfrak{A})}{nn}$$

$$B' = 4\alpha B - \frac{A(2\kappa A + \mathfrak{A})}{nn}$$

$$a' = \alpha a - \frac{A(2\kappa a + \mathfrak{a})}{nn} + \frac{A(\kappa b + \mathfrak{b})}{nn}$$

$$b' = 3\alpha b - \frac{A(2\kappa a + \mathfrak{a})}{nn}$$

§. 65. Hinc cum sit:

$$\frac{d.\sin 2\eta}{dr} = \cos 2\eta . \frac{2d\eta}{dr} = 2\alpha \cos 2\eta - \frac{(2\kappa B + \mathfrak{B})}{nn} \cos 2\eta - \frac{(2\kappa A + \mathfrak{A})}{nn} \cos 4\eta$$
$$- \frac{(2\kappa A + \mathfrak{A})}{nn} - \frac{(2\kappa a + \mathfrak{a})}{nn} v \cos \eta - \frac{(2\kappa b + \mathfrak{b})}{nn} v \cos \eta - \frac{(2\kappa a + \mathfrak{a})}{nn} v \cos 3\eta$$

$$\frac{d.\sin 4\eta}{dr} = \cos 4\eta . \frac{4d\eta}{dr} = 4\alpha \cos 4\eta - \frac{2(2\kappa A + \mathfrak{A})}{nn} \cos 2\eta - \frac{2(2\kappa B + \mathfrak{B})}{nn}$$

$$\frac{d.\sin \eta}{dr} = \cos \eta . \frac{d\eta}{dr} = \alpha \cos \eta; \text{ et } \frac{d.\sin 3\eta}{dr} = \cos 3\eta . \frac{3d\eta}{dr} = 3\alpha \cos 3\eta$$

prodibit

$$\frac{ddv}{dr^2} = \begin{cases} + \frac{A'(2\kappa A + \mathfrak{A})}{nn} + \frac{2B'(2\kappa B + \mathfrak{B})}{nn} \\ \left(-2\alpha A' + \frac{A'(2\kappa B + \mathfrak{B})}{nn} + \frac{2B'(2\kappa A + \mathfrak{A})}{nn}\right) \cos 2\eta \\ \left(-4\alpha B' + \frac{A'(2\kappa A + \mathfrak{A})}{nn}\right) \cos 4\eta \\ \left(-\alpha a' + \frac{A'(2\kappa a + \mathfrak{a})}{nn}\right) v \cos \eta \\ \left(-3\alpha b' + \frac{A'(2\kappa a + \mathfrak{a})}{nn}\right) v \cos 3\eta \end{cases}$$

seu

ſeu ſubſtitutis ſuperioribus valoribus:

$$\frac{ddv}{dr^2} = \begin{cases} +\dfrac{2A\alpha(2\kappa A+\mathfrak{A})}{nn} + \dfrac{8\alpha B(2\kappa B+\mathfrak{B})}{nn} \\ \left(-4\alpha\alpha A + \dfrac{12\alpha B(2\kappa A+\mathfrak{A})}{nn}\right)\cos 2\eta \\ \left(-16\alpha\alpha B + \dfrac{8\alpha A(2\kappa A+\mathfrak{A})}{nn}\right)\cos 4\eta \\ \left(-\alpha\alpha a + \dfrac{3\alpha A(2\kappa a+\mathfrak{a})}{nn} - \dfrac{\alpha A(2\kappa b+\mathfrak{b})}{nn}\right) v\cos\eta \\ \left(-9\alpha\alpha b + \dfrac{5\alpha A(2\kappa a+\mathfrak{a})}{nn}\right) v\cos 3\eta \end{cases}$$

§ 66. Hi iam termini ſingulatim illis, qui §. 63. ſunt exhibiti, aequales ſtatuantur, atque ſequentes prodibunt determinationes,

$$\tfrac{1}{2}\delta-\gamma+\frac{3AA+6A\mathfrak{A}+\mathfrak{A}\mathfrak{A}}{2nn}+\frac{3A}{4nn}=\frac{2\alpha A(2\kappa A+\mathfrak{A})}{nn}+\frac{8\alpha B(2\kappa B+\mathfrak{B})}{nn}$$

$$-A+\tfrac{3}{2}-2\kappa\mathfrak{A}+\frac{(4\gamma-3\delta)}{2nn}A+\frac{3B}{4nn}=-4\alpha\alpha A+\frac{12\alpha B(2\kappa A+\mathfrak{A})}{nn}$$

$$-B-2\kappa\mathfrak{B}+\frac{3AA+6A\mathfrak{A}+\mathfrak{A}\mathfrak{A}}{2nn}+\frac{3A}{4nn}+\frac{(4\gamma-3\delta)}{2nn}B= -16\alpha\alpha B+\frac{8\alpha A(2\kappa A+\mathfrak{A})}{nn}$$

$$-a+\tfrac{9}{8}-2\kappa\mathfrak{a}+\frac{3a+3b}{4nn}=-\alpha\alpha a+\frac{3\alpha A(2\kappa a+\mathfrak{a})}{nn}-\frac{\alpha A(2\kappa b+\mathfrak{b})}{nn}$$

$$-b+\tfrac{15}{8}-2\kappa\mathfrak{b}+\frac{3a}{4nn}=-9\alpha\alpha b+\frac{5\alpha A(2\kappa a+\mathfrak{a})}{nn}$$

vnde primum quaeri debent valores vero proximi, qui ſunt:

$$\mathfrak{A}=-\frac{3}{4\alpha};\quad \mathfrak{a}=-\frac{3}{8\alpha};\quad \mathfrak{b}=-\frac{5}{8\alpha};$$

$$A=-\frac{\frac{3}{2}+2\kappa\mathfrak{A}}{4\alpha\alpha-1};\quad a=\frac{\frac{9}{8}-2\kappa\mathfrak{a}}{1-\alpha\alpha};\quad b=-\frac{\frac{15}{8}+2\kappa\mathfrak{b}}{9\alpha\alpha-1}$$

§. 67.

§. 67. Calculus ergo sequenti modo instituatur:

$\alpha = 0,933089$; $l\alpha = 9,969923$
$n = 1,008527$; $ln = 0,003687$
$l2n = 0,304717$
Iam est $l8\alpha = 0,873013$
subtr. a $\begin{cases} l3 = 0,477121 \\ l5 = 0,698970 \end{cases}$

$\mathfrak{a} = -0,402$; erit $l-\mathfrak{a} = 9,604\cdot08$
$\mathfrak{b} = -0,635$; $l-\mathfrak{b} = 9,825957$
$\mathfrak{A} = -0,804$; $l-\mathfrak{A} = 9,905138$

atque hinc conficietur:

$$A = -\frac{3,121}{4\alpha\alpha-1};\quad a = +\frac{1,936}{1-\alpha\alpha};\quad b = -\frac{3,156}{9\alpha\alpha-1}$$

quarum ergo litterarum valores proximi sunt

$A = -1,2583$; $a = +14,968$; $b = -0,4613$.

§. 68. Quaeramus hinc primum valores litterarum $\mathfrak{B}$ et B.

$\mathfrak{A} = -0,804$; $l-\mathfrak{A} = 9,905138$
$2nA + \mathfrak{A} = -3,341$; vnde colligitur

$4\alpha\mathfrak{B} = +\frac{4,573}{nn}$ $\quad l4,573 = 0,660201$

hinc erit $\quad lnn = 2,244816$
$\quad 8,415385$
$\quad l4\alpha = 0,571983$

$\mathfrak{B} = +0,00697$ $\quad l\mathfrak{B} = 7,843402$

Deinde est

$$(16\alpha\alpha-1)B = 2n\mathfrak{B} - \frac{3A}{4nn} - \frac{3AA-6A\mathfrak{A}-\mathfrak{A}\mathfrak{A}}{2nn} + \frac{8\alpha A(2nA+\mathfrak{A})}{nn}$$

seu $B = +\frac{0,16819}{16\alpha\alpha-1}$; vnde reperitur

$B = +0,012792$ et $lB = 8,106947$

 §. 69.

§. 69. His iam valoribus proxime veris inuentis quaerantur exacti, ac primo quidem

$$2\alpha\mathfrak{A} = -\tfrac{3}{8} + \frac{\frac{3}{4}B - \mathfrak{A}(2\kappa B+\mathfrak{B}) + 2\mathfrak{B}(2\kappa A+\mathfrak{A})}{nn}$$

vnde reperitur vt ante:

$$\mathfrak{A} = -0,80378 \quad \ldots \quad l-\mathfrak{A} = 9,905138$$

$$\alpha\mathfrak{a} = -\tfrac{3}{8} - \frac{\frac{3}{4}(a-b) + \mathfrak{A}(2\kappa a+\mathfrak{a}) - \mathfrak{A}(2\kappa b+\mathfrak{b})}{nn} = -0,65361$$

$$\mathfrak{a} = -0,70048 \quad \ldots \quad l-\mathfrak{a} = 9,845396$$

$$3\alpha\mathfrak{b} = -\tfrac{15}{8} - \frac{\frac{3}{2}a + \mathfrak{A}(2\kappa a+\mathfrak{a})}{nn} = -2,13900$$

$$\mathfrak{b} = -0,76413 \quad \ldots \quad l-\mathfrak{b} = 9,883167$$

$$(4\alpha\alpha-1)A = -\tfrac{3}{2} + 2\kappa\mathfrak{A} - \frac{\frac{3}{4}B + 12\alpha B(2\kappa A+\mathfrak{A})}{nn} = -3,12379$$

$$A = -1,25826 \quad \ldots \quad l-A = 0,099771$$

$$(1-\alpha\alpha)a = \tfrac{3}{8} - 2\kappa\mathfrak{a} + \frac{\frac{3}{4}(a+b) - 3\alpha A(2\kappa a+\mathfrak{a}) + \alpha A(2\kappa b+\mathfrak{b})}{nn} \quad \text{vel}$$

$$\left(1-\alpha\alpha - \frac{3}{4nn} + \frac{6\alpha\kappa A}{nn}\right) a = \tfrac{3}{8} - 2\kappa\mathfrak{a}$$

$$+ \frac{\frac{3}{4}b - 3\alpha A\mathfrak{a} + \alpha A(2\kappa b+\mathfrak{b})}{nn} = -2,53335$$

hinc $a = +30,989$ et $l\,a = 1,491207$

Vnde patet valorem ipsius $\mathfrak{a}$ ante inuentum non satis esse exactum, exactior ergo prodibit ex hac formula

$$\left(\alpha - \frac{\mathfrak{A}}{nn}\right)\mathfrak{a} = -\tfrac{3}{8} - \frac{\frac{3}{4}(a-b) + \mathfrak{A}(2\kappa a - 2\kappa b - \mathfrak{b})}{nn} = -0,93709$$

hinc $\mathfrak{a} = -0,99939$ et $l-\mathfrak{a} = 9,999735$

vnde etiam exactius valor ipsius a reperitur, ex quo denuo

denuo valor ipsius a corrigetur, sicque tandem satis exacte obtinebitur

$$\mathfrak{a} = -1,2537 \ldots\ l{-}\mathfrak{a} = 0,098200$$
$$a = +44,48 \ldots\ l\,a = 1,648165$$
$$\mathfrak{b} = -0,95003 \ldots\ l{-}\mathfrak{b} = 9,977736$$

§. 70. Hinc iam accuratius quaeramus valorem ipsius $\mathfrak{b}$

$$(9\alpha\alpha - 1)b = -\Psi + 2\alpha\mathfrak{b} - \frac{\frac{1}{2}a + 5\alpha A(2\alpha a + \mathfrak{a})}{\alpha\alpha}$$

$$b = -1,0146 \ldots\ l{-}b = 0,006314$$

vnde si denuo praecedentes valores corrigantur, fiet

$$\mathfrak{a} = -1,2630 \ldots\ l{-}\mathfrak{a} = 0,101403$$
$$\mathfrak{b} = -0,9500 \ldots\ l{-}\mathfrak{b} = 9,977736$$
$$a = +44,525 \ldots\ l\,a = 1,648604$$
$$b = -1,015 \ldots\ l{-}b = 0,006400$$
$$\mathfrak{A} = -0,80378 \ldots\ l{-}\mathfrak{A} = 9,905138$$
$$\mathfrak{B} = +0,00697 \ldots\ l\,\mathfrak{B} = 7,843402$$
$$A = -1,25826 \ldots\ l{-}A = 0,099771$$
$$B = +0,01279 \ldots\ l\,B = 8,106947$$

His autem valoribus inuentis colligitur fore

$$\tfrac{1}{2}\delta - \gamma = +0,01742.$$

Hic autem valor partem insuper accipit cum ab excentricitate vtriusque orbitae, tum ab inclinatione oriundam, quam deinceps determinabimus.

§. 71. Ex his ergo valoribus habebimus:

$$\int R\,dv = -\underset{9,905138}{0,80378}\cos 2\eta + \underset{7,843402}{0,00697}\cos 4\eta$$
$$-\underset{0,101403}{1,2630}\,v\cos\eta - \underset{9,977736}{0,9500}\,v\cos 3\eta$$

 $v =$

$$\begin{array}{llll} v = & -1,25826\cos 2\eta & + & 0,01279\cos 4\eta \\ & \quad 0,099771 & & \quad 8,106947 \\ & +44,525\,\nu\cos\eta & - & 1,015\,\nu\cos 3\eta \\ & \quad 1,648604 & & \quad 0,006400 \end{array}$$

hincque porro

$$\begin{array}{llll} \frac{d\Phi}{dr} = \varkappa & + \; 0,019015\cos 2\eta & - & 0,0000762\cos 4\eta \\ & \quad 8,279096 & & \quad 5,881955 \\ +0,0001103 & & & \\ & - \; 0,50381\,\nu\cos\eta & + & 0,017068\,\nu\cos 3\eta \\ & \quad 9,702270 & & \quad 8,232184 \end{array}$$

at est $\frac{d\eta}{dr} = \frac{d\Phi}{dr} - \frac{1}{n}$, posuimusque $\varkappa - \frac{1}{n} = \alpha$, existente $\varkappa = \sqrt{\left(1 + \frac{3+4\mu+\delta}{2nn}\right)}$

§. 72. Ponatur breuitatis gratia

$$\frac{d\Phi}{dr} = \mathfrak{O} + \mathfrak{P}\cos 2\eta - \mathfrak{Q}\cos 4\eta - \mathfrak{R}\nu\cos\eta + \mathfrak{S}\nu\cos 3\eta$$

erit $\frac{d\eta}{dr} = \alpha + \mathfrak{P}\cos 2\eta - \mathfrak{Q}\cos 4\eta - \mathfrak{R}\nu\cos\eta + \mathfrak{S}\nu\cos 3\eta$

sitque ad integrandum:

$$\Phi = \mathfrak{o}r + \mathfrak{p}\sin 2\eta - \mathfrak{q}\sin 4\eta - \mathfrak{r}\nu\sin\eta + \mathfrak{s}\nu\sin 3\eta$$

vnde per differentiationem elicitur:

$$\begin{array}{lllll} \frac{d\Phi}{dr} = \mathfrak{o} & + 2\alpha\mathfrak{p}\cos 2\eta & - 4\alpha\mathfrak{q}\cos 4\eta & - \alpha\mathfrak{r}\nu\cos\eta & + 3\alpha\mathfrak{s}\nu\cos 3\eta \\ \quad \mathfrak{P}\mathfrak{p} & - \mathfrak{Q}\mathfrak{p}\cos 2\eta & + \mathfrak{P}\mathfrak{p}\cos 4\eta & - \mathfrak{R}\mathfrak{p}\nu\cos\eta & - \mathfrak{R}\mathfrak{p}\nu\cos 3\eta \\ + 2\mathfrak{Q}\mathfrak{q} & - 2\mathfrak{P}\mathfrak{q}\cos 2\eta & & & + 2\mathfrak{R}\mathfrak{q}\nu\cos 3\eta \end{array}$$

hinc ergo fit:

$$\mathfrak{o} = \mathfrak{O} - \mathfrak{P}\mathfrak{p} - 2\mathfrak{Q}\mathfrak{q} = \varkappa + 0,0001103 - \mathfrak{P}\mathfrak{p} - 2\mathfrak{Q}\mathfrak{q}$$

$$(2\alpha - \mathfrak{Q})\mathfrak{p} = \mathfrak{P} + 2\mathfrak{P}\mathfrak{q}; \quad 4\alpha\mathfrak{q} = \mathfrak{Q} + \mathfrak{P}\mathfrak{p};$$

$$\alpha\mathfrak{r} = \mathfrak{R} - \mathfrak{R}\mathfrak{p}; \quad 3\alpha\mathfrak{s} = \mathfrak{S} + \mathfrak{R}(\mathfrak{p} - 2\mathfrak{q})$$

Ergo

Ergo $p = 0,010191$ . . . . $lp = 8,008208$
$q = 0,000072$ . . . $lq = 5,859381$
$r = 0,53453$ . . . $lr = 9,727977$
$s = 0,00790$ . . . $ls = 7,897466$
$b = \varkappa - 0,000080$

§. 73. Longitudo igitur lunae Φ quatenus pendet a sola distantia lunae a sole erit

$$\Phi = (\varkappa - 0,000080)r + 0,010191 \sin 2\eta - 0,53453\, v \sin \eta - 0,000072 \sin 4\eta + 0,00790\, v \sin 3\eta$$

Simili modo cum distantia lunae a terra posita sit $= \frac{a(1-kk)u}{1-k\cos r}$, ob $u = 1 + \frac{v}{nn}$, quatenus valor ipsius u a sola phasi lunae pendet, erit

$$u = 1 - 0,00716 \cos 2\eta + 0,00287\, v \cos \eta + 0,00007 \cos 4\eta - 0,00009\, v \cos 3\eta$$

Verum tamen hic valor litterae a ac praecipue ipsius $\varkappa$ non admodum certus videtur, cum a terminis neglectis licet minimis insignem mutationem perpeti queat. Hic enim pro $\varkappa$ non solum $\varkappa - \frac{1}{n}$ sed $\varkappa - \frac{1}{n} + 0,0001103$ accipi debuisset; quare cum valorem ipsius $\varkappa$ propius cognoscimus, hanc determinationem repeti conueniet.

CAPUT V.

INUESTIGATIO INAEQUALITATUM LUNAE AB EIUS EXCENTRICITATE SIMPLICI SOLUM PEDENTIUM.

§. 74.

Quemadmodum in praecedenti capite inaequalitas abſoluta ſeu variatio duabus partibus conſtans eſt inuenta, quarum poſterior a littera ν ſeu a parallaxi ſolis pendebat, ac maiorem curam requirebat; ita etiam inaequalitates, quas hoc capite ſcrutamur, partes continent ab eadem parallaxi ſolis pendentes; quarum indagatio quoque accuratiorem cognitionem quorundam elementorum exigit. Hancobrem et praecedentis capitis et huius partes, quae litteram ν inuoluunt deinceps, cum reliquas inaequalitates, a parallaxi ſolis non pendentes determinauerimus, ſeorſim inueſtigabimus, atque titulo inaequalitatum parallacticarum complectemur.

§. 75. In hoc ergo capite ac ſequentibus, donec ad parallaxin ſolis perueniamus, terminos formularum noſtrarum per ν multiplicatos tantisper remouebimus; et quoniam hoc loco tantum propoſitum eſt in motus lunae inaequalitates a ſola excentricitate orbitae lunaris ortas inquirere, eos terminos qui vel excentricitatem ſolis e vel inclinationem ρ continent, praetermittemus. Cum autem in formulis noſtris duplicis generis termini relinquantur, quorum alteri per k, alteri per kk ſunt affecti, inaequalitates ab excentricitate lunae k pendentes

in

in duas partes distribui conueniet; quarum altera excentricitatem tantum simplicem k implicet, cui hoc caput destinatur, altera vero excentricitatis huius quadrato kk afficiatur, de quo in sequenti capite agemus.

§. 76. Verum tam in huius generis inaequalitates, quam in sequentes, omnes inaequalitates absolutae in praecedenti capite erutae praecipue ingrediuntur; ex quo eas quoque in calculum introduci oportebit. Retinendae ergo erunt in calculo litterae $\mathfrak{A}$, $\mathfrak{B}$ et A, B, quarum valores cum iam constent, calculus vehementer contrahetur: imprimis autem quia valores litterarum $\mathfrak{B}$ et B per se sunt admodum parui, quatenus illi in valores sequentium terminorum influunt, effectum pro nihilo habendum praestabunt. Inuestigationem ergo nostram ita incipiemus, vt pro $\int R\,dr$ et v valores fictos assumamus, et quoniam $\int R\,dr$ nullum terminum constantem, v vero neque constantem neque terminum huius formae $\alpha \cos r$ continere debet, ponamus:

$$\begin{aligned}\int R\,dr = {} & \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos 4\eta + \mathfrak{C}k\cos r \\ & + \mathfrak{D}k\cos(2\eta - r) + \mathfrak{F}k\cos(4\eta - r) \\ & + \mathfrak{E}k\cos(2\eta + r) + \mathfrak{G}k\cos(4\eta + r)\end{aligned}$$

$$\begin{aligned}v = {} & A\cos 2\eta + B\cos 4\eta \\ & + Dk\cos(2\eta + r) + Fk\cos(4\eta - r) \\ & + Ek\cos(2\eta + r) + Gk\cos(4\eta + r)\end{aligned}$$

vbi quidem facile colligere licet, coefficientes $\mathfrak{F}$, $\mathfrak{G}$, F et G fore minimos.

§. 77. Ex his autem valoribus assumtis obtinebimus ex (§. 52.) sequentes expressiones.

$$\frac{d\Phi}{dr} = \begin{cases} \varkappa + \frac{A(3\varkappa A + 2\mathfrak{A})}{2\varkappa^4} - \frac{(2\varkappa A + \mathfrak{A})}{\varkappa\varkappa}\cos 2\eta \\ \left(-\frac{(2\varkappa B + \mathfrak{B})}{\varkappa\varkappa} + \frac{A(3\varkappa A + 2\mathfrak{A})}{2\varkappa^4}\right)\cos 4\eta \\ \left(-\frac{\mathfrak{C}}{\varkappa\varkappa} + \frac{3\varkappa AD}{\varkappa^4} + \frac{A\mathfrak{D} + \mathfrak{A}D}{\varkappa^4}\right)k\cos r \\ -\frac{(2\varkappa D + \mathfrak{D})}{\varkappa\varkappa}k\cos(2\eta - r) - \frac{(2\varkappa E + \mathfrak{E})}{\varkappa\varkappa}k\cos(2\eta + r) \\ -\frac{(2\varkappa F + \mathfrak{F})}{\varkappa\varkappa}k\cos(4\eta - r) - \frac{(2\varkappa G + \mathfrak{G})}{\varkappa\varkappa}k\cos(4\eta + r) \\ + \frac{(3\varkappa AD + A\mathfrak{D} + \mathfrak{A}D)}{\varkappa^4}k\cos(4\eta - r) \end{cases}$$

Patebit enim ex valoribus qui inuenientur, litteras D et $\mathfrak{D}$ tantum prae reliquis fore notabiles, vnde terminos ex combinatione reliquarum litterarum oriundos tuto omittere licet. Pro valore autem ipsius $\frac{d\eta}{dr}$ etiam hi termini ex combinatione orti omitti poterunt. Posito ergo $\varkappa + \frac{A(3\varkappa A + 2\mathfrak{A})}{2\varkappa^4} - \frac{1}{n}$ seu $\varkappa + 0,000103 - \frac{1}{n} = \alpha$ erit:

$$\frac{d\eta}{dr} = \alpha - \frac{(2\varkappa A + \mathfrak{A})}{\varkappa\varkappa}\cos 2\eta - \frac{2k}{n}\cos r$$
$$- \frac{(2\varkappa D + \mathfrak{D})}{\varkappa\varkappa}k\cos(2\eta - r) - \frac{(2\varkappa E + \mathfrak{E})}{\varkappa\varkappa}k\cos(2\eta + r)$$

Cum enim haec formula differentiationibus instituendis inseruiat, reliqui termini post primum cum aliis angulis combinantur, sicque tanto minores terminos producunt, ount, qui ex calculo ſine errore expungi poterunt: atque ob hanc cauſam in expreſſione valoris $\frac{d\eta}{dr}$, ſtatim terminos prae reliquis admodum paruos praetermittere viſum eſt.

§. 78. Valorem autem ipſius R atque $\frac{ddv}{dr^2}$ accuratiſſime exhiberi oportet, propterea quod his expreſſionibus totus calculus praecipue innititur, dum valor $\frac{d\eta}{dr}$ formulam tantum ſubſidiariam ſuppeditat. Erit ergo

$$
\begin{aligned}
R = & \tfrac{3}{2} \sin 2\eta + \frac{3A}{2nn} \sin 4\eta + 3k \sin(2\eta - r) + 3k \sin(2\eta + r) \\
& + \frac{3A}{nn} k \sin(4\eta - r) \quad + \frac{3A}{nn} k \sin(4\eta + r) \\
& + \frac{3D}{2nn} k \sin r \quad - \frac{3E}{2nn} k \sin r \\
& + \frac{3D}{2nn} k \sin(4\eta - r) \quad + \frac{3E}{2nn} k \sin(4\eta + r) \\
& - \frac{3A}{nn} k \sin r \quad + \frac{3A}{nn} k \sin r
\end{aligned}
$$

vbi quidem terminos ab k non pendentes omittere poſſumus, quia illorum iam habuimus rationem, ita vt ſit

$$
\begin{aligned}
R = \ldots\ldots & + \frac{3(D-E)}{2nn} k \sin r + 3k \sin(2\eta - r) + 3k \sin(2\eta + r) \\
& + \frac{3D}{2nn} k \sin(4\eta - r) + \frac{3E}{2nn} k \sin(4\eta + r) \\
& + \frac{3A}{nn} k \sin(4\eta - r) + \frac{3A}{nn} k \sin(4\eta + r)
\end{aligned}
$$

§. 79. Simili modo terminis a k non pendentibus omittendis habebitur:

$$\frac{ddv}{dr^2} = -\gamma k\,\mathrm{cs}\,r + 3k\,\mathrm{cs}(2\eta - r) + 3k\,\mathrm{cs}(2\eta + r) - 2\kappa\mathfrak{F}k\,\mathrm{cs}(4\eta - r) - 2\kappa\mathfrak{G}k\,\mathrm{cs}(4\eta + r)$$

$$\begin{array}{lllll}
-2\kappa\mathfrak{E} & -2\kappa\mathfrak{D} & -2\kappa\mathfrak{E} & +\frac{\mathfrak{A}\mathfrak{D}}{2nn} & +\frac{\mathfrak{A}\mathfrak{E}}{2nn} \\
+\frac{\mathfrak{A}\mathfrak{D}}{2nn} & +\frac{\mathfrak{A}\mathfrak{E}}{2nn} & +\frac{\mathfrak{A}\mathfrak{E}}{2nn} & & \\
+\frac{\mathfrak{A}\mathfrak{E}}{2nn} & -D & -E & -F & -G \\
+\frac{3AD}{nn} & +\frac{3}{2}A & +\frac{1}{2}A & +\frac{3}{2}B & +\frac{1}{2}B \\
+\frac{3AE}{nn} & +\frac{(2\gamma-\frac{3}{2}\delta)}{nn}D & +\frac{(2\gamma-\frac{3}{2}\delta)}{nn}E & +\frac{3AD}{nn} & +\frac{3AE}{nn} \\
-\frac{3AA}{2nn} & +\frac{(3+3\mu+\gamma)}{nn}A & +\frac{(3+3\mu+\gamma)}{nn}A & -\frac{3AA}{4nn} & -\frac{3AA}{4nn} \\
+\frac{3A\mathfrak{D}}{nn} & & & +\frac{3A\mathfrak{D}}{nn} & +\frac{3A\mathfrak{E}}{nn} \\
+\frac{3A\mathfrak{E}}{nn} & & & +\frac{3\mathfrak{A}D}{nn} & +\frac{3\mathfrak{A}E}{nn} \\
+\frac{3\mathfrak{A}D}{nn} & & & & \\
+\frac{3\mathfrak{A}E}{nn} & & & &
\end{array}$$

Hic ſcilicet plures terminos, qui nullius futuri eſſent momenti, omiſimus, ne calculus nimium implicaretur: notandum autem eſt eſſe $\kappa = \sqrt{\left(1 + \frac{3+4\mu+\delta}{2nn}\right)} = 1 + \frac{3+4\mu+\delta}{4nn}$ proxime; vnde $\mu = (\kappa - 1)nn - \frac{3-\delta}{4}$ et $\frac{3+3\mu+\gamma}{4nn}$ $= 3(\kappa - 1) + \frac{3-3\delta+4\gamma}{4nn}$.

§. 80.

§. 80. Quaeramus nunc quoque ex forma pro $\int R\, dr$ ficta valorem ipsius R, atque exclusis terminis ab k non pendentibus reperiemus:

$$
R = \begin{cases}
\left(-\frac{\mathfrak{A}(2\kappa E+\mathfrak{E})}{nn}+\frac{\mathfrak{A}(2\kappa D+\mathfrak{D})}{nn}-\mathfrak{C}-\frac{\mathfrak{D}(2\kappa A+\mathfrak{A})}{nn}+\frac{\mathfrak{E}(2\kappa A+\mathfrak{A})}{nn}\right) k \sin r \\
\left(+\frac{2\mathfrak{A}}{n}-(2\alpha-1)\mathfrak{D}\right) k \sin(2\eta-r) \\
\left(+\frac{2\mathfrak{A}}{n}-(2\alpha+1)\mathfrak{E}\right) k \sin(2\eta+r) \\
\left(+\frac{\mathfrak{A}(2\kappa D+\mathfrak{D})}{nn}+\frac{4\mathfrak{B}}{n}+\frac{\mathfrak{D}(2\kappa A+\mathfrak{A})}{nn}-(4\alpha-1)\mathfrak{F}\right) k \sin(4\eta-r) \\
\left(+\frac{\mathfrak{A}(2\kappa E+\mathfrak{E})}{nn}+\frac{4\mathfrak{B}}{n}+\frac{\mathfrak{E}(2\kappa A+\mathfrak{A})}{nn}-(4\alpha+1)\mathfrak{G}\right) k \sin(4\eta+r)
\end{cases}
$$

atque instituta comparatione inuenietur:

$$\mathfrak{C} = \frac{\mathfrak{A}(2\kappa D+\mathfrak{D})-(\mathfrak{D}-\mathfrak{E})(2\kappa A+\mathfrak{A})-\frac{3}{2}(D-E)-\mathfrak{A}(2\kappa E+\mathfrak{E})}{nn}$$

$$(2\alpha-1)\mathfrak{D} = \frac{2\mathfrak{A}}{n} - 3: \quad (2\alpha+1)\mathfrak{E} = \frac{2\mathfrak{A}}{n} - 3;$$

$$(4\alpha-1)\mathfrak{F} = \frac{4\mathfrak{B}}{n} + \frac{\mathfrak{A}(2\kappa D+\mathfrak{D})+\mathfrak{D}(2\kappa A+\mathfrak{A})-\frac{3}{2}(2A+D)}{nn}$$

$$(4\alpha+1)\mathfrak{G} = \frac{4\mathfrak{B}}{n} + \frac{\mathfrak{A}(\kappa E+\mathfrak{E})-\frac{3}{2}(2A+E)+\mathfrak{E}(2\kappa A+\mathfrak{A})}{nn}$$

§. 81. Pro differentiali $\frac{dv}{dr}$ inueniendo, praeter terminos supra inventos habebimus:

$$\frac{dv}{dr} = -A' \sin 2\eta - B' \sin 4\eta$$

$$\left(+\frac{A(2\kappa D+\mathfrak{D})}{nn}-\frac{A(2\kappa E+\mathfrak{E})}{nn}-\frac{D(2\kappa A+\mathfrak{A})}{nn}+\frac{E(2\kappa A+\mathfrak{A})}{nn}\right) k \sin r$$

$$\left(+\frac{2A}{n}-(2\alpha-1)D\right) k \sin(2\eta-r) + \left(\frac{2A}{n}-(2\alpha+1)E\right) k \sin(2\eta+r)$$

 +

$$(+\frac{A(2\varkappa D+\mathfrak{D})}{n}+\frac{4B}{n}+\frac{D(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha-1)F)k\sin(4\eta-r)$$

$$(+\frac{A(2\varkappa E+\mathfrak{E})}{nn}+\frac{4B}{n}+\frac{E(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha+1)G)k\sin(4\eta+r)$$

Ponatur autem breuitatis gratia:

$$\frac{dv}{dr}=-A'\sin 2\eta-B'\sin 4\eta-C'k\sin r-D'k\sin(2\eta-r)$$
$$-E'k\sin(2\eta+r)-F'k\sin(4\eta-r)-G'k\sin(4\eta+r)$$

vt sit:

$$A'=2A\alpha+\frac{A(2\varkappa B+\mathfrak{B})-2B(2\varkappa A+\mathfrak{A})}{nn};\ B'=4\alpha B-\frac{A(2\varkappa A+\mathfrak{A})}{nn}$$

$$C'=\frac{-A(2\varkappa D+\mathfrak{D})+A(2\varkappa E+\mathfrak{E})+(D-E)(2\varkappa A+\mathfrak{A})}{nn}$$

siue $C'=-\frac{A(\mathfrak{D}-\mathfrak{E})+\mathfrak{A}(D-E)}{nn}$

$$D'=(2\alpha-1)D-\frac{2A}{n};\quad E'=(2\alpha+1)E-\frac{2A}{n}$$

$$F'=(4\alpha-1)F-\frac{4B}{n}-\frac{A(2\varkappa D+\mathfrak{D})-D(2\varkappa A+\mathfrak{A})}{nn}$$

$$G'=(4\alpha+1)G-\frac{4B}{n}-\frac{A(2\varkappa E+\mathfrak{E})-E(2\varkappa A+\mathfrak{A})}{nn}$$

§. 82. Hinc denuo differentiando obtinebitur terminis tantum per k multiplicatis scribendis:

$$\frac{ddv}{dr^2}=(-C'+\frac{A'(2\varkappa D+\mathfrak{D})}{nn}+\frac{A'(2\varkappa E+\mathfrak{E})}{nn}+\frac{D'(2\varkappa A+\mathfrak{A})}{nn}+\frac{E'(2\varkappa A+\mathfrak{A})}{nn})k\cos r$$

$$(+\frac{2A'}{n}-(2\alpha-1)D')k\cos(2\eta-r)$$

$$(+\frac{2A'}{n}-(2\alpha+1)E')k\cos(2\eta+r)$$

+

$$\left(+\frac{4B'}{n}+\frac{A'(2\varkappa D+\mathfrak{D})}{nn}+\frac{D'(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha-1)F'\right)k\cos(4\eta-r)$$

$$\left(+\frac{4B'}{n}+\frac{A'(2\varkappa E+\mathfrak{E})}{nn}+\frac{E'(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha+1)G'\right)k\cos(4\eta+r)$$

vnde comparatione instituta orietur:

$$\gamma=-2\varkappa\mathfrak{G}-\frac{\frac{3}{4}AA+3A(D+E)+2A(\mathfrak{D}+2\mathfrak{E})+2\mathfrak{A}(2D+E)+\frac{1}{2}\mathfrak{A}(\mathfrak{D}+\mathfrak{E})}{nn}$$

$$-\frac{A'(2\varkappa D+\mathfrak{D})-A'(2\varkappa E+\mathfrak{E})-(D'+E')(2\varkappa A+\mathfrak{A})}{nn}$$

$$\left.\begin{array}{l}(2\alpha-1)^2D-\frac{2(2\alpha-1)}{n}A-\frac{2A'}{n}+3-2\varkappa\mathfrak{D}-D\\ +\frac{3}{2}A+\frac{\frac{1}{2}\mathfrak{A}\mathfrak{E}+(2\gamma-\frac{3}{2}\delta)D+(3+3\mu+\gamma)A}{nn}\end{array}\right\}=0$$

$$\left.\begin{array}{l}(2\alpha+1)^2E-\frac{2(2\alpha+1)}{n}A-\frac{2A'}{n}+3-2\varkappa\mathfrak{E}-E\\ +\frac{3}{2}A+\frac{\frac{1}{2}\mathfrak{A}\mathfrak{E}+(2\gamma-\frac{3}{2}\delta)E+(3+3\mu+\gamma)A}{nn}\end{array}\right\}=0$$

$$\left.\begin{array}{l}(4\alpha-1)^2F-\frac{4(4\alpha-1)}{n}B-\frac{(4\alpha-1)A(2\varkappa D+\mathfrak{D})-(4\alpha-1)D(2\varkappa A+\mathfrak{A})}{nn}\\ -\frac{4B'}{n}-\frac{A'(2\varkappa D+\mathfrak{D})-D'(2\varkappa A+\mathfrak{A})}{nn}-2\varkappa\mathfrak{F}-F\\ +\frac{3}{2}B+\frac{\frac{1}{2}\mathfrak{A}\mathfrak{D}+3AD-\frac{3}{4}AA+3A\mathfrak{D}+3\mathfrak{A}D}{nn}\end{array}\right\}=0$$

$$\left.\begin{array}{l}(4\alpha+1)^2G-\frac{4(4\alpha+1)}{n}B-\frac{(4\alpha+1)A(2\varkappa E+\mathfrak{E})-(4\alpha+1)E(2\varkappa A+\mathfrak{A})}{nn}\\ -\frac{4B'}{n}-\frac{A'(2\varkappa E+\mathfrak{E})-E'(2\varkappa A+\mathfrak{A})}{nn}-2\varkappa\mathfrak{G}-G\\ +\frac{3}{2}B+\frac{\frac{1}{2}\mathfrak{A}\mathfrak{E}+3AE-\frac{3}{4}AA+3A\mathfrak{E}+3\mathfrak{A}E}{nn}\end{array}\right\}=0$$

§. 83. Incipiamus a coefficientibus $\mathfrak{D}$, $\mathfrak{E}$, et D,E; et quia $\mathfrak{E}$ est quantitas admodum exigua, erit:

$$(2\alpha-1)\mathfrak{D} = -3 + \frac{2\mathfrak{A}}{n}; \quad (2\alpha+1)\mathfrak{E} = -3 + \frac{2\mathfrak{A}}{n}$$

$$\left((2\alpha-1)^2 - 1 + \frac{2\gamma-\frac{3}{2}\delta}{nn}\right)D = -3 - \tfrac{1}{2}A + 2\alpha\mathfrak{D}$$
$$+ 2\frac{(2\alpha-1)}{n}A + \frac{2A'}{n} - \left(3\alpha - 3 + \frac{3-3\delta+4\gamma}{4nn}\right)A$$

$$\left((2\alpha+1)^2 - 1 + \frac{2\gamma-\frac{3}{2}\delta}{nn}\right)E = -3 - \tfrac{1}{2}A + 2\alpha\mathfrak{E}$$
$$+ 2\frac{(2\alpha+1)}{n}A + \frac{2A'}{n} - \left(3\alpha - 3 + \frac{3-3\delta+4\gamma}{4nn}\right)A$$

vnde reperitur:

$\mathfrak{D} = -3,6035$ . . . $l-\mathfrak{D} = 0,556724$

$\mathfrak{E} = -1,0890$ - - - $l-\mathfrak{E} = 0,037028$

ac porro

$$\left.\begin{array}{l}\left(-0,24973+\frac{(2\gamma-\frac{3}{2}\delta)}{nn}\right)D = -1,40048-7,4315 \\ \left(+7,21497+\frac{(2\gamma-\frac{3}{2}\delta)}{nn}\right)E = -1,40048-2,7403\end{array}\right\} + \frac{(2\gamma-\frac{3}{2}\delta)}{nn}.0,629$$

§. 84. Quoniam autem valorem ipsius $\frac{2\gamma-\frac{3}{2}\delta}{nn}$ nondum nouimus, hunc terminum, cum certo sit valde paruus, reiiciamus. Postmodum vero cum istum terminum cognouerimus, facile erit correctionem inde oriundam, si operae pretium videbitur, inuenire.

$D = +35,3662$ . . . $l\,D = 1,548588$

$E = -0,5739$ . . . $l-E = 9,758848$

Porro autem litterae $\mathfrak{F}$ et $\mathfrak{G}$ ita elicientur, vt sit.

$(4\alpha-1)\mathfrak{F} = 0,000529 - 0,30976 + 0,06852 - 0,28042$

$(4\alpha+1)\mathfrak{G} = 0,000529 + 0,01028 + 0,02071 + 0,02638$

$\mathfrak{F} =$

$\mathfrak{F} = -0,1907 \quad \ldots \quad l-\mathfrak{F} = 9,280416$

$\mathfrak{G} = +0,0122 \quad \ldots \quad l\,\mathfrak{G} = 8,087607$

Praeterea autem colligimus fore

$\mathfrak{E} = -0,67465 \quad \ldots \quad l-\mathfrak{E} = 9,829072$

vnde erit proxime $\frac{\mathfrak{A}\mathfrak{E}}{2mm} = 0,09154$, ex quo accuratius concluditur fore

$D = +35,3724 \quad \ldots \quad l\,D = 1,548664$

$E = -0,5741 \quad \ldots \quad l-E = 9,758988$

§. 85. Reliquae aequationes nobis praebebunt

$$6,4655\,F + 3,67820 = 0$$

$$21,3946\,G - 0,29574 = 0$$

vnde obtinebitur

$F = -0,56890 \quad ; \quad l-F = 9,755033$

$G = +0,01382 \quad . \quad l\,G = 8,140620$

ac denique $\gamma = 1,40673$.

Supra autem iam inuenimus $\frac{1}{2}\delta - \gamma = 0,01742$, vnde ambas istas quantitates γ et δ, quas initio ad veros valores constantium litterarum m et n determinandos assumsimus, nunc cognitas habemus, erit enim:

$$\delta = 2,84830, \quad \text{et} \quad 2\gamma - \tfrac{1}{2}\delta = -1,45899$$

ac propterea particulae illius $\frac{2\gamma - \frac{1}{2}\delta}{nn}$ hactenus neglectae valor erit $\frac{2\gamma - \frac{1}{2}\delta}{nn} = -0,00832$, cuius ope iam litterae D et E accuratius definiri poterunt.

§. 86.

§. 86. Hinc autem potissimum valor ipsius D mutationem patitur, fiet enim re vera

$$-0,25805\ D = -8,83698 \text{ seu}$$

$$D = 34,24520 \quad \ldots \quad l\,D = 1,534600$$

$$7,20665\ E = -4,14578 \text{ seu}$$

$$E = -0,57527 \quad \ldots \quad l{-}E = 9,759874$$

et quoniam D parte sua tricesima diminuitur, in eadem fere ratione diminuentur valores litterarum $\mathfrak{C}$ et γ, ita vt exactius sit:

$$\mathfrak{C} = -0,65217 \quad \ldots \quad l{-}\mathfrak{C} = 9,814361$$

$$\gamma = +1,35984 \quad \ldots \quad l\,\gamma = 0,133490$$

$$\delta = +2,75336 \quad \ldots \quad l\,\delta = 0,439863$$

$$\text{et } \frac{2\gamma - \frac{3}{2}\delta}{nn} = -0,00804$$

Deinceps autem operae erit pretium in hos valores adhuc diligentius inquirere.

§. 87. Cum igitur finxerimus sequentes valores:

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos 4\eta + \mathfrak{C}\,k\cos r$$
$$+ \mathfrak{D}\,k\cos(2\eta - r) + \mathfrak{F}\,k\cos(4\eta - r)$$
$$+ \mathfrak{E}\,k\cos(2\eta + r) + \mathfrak{G}\,k\cos(4\eta + r)$$

$$v = A\cos 2\eta + B\cos 4\eta$$
$$+ D\,k\cos(2\eta - r) + F\,k\cos(4\eta - r)$$
$$+ E\,k\cos(2\eta + r) + G\,k\cos(4\eta + r)$$

horum coefficientium valores sunt.

$\mathfrak{A} = -0,80378$	$A = -1,25826$
$\mathfrak{B} = +0,00697$	$B = -0,01279$
$\mathfrak{C} = -0,65217$	. .
$\mathfrak{D} = -3,60350$	$D = +34,24520$
$\mathfrak{E} = -1,08900$	$E = -0,57527$
$\mathfrak{F} = -0,19070$	$F = -0,56890$
$\mathfrak{G} = +0,01220$	$G = +0,01382$

vnde

vnde pro distantia lunae a terra $x = \frac{(1-kk)aa}{1-k\cos r}$ fit

$$x = 1 - 0{,}007161 \cos 2\eta + 0{,}000073 \cos 4\eta$$
$$+ 0{,}194888\, k\cos(2\eta - r) - 0{,}003274\, k\cos(2\eta + r)$$
$$- 0{,}003238\, k\cos(4\eta - r) + 0{,}000078\, k\cos(4\eta + r)$$

§. 88. His valoribus in §. 77. substitutis obtinebimus:

$$\frac{d\Phi}{dr} = \quad x \quad + 0{,}019015 \cos 2\eta - 0{,}001255\, k\cos r$$
$$+ 0{,}0001103 - 0{,}000076 \cos 4\eta$$
$$- 0{,}38410\, k\cos(2\eta - r) + 0{,}01278\, k\cos(2\eta + r)$$
$$+ 0{,}002647\, k\cos(4\eta - r) - 0{,}000229\, k\cos(4\eta + r)$$

ad cuius integrale inueniendum ponamus:

$$\Phi = \mathfrak{O} r + \mathfrak{A}' \sin 2\eta + \mathfrak{B}' \sin 4\eta + \mathfrak{C}' k \sin r$$
$$+ \mathfrak{D}' k \cos(2\eta - r) + \mathfrak{E}' k \cos(2\eta + r)$$
$$+ \mathfrak{F}' k \cos(4\eta - r) + \mathfrak{G}' k \cos(4\eta + r)$$

eritque differentiando et terminis iam cognitis omittendis.

$$\frac{d\Phi}{dr} = \left(\mathfrak{C}' - \frac{\mathfrak{A}'(2x\mathrm{D} + \mathfrak{D})}{nn} - \frac{\mathfrak{A}'(2x\mathrm{E} + \mathfrak{E})}{nn} - \frac{\mathfrak{D}'(2x\mathrm{A} + \mathfrak{A})}{nn} - \frac{\mathfrak{E}'(2x\mathrm{A} + \mathfrak{A})}{nn}\right) k\cos r$$

$$\left(-\frac{2\mathfrak{A}'}{n} + (2n - 1)\mathfrak{D}'\right) k\cos(2\eta - r)$$

$$\left(-\frac{2\mathfrak{A}'}{n} + (2n + 1)\mathfrak{E}'\right) k\cos(2\eta + r)$$

$$\left(-\frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2x\mathrm{D} + \mathfrak{D})}{nn} - \frac{\mathfrak{D}'(2x\mathrm{A} + \mathfrak{A})}{nn} + (4n - 1)\mathfrak{F}'\right) k\cos(4\eta - r)$$

$$\left(-\frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2x\mathrm{E} + \mathfrak{E})}{nn} - \frac{\mathfrak{E}'(2x\mathrm{A} + \mathfrak{A})}{nn} + (4n + 1)\mathfrak{G}'\right) k\cos(4\eta + r)$$

Pro terminis autem iam inuentis est

$$\mathfrak{O} = x - 0{,}000080; \quad \mathfrak{A}' = 0{,}010191; \quad \mathfrak{B}' = -0{,}000072$$
$$l\mathfrak{A}' = 8{,}008208; \quad l\mathfrak{B}' = -5{,}859381$$

§. 89. Comparatione iam instituta fiet:

$$(2\alpha-1)\mathfrak{D}' = -0,38410 + \frac{2\mathfrak{A}'}{n}; \quad (2\alpha+1)\mathfrak{E}' = +0,01278 + \frac{2\mathfrak{A}'}{n}$$

$$(4\alpha-1)\mathfrak{F}' = +0,002647 + \frac{4\mathfrak{B}'}{n} + \frac{\mathfrak{A}'(2\kappa D + \mathfrak{D}) + \mathfrak{D}'(2\kappa A + \mathfrak{A})}{nn}$$

$$(4\alpha+1)\mathfrak{G}' = -0,000229 + \frac{4\mathfrak{B}'}{n} + \frac{\mathfrak{A}'(2\kappa E + \mathfrak{E}) + \mathfrak{E}'(2\kappa A + \mathfrak{A})}{nn}$$

$$\mathfrak{C}' = -0,001255 + \frac{\mathfrak{A}'(2\kappa D + \mathfrak{D}) + \mathfrak{D}'(2\kappa A + \mathfrak{A})}{nn} + \frac{\mathfrak{A}'(2\kappa E + \mathfrak{E}) + \mathfrak{E}'(2\kappa A + \mathfrak{A})}{nn}$$

vnde colligitur fore

$\mathfrak{C}' = +0,01083$

$\mathfrak{D}' = -0,44167$ . . . $l-\mathfrak{D}' = 9,645092$

$\mathfrak{E}' = +0,00499$ . . . $l\,\mathfrak{E}' = 7,698640$

$\mathfrak{F}' = +0,00546$ . . . $l\,\mathfrak{F}' = 7,737733$

$\mathfrak{G}' = -0,00010$ . . . $l-\mathfrak{G}' = 6,002537$

ita vt sit

$$\begin{aligned}\Phi = (\kappa - 0,000080 r) &+ 0,010191 \sin 2\eta + 0,01083\, k \sin r \\ &- 0,000072 \sin 4\eta \\ &- 0,44167\, k \sin(2\eta - r) + 0,00499\, k \sin(2\eta + r) \\ &+ 0,00546\, k \sin(4\eta - r) - 0,00010\, k \sin(4\eta + r)\end{aligned}$$

vnde ex comparatione motus medii ad modum anomaliae erit $\kappa = 1,008607$, et $\alpha = 0,933279$, qui valores iam propius ad veritatem accedunt, quam hactenus vsurpati.

CAPUT

CAPUT VI.

INUESTIGATIO INAEQUALITATUM LUNAE A QUADRATO EXCENTRICITATIS IPSIUS ORTARUM.

§. 90.

Peruenimus nunc ad alteram partem inaequalitatum in motu Lunae, quae ab eius excentricitate k pendent, eiusque quadratum inuoluunt, ita vt hic nonnisi eos terminos simus contemplaturi, qui per quadratum excentricitatis lunae kk sunt multiplicati. Hic autem tam in valorem ipsius $\int R dr$, quam ipsius v termini formae $kk \cos 2\eta$ et $kk \cos 4\eta$ ingredientur, qui postquam fuerint inuenti, terminis huius generis iam ante inuentis adiici debent: praeterea vero vtrinque etiam termini formae $kk \cos 2r$ accedent. Hinc ponamus:

$$\begin{aligned} \int R dr = {} & \mathfrak{A} \cos 2\eta + \mathfrak{a}\, kk \cos 2\eta + \mathfrak{B} \cos 4\eta + \mathfrak{b}\, kk \cos 4\eta \\ & + \mathfrak{C}\, k \cos r + \mathfrak{D}\, k \cos(2\eta - r) + \mathfrak{E}\, k \cos(2\eta + r) \\ & + \mathfrak{F}\, k \cos(4\eta - r) + \mathfrak{G}\, k \cos(4\eta + r) \\ & + \mathfrak{H}\, kk \cos 2r + \mathfrak{I}\, kk \cos(2\eta - 2r) + \mathfrak{K}\, kk \cos(2\eta + 2r) \\ & + \mathfrak{L}\, kk \cos(4\eta - 2r) + \mathfrak{M}\, kk \cos(4\eta + 2r) \end{aligned}$$

$$\begin{aligned} v = {} & A \cos 2\eta + a\, kk \cos 2\eta + B \cos 4\eta + b\, kk \cos 4\eta \\ & + D\, k \cos(2\eta - r) + E\, k \cos(2\eta + r) \\ & + F\, k \cos(4\eta - r) + G\, k \cos(4\eta + r) \\ & + H\, kk \cos 2r + J\, kk \cos(2\eta - 2r) + K\, kk \cos(2\eta + 2r) \\ & + L\, kk \cos(4\eta - 2r) + M\, kk \cos(4\eta + 2r) \end{aligned}$$

§. 91. Nunc ad terminos, quibus ante valorem ipſius $\frac{d\Phi}{dr}$ exprimi inuenimus, inſuper ſequentes per kk multiplicati accedent:

$$\frac{d\Phi}{dr} = \ldots\ldots + \frac{D(3\kappa D + 2\mathfrak{D})}{2n^4}k^2 + \left(-\frac{(2\kappa a + \mathfrak{a})}{nn} + \frac{\mathfrak{E}D}{n^4}\right)k^2 \operatorname{cof} 2\eta$$

$$\left(-\frac{(2\kappa b + \mathfrak{b})}{nn} + \frac{D(3\kappa E + 2\mathfrak{E})}{2n^4} + \frac{E(3\kappa D + 2\mathfrak{D})}{2n^4}\right)k^2 \operatorname{cof} 4\eta$$

$$\left\{\begin{matrix} -\frac{(2\kappa H + \mathfrak{H})}{nn} + \frac{D(3\kappa E + 2\mathfrak{E})}{2n^4} + \frac{E(3\kappa D + 2\mathfrak{D})}{2n^4} \\ + \frac{A(3\kappa J + 2\mathfrak{J})}{2n^4} + \frac{J(3\kappa A + 2\mathfrak{A})}{2n^4} \end{matrix}\right\}k^2 \operatorname{cof} 2r$$

$$\left(-\frac{(2\kappa J + \mathfrak{J})}{nn} + \frac{A(3\kappa H + 2\mathfrak{H})}{2n^4} + \frac{H(3\kappa A + 2\mathfrak{A})}{2n^4} + \frac{\mathfrak{E}D}{n^4}\right)k^2 \operatorname{cof}(2\eta - 2r)$$

$$\left(-\frac{(2\kappa K + \mathfrak{K})}{nn} + \frac{A(3\kappa H + 2\mathfrak{H})}{2n^4} + \frac{H(3\kappa A + 2\mathfrak{A})}{2n^2}\right)k \operatorname{cf}(42\eta + 2r)$$

$$\left(-\frac{(2\kappa L + \mathfrak{L})}{nn} + \frac{A(3\kappa J + 2\mathfrak{J})}{2n^4} + \frac{J(3\kappa A + 2\mathfrak{A})}{2n^4} + \frac{D(3\kappa D + 2\mathfrak{D})}{2n^4}\right)k^2 \operatorname{cf}(4\eta - 2r)$$

$$-\frac{(2\kappa M + \mathfrak{M})}{nn}k^2 \operatorname{cof}(4\eta + 2r)$$

vbi quidem terminos, quos minimos fore facile eſt praeuidere, omiſimus.

§. 92. Terminus autem conſtans $\frac{D(3\kappa D + 2\mathfrak{D})kk}{2n^4}$ reperitur $= 0{,}000175$, vnde poſito $\kappa + 0{,}000285 - \frac{1}{n} = \alpha$, quoniam valorem ipſius $\frac{d\eta}{dr}$ non opus eſt tam exacte noſſe, ſumamus:

$\frac{d\eta}{dr}$

$$\frac{d\eta}{dr} = \alpha - \frac{(2nA+\mathfrak{A})}{nn}\cos 2\eta - \left(\frac{2k}{n} + \frac{\mathfrak{C}k}{nn}\right)\cos r$$

$$- \frac{(2nD+\mathfrak{D})}{nn}k\cos(2\eta - r) - \left(\frac{3kk}{2n} \mp \frac{(2nH \mp \mathfrak{H})}{nn}\right)\cos 2r$$

$$- \frac{(2nJ+\mathfrak{J})}{nn}k^2\cos(2\eta - 2r)$$

Deinde vero praeter terminos iam tractatos habebitur:

$$R = \ldots 3k^2 \sin 2\eta + \frac{3D}{nn}k^2 \sin 4\eta + \frac{3(2D+J)}{2nn}k^2 \sin 2r$$

$$+ \left(\tfrac{15}{4}k^2 + \frac{3(H-L)}{2nn}\right)\sin(2\eta - 2r) + \left(\tfrac{15}{4}k^2 + \frac{3H}{2nn}\right)\sin(2\eta + 2r)$$

$$+ \frac{3(2D+J)}{2nn}k^2 \sin(4\eta - 2r)$$

atque simili modo:

$$\frac{ddv}{dr^2} = \tfrac{1}{2}\delta - \gamma + \frac{\mathfrak{A}\mathfrak{a} + \tfrac{1}{2}\mathfrak{D}\mathfrak{D} + 3D\mathfrak{D} + 3DD + 3A\alpha + 3\mathfrak{A}\alpha + 3A\mathfrak{a}}{nn}kk$$

$$+ \left(3 + \tfrac{3}{2}(A + D + E) - a - 2n\mathfrak{a}\right)kk\cos 2\eta$$

$$+ \left(\tfrac{3}{2}(B + F + G) - b - 2n\mathfrak{b}\right)kk\cos 4\eta$$

$$+ \left(\tfrac{3}{2} - 2n\mathfrak{H} - H + \frac{\mathfrak{A}\mathfrak{J} + 3\mathfrak{A}J + 3A\mathfrak{J} + 3AJ}{nn}\right)kk\cos 2r$$

$$+ \left(\tfrac{15}{4} - 2n\mathfrak{J} - J + \tfrac{3}{4}A + \tfrac{3}{2}D\right)kk\cos(2\eta - 2r)$$

$$+ \left(\tfrac{15}{4} - 2n\mathfrak{K} - K + \tfrac{3}{4}A + \tfrac{3}{2}E\right)k^2\cos(2\eta + 2r)$$

$$+ \left(-2n\mathfrak{L} - L + \tfrac{3}{4}B + \tfrac{3}{2}F\right)kk\cos(4\eta - 2r)$$

$$+ \left(-2n\mathfrak{M} - M + \tfrac{3}{4}B + \tfrac{3}{2}G\right)k^2\cos(4\eta + 2r)$$

$$+ \frac{\mathfrak{A}\mathfrak{J} + 3\mathfrak{A}\mathfrak{J} + 3AJ + \tfrac{1}{2}\mathfrak{D}\mathfrak{D} + 3D\mathfrak{D} + 3DD)}{nn}kk\cos(4\eta - 2r)$$

§. 93. Eliciamus nunc quoque valorem ipsius R per differentiationem ex formula $\int R dr$, ac terminis apte dispositis habebimus

 R =

R =

$kk \sin 2\eta$	$kk \sin 4\eta$	$kk \sin 2r$	$k^2 \sin(2\eta - 2r)$
$-2\alpha\mathfrak{a}$	$+\frac{\mathfrak{a}(2\kappa A+\mathfrak{A})}{nn}$	$+\frac{\mathfrak{A}(2\kappa J+\mathfrak{J})}{nn}$	$+\frac{3\mathfrak{A}}{2n}$
$+\frac{2\mathfrak{b}(2\kappa A+\mathfrak{A})}{nn}$	$-4\alpha\mathfrak{b}$	$-2\mathfrak{H}$	$+\frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn}$
$+\frac{\mathfrak{D}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{E}(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{\mathfrak{E}(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{\mathfrak{D}(2n+\mathfrak{C})}{nn}$
$+\frac{\mathfrak{E}(2n+\mathfrak{C})}{nn}$	$+\frac{2\mathfrak{F}(2n+\mathfrak{C})}{nn}$	$-\frac{\mathfrak{J}(2\kappa A+\mathfrak{A})}{nn}$	$-2(\alpha-1)\mathfrak{J}$
$+\frac{2\mathfrak{F}(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{2\mathfrak{G}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{K}(2\kappa A+\mathfrak{A})}{nn}$	$+\frac{2\mathfrak{L}(2\kappa A+\mathfrak{A})}{nn}$

$kk \sin(2\eta+2r)$	$kk \sin(4\eta-2r)$	$kk \sin(4\eta+2r)$
$+\frac{3\mathfrak{A}}{2n}$	$+\frac{\mathfrak{A}(2\kappa J+\mathfrak{J})}{nn}$	
$+\frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn}$	$+\frac{3\mathfrak{B}}{n}$	$+\frac{3\mathfrak{B}}{n}$
$+\frac{2\mathfrak{B}(2\kappa J+\mathfrak{J})}{nn}$	$+\frac{2\mathfrak{B}(2\kappa H+\mathfrak{H})}{nn}$	$+\frac{2\mathfrak{B}(2\kappa H+\mathfrak{H})}{nn}$
$+\frac{\mathfrak{E}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{D}(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{\mathfrak{G}(2n+\mathfrak{C})}{nn}$
$+\frac{2\mathfrak{G}(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{2\mathfrak{F}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{K}(2\kappa A+\mathfrak{A})}{nn}$
$-2(\alpha+1)\mathfrak{K}$	$+\frac{\mathfrak{J}(2\kappa A+\mathfrak{A})}{nn}$	$-2(2\alpha+1)\mathfrak{M}$
$+\frac{2\mathfrak{M}(2\kappa A+\mathfrak{A})}{nn}$	$-2(2\alpha-1)\mathfrak{L}$	

vnde

vnde oriuntur sequentes determinationes:

$$3 = -2\alpha\mathfrak{a} + \frac{2\mathfrak{b}(2\kappa A+\mathfrak{A})+(\mathfrak{D}+\mathfrak{E})(2n+\mathfrak{C})+2\mathfrak{F}(2\kappa D+\mathfrak{D})}{nn}$$

$$\frac{3D}{nn} = -4\alpha\mathfrak{b} + \frac{\mathfrak{a}(2\kappa A+\mathfrak{A})+\mathfrak{E}(2\kappa D+\mathfrak{D})+2(\mathfrak{F}+\mathfrak{G})(2n+\mathfrak{C})}{nn}$$

$$\frac{3(2D+J)}{2nn} = -2\mathfrak{H} + \frac{\mathfrak{A}(2\kappa J+\mathfrak{J})+\mathfrak{E}(2\kappa D+\mathfrak{D})-(\mathfrak{J}-\mathfrak{K})(2\kappa A+\mathfrak{A})}{nn}$$

$$\frac{15}{4} + \frac{3(H-L)}{2nn} = -2(\alpha-1)\mathfrak{J} + \frac{3\mathfrak{A}}{nn} + \frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn}$$
$$+ \frac{\mathfrak{D}(2n+\mathfrak{C}) + 2\mathfrak{L}(2\kappa A+\mathfrak{A})}{nn}$$

$$\frac{15}{4} + \frac{3H}{2nn} = 2(\alpha+1)\mathfrak{K} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn}$$
$$+ \frac{2\mathfrak{B}(2\kappa J+\mathfrak{J})+2\mathfrak{G}(2\kappa D+\mathfrak{D})+\mathfrak{E}(2n+\mathfrak{C})+2\mathfrak{M}(2\kappa A+\mathfrak{A})}{nn}$$

$$+\frac{3(2D+J)}{2nn} = -2(2\alpha-1)\mathfrak{L} + \frac{3\mathfrak{B}}{n} + \frac{\mathfrak{A}(2\kappa J+\mathfrak{J})}{nn}$$
$$+ \frac{2\mathfrak{B}(2\kappa H+\mathfrak{H})+\mathfrak{D}(2\kappa D+\mathfrak{D})+2\mathfrak{F}(2n+\mathfrak{C})+\mathfrak{J}(2\kappa A+\mathfrak{A})}{nn}$$

$$0 = -2(2\alpha+1)\mathfrak{M} + \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\kappa H+\mathfrak{H})+2\mathfrak{G}(2n+\mathfrak{C})}{nn}$$
$$+ \frac{\mathfrak{K}(2\kappa A+\mathfrak{A})}{nn}$$

§. 94. Deinde simili modo si ponatur:

$$\frac{dv}{dr} = -A'\sin 2\eta - a'k^2\sin 2\eta - B'\sin 4\eta - b'k^2\sin 4\eta$$
$$- C'k\sin r - D'k\sin(2\eta-r) - F'k\sin(4\eta-r)$$
$$- E'k\sin(2\eta+r) - G'k\sin(2\eta+r)$$
$$- H'k^2\sin 2r - J'k^2\sin(2\eta-2r) - L'k^2\sin(4\eta-2r)$$
$$- K'k^2\sin(2\eta+2r) - M'k^2\sin(4\eta+2r)$$

erit

erit praeter valores §. 81. datos:

$$a' = 2\alpha a - \frac{2b(2xA \pm \mathfrak{A}) - (D \pm E)(2n + \mathfrak{E}) - 2F(2xD + \mathfrak{D})}{nn}$$

$$b' = 4ab - \frac{a(2xA + \mathfrak{A}) - E(2xD + \mathfrak{D})}{nn} \\ - \frac{2(F + G)(2n + \mathfrak{E}) - D(2xE + \mathfrak{E})}{nn}$$

$$H' = 2H - \frac{A(2xJ + \mathfrak{J}) - E(2xD + \mathfrak{D})}{nn} \\ + \frac{(J - K)(2xA + \mathfrak{A}) + D(2xE + \mathfrak{E})}{nn}$$

$$J' = 2(\alpha - 1)J - \frac{3A}{2n} - \frac{A(2xH + \mathfrak{H}) - D(2n + \mathfrak{E}) - 2L(2xA + \mathfrak{A})}{nn}$$

$$K' = 2(\alpha + 1)K - \frac{3A}{2n} - \frac{A(2xH + \mathfrak{H}) - 2B(2xJ + \mathfrak{J}) - 2G(2xD + \mathfrak{D})}{nn} \\ - \frac{E(2n + \mathfrak{E}) - 2M(2xA + \mathfrak{A})}{nn}$$

$$L' = 2(2\alpha - 1)L - \frac{3B}{n} - \frac{A(2xJ + \mathfrak{J}) - 2B(2xH + \mathfrak{H}) - D(2xD + \mathfrak{D})}{nn} \\ - \frac{2F(2n + \mathfrak{E}) - J(2xA + \mathfrak{A})}{nn}$$

$$M' = 2(2\alpha + 1)M - \frac{3B}{n} - \frac{2B(2xH + \mathfrak{H}) - G(2n + \mathfrak{E}) - K(2xA + \mathfrak{A})}{nn}$$

vbi quidem plures terminos, quos admodum paruos fore praeuidimus, omisimus.

§. 95.

§. 95. Hinc autem denuo differentiando obtinemus

valorem ipsius $\frac{ddv}{dr^2}$ =

kk	$kk\cos 2\eta$	$kk\cos 4\eta$	$kk\cos 2r$
$+\frac{a'(2\kappa A+\mathfrak{A})}{nn}$	$-2\alpha a'$	$+\frac{a'(2\kappa A+\mathfrak{A})}{nn}$	$+\frac{A'(2\kappa J+\mathfrak{J})}{nn}$
$+\frac{D'(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{2b'(2\kappa A+\mathfrak{A})}{nn}$	$-4\alpha b'$	$+\frac{E'(2\kappa D+\mathfrak{D})}{nn}$
	$+\frac{D'(2\kappa+\mathfrak{C})}{nn}$	$+\frac{E'(2\kappa D+\mathfrak{D})}{nn}$	$-2H'$
	$+\frac{E'(2\kappa+\mathfrak{C})}{nn}$	$+\frac{2F'(2\kappa+\mathfrak{C})}{nn}$	$+\frac{J'(2\kappa A+\mathfrak{A})}{nn}$
	$+\frac{2F'(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{2G'(2\kappa+\mathfrak{C})}{nn}$	$+\frac{K'(2\kappa A+\mathfrak{A})}{nn}$

$kk\cos(2\eta-2r)$	$kk\cos(2\eta+2r)$	$kk\cos(4\eta-2r)$	$kk\cos(4\eta+2r)$
$+\frac{3A'}{2n}$	$+\frac{3A'}{2n}$	$+\frac{A'(2\kappa J+\mathfrak{J})}{nn}$	
$+\frac{A'(2\kappa H+\mathfrak{H})}{nn}$	$+\frac{A'(2\kappa H+\mathfrak{H})}{nn}$	$+\frac{3B'}{n}$	$+\frac{3B'}{n}$
$+\frac{D'(2\kappa+\mathfrak{C})}{nn}$	$+\frac{2G'(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{2B'(2\kappa H+\mathfrak{H})}{nn}$	$+\frac{2B'(2\kappa H+\mathfrak{H})}{nn}$
$-2(\alpha-1)J'$	$-2(\alpha+1)K'$	$+\frac{D'(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{2G'(2\kappa+\mathfrak{C})}{nn}$
$+\frac{2L'(2\kappa A+\mathfrak{A})}{nn}$	$+\frac{2M'(2\kappa A+\mathfrak{A})}{nn}$	$+\frac{2F(2\kappa+\mathfrak{C})}{nn}$	$-2(2\alpha+1)M'$
		$-2(2\alpha-1)L'$	$+\frac{K'(2\kappa A+\mathfrak{A})}{nn}$
		$+\frac{J'(2\kappa A+\mathfrak{A})}{nn}$	

 vnde

vnde tandem nanciscimur has determinationes:

$$\tfrac{1}{2}\delta-\gamma+\frac{\mathfrak{A}\mathfrak{a}+\frac{1}{2}\mathfrak{D}\mathfrak{D}+3D\mathfrak{D}+\frac{1}{2}DD+3Aa+3\mathfrak{A}a+3A\mathfrak{a}}{nn}kk = \frac{a'(2nA+\mathfrak{A})+D'(2nD+\mathfrak{D})}{nn}kk$$

$$3+\tfrac{1}{2}(A+D+E)-a-2n\mathfrak{a}=-2aa'+\frac{2b'(2nA+\mathfrak{A})}{nn} + \frac{(D'+E')(2n+\mathfrak{C})+2F'(2nD+\mathfrak{D})}{nn}$$

$$\tfrac{1}{2}(B+F+G)-b-2n\mathfrak{b}=-4ab'+\frac{a'(2nA+\mathfrak{A})}{nn} + \frac{E'(2nD+\mathfrak{D})+2(F'+G')(2n+\mathfrak{C})}{nn}$$

$$\tfrac{1}{2}-2n\mathfrak{H}-H+\frac{\mathfrak{A}\mathfrak{J}+3\mathfrak{A}J+3A\mathfrak{J}+3AJ}{nn}=-2H'+\frac{A'(2nJ+\mathfrak{J})}{nn} + \frac{E'(2nD+\mathfrak{D})+(J'+K')(2nA+\mathfrak{A})}{nn}$$

$$\tfrac{15}{4}-2n\mathfrak{J}-J+\tfrac{3}{4}A+\tfrac{1}{2}D=-2(a-1)J'+\frac{3A'}{2n}+\frac{A'(2nH+\mathfrak{H})}{nn} + \frac{D'(2n+\mathfrak{C})+2L'(2nA+\mathfrak{A})}{nn}$$

$$\tfrac{15}{4}-2n\mathfrak{K}-K+\tfrac{3}{4}A+\tfrac{1}{2}E=-2(a+1)K'+\frac{3A'}{2n}+\frac{A'(2nH+\mathfrak{H})}{nn} + \frac{2G'(2nD+\mathfrak{D})+2M'(2aA+\mathfrak{A})}{nn}$$

$$-2n\mathfrak{L}-L+\tfrac{3}{4}B+\tfrac{1}{2}F+\frac{\mathfrak{A}\mathfrak{J}+3\mathfrak{A}J+3A\mathfrak{J}+3AJ+\frac{1}{2}\mathfrak{D}\mathfrak{D}+3D\mathfrak{D}+3DD}{nn}= -2(2a-1)L'+\frac{3B'}{n}+\frac{A'(2nJ+\mathfrak{J})+2B'(2nH+\mathfrak{H})}{nn} + \frac{D'(2nD+\mathfrak{D})+2F'(2n+\mathfrak{C})+J'(2nA+\mathfrak{A})}{nn}$$

$$-2\varkappa\mathfrak{M}-M+\tfrac{3}{4}B+\tfrac{1}{4}G=-2(2\alpha+1)M'+\frac{3B'}{\varkappa}+\frac{2B'(2\varkappa H+\mathfrak{H})}{\varkappa\varkappa}$$
$$+\frac{2G'(2\varkappa+\mathfrak{C})+K'(2\varkappa A+\mathfrak{A})}{\varkappa\varkappa}$$

§. 96. Primum autem valoribus iam cognitis substituendis, reperitur:

$2\alpha\mathfrak{a}=-3,837-0,038\,\mathfrak{b}$ et $4\alpha\mathfrak{b}=-1,073-0,019\,\mathfrak{a}$
hincque $\mathfrak{a}=-2,051$ et $\mathfrak{b}=-0,277$
ex quibus porro elicimus: $a=-12,595$ et $b=-0,086$
et $a'=-23,510$. Deinde pro reliquis litteris

$\mathfrak{J}=+32,663$ . . . $l\,\mathfrak{J}=1,514059$
$\mathfrak{K}=-1,035$ $l-\mathfrak{K}=0,014776$
$J'=-2(1-\alpha)\,J-5,060;$
$K'=2(1+\alpha)\,K+0,227;$
$J=-15,555$ $l-J=1,191891$
$K=-0,370$ $l-K=9,568589$
Porro $\mathfrak{L}=-1,453$ . . . $l-\mathfrak{L}=0,162070$
$\mathfrak{M}=-0,000$ . . .
$L'=2(2\alpha-1)\;L-12,786$
$M'=2(2\alpha+1)\;M$
$L=+6,252$ $l\,L=0,796019$
$M=-0,001$ $l-M=7,000000$
Denique $\mathfrak{H}=-0,123$ et $H=-1,033$
atque $\frac{1}{2}\,\delta-\gamma=-7,459\,kk$

§. 97. His igitur valoribus inuentis innotescet primum distantia Lunae a terra curtata, quatenus a sola

 excen-

excentricitate orbitae lunaris k pendet. Cum enim haec distantia posita sit $x = \frac{(1-kk)au}{1-k\cos r}$ ob $u = 1 + \frac{v}{nn}$, erit

$$\begin{aligned}
u = 1 &- 0,007161 \cos 2\eta &&- 0,0719\, kk \cos 2\eta \\
&+ 0,000073 \cos 4\eta &&- 0,0005\, kk \cos 4\eta \\
&+ 0,194888\, k \cos(2\eta - r) &&- 0,003274\, k \cos(2\eta + r) \\
&- 0,003238\, k \cos(4\eta - r) &&+ 0,000078\, k \cos(4\eta + r) \\
&- 0,0059\, kk \cos 2r && \\
&- 0,0889\, kk \cos(2\eta - 2r) &&- 0,0021\, kk \cos(2\eta + 2r) \\
& &&+ 0,0357\, kk \cos(4\eta - 2r)
\end{aligned}$$

At pro longitudine Lunae, quatenus a sola excentricitate k pendet, prodibit $\frac{d\Phi}{dr} =$

$$\begin{aligned}
u &+ 0,000285 + 0,019015 \cos 2\eta + 0,000076 \cos 4\eta \\
&\quad + 0,1562\, kk \cos 2\eta + 0,0008\, kk \cos 4\eta \\
&- 0,001255\, k \cos r - 0,38410\, k \cos(2\eta - r) + 0,002647\, k \cos(4\eta - r) \\
&\quad + 0,01278\, k \cos(2\eta + r) - 0,000229\, k \cos(4\eta + r) \\
&+ 0,0118\, kk \cos 2r - 0,0081\, kk \cos(2\eta - 2r) - 0,0076\, kk \cos(4\eta - 2r) \\
&\quad + 0,0102\, kk \cos(2\eta + 2r)
\end{aligned}$$

§. 98. Ponatur nunc:

$$\begin{aligned}
\Phi = Or &+ \mathfrak{A}' \sin 2\eta + \mathfrak{a}' kk \sin 2\eta + \mathfrak{B}' \sin 4\eta + \mathfrak{b}' kk \sin 4\eta \\
&+ \mathfrak{C}' k \sin r + \mathfrak{D}' k \sin(2\eta - r) + \mathfrak{F}' k \sin(4\eta - r) \\
&\qquad + \mathfrak{E}' k \sin(2\eta + r) + \mathfrak{G}' k \sin(4\eta + r) \\
&+ \mathfrak{H}' kk \sin 2r - \mathfrak{J}' kk \sin(2\eta - 2r) + \mathfrak{L}' kk \sin(4\eta - 2r) \\
&\qquad + \mathfrak{K}' kk \sin(2\eta + 2r) + \mathfrak{M}' kk \sin(4\eta + 2r)
\end{aligned}$$

atque

atque differentiando orientur sequentes comparationes

$$x + 0{,}000285 = O - \frac{\mathfrak{a}'(2xA+\mathfrak{A}) - \mathfrak{D}'(2\alpha D+\mathfrak{D})}{nn}\,kk + 0{,}000190$$

$$+0{,}1562 = 2\alpha\,\mathfrak{a}' - \frac{(\mathfrak{D}'+\mathfrak{E}')(2n+\mathfrak{C}) - 2\mathfrak{F}'(2xD+\mathfrak{D})}{nn}$$

$$+0{,}0008 = 4\alpha\,\mathfrak{b}' - \frac{\mathfrak{a}'(2xA+\mathfrak{A}) - \mathfrak{E}')2xD+\mathfrak{D}) - 2(\mathfrak{F}'+\mathfrak{G}')(2n+\mathfrak{C})}{nn}$$

$$+0{,}0118 = 2\mathfrak{H}' - \frac{\mathfrak{E}'(2xD+\mathfrak{D}) - \mathfrak{A}'(2xJ+\mathfrak{J}) - (\mathfrak{J}'+\mathfrak{K}')(2xA+\mathfrak{A})}{nn}$$

$$-0{,}0081 = 2(\alpha-1)\mathfrak{J}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{D}'(2n+\mathfrak{C}) - \mathfrak{A}'(2xH+\mathfrak{H})}{nn}$$

$$- \frac{2\mathfrak{L}'(2xA+\mathfrak{A})}{nn}$$

$$+0{,}0102 = 2(\alpha+1)\mathfrak{K}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{A}'(2xH+\mathfrak{H}) - 2\mathfrak{G}'(2xD+\mathfrak{D})}{nn}$$

$$-0{,}0076 = 2(2\alpha-1)\mathfrak{L}' - \frac{3\mathfrak{B}'}{2n} - \frac{\mathfrak{A}'(2xJ+\mathfrak{J}) - \mathfrak{J}'(2xA+\mathfrak{A})}{nn}$$

$$- \frac{\mathfrak{D}'(2xD+\mathfrak{D}) - 2\mathfrak{F}'(2n+\mathfrak{C})}{nn}$$

$$0 = 2(2\alpha+1)\mathfrak{M}' - \frac{3\mathfrak{B}'}{n} - \frac{2\mathfrak{G}'(2n+\mathfrak{C})}{nn}.$$

§. 99. Ex his comparationibus elicimus:

$\mathfrak{a}' = +\,0{,}0509$; $\mathfrak{b}' = 0{,}0008$
$\mathfrak{J}' = +\,0{,}5385$; $\mathfrak{K}' = 0{,}0028$
$\mathfrak{L}' = -\,0{,}1055$; $\mathfrak{M}' = 0{,}0000$
$\mathfrak{H}' = +\,0{,}0021$; et $O = x - 0{,}000429$

L 3 posito

posito $k = 0,05445$. Hinc autem erit $\frac{1}{2}\delta - \gamma = -0,02302$
Cum autem iam ante inuentum esset $\frac{1}{2}\delta - \gamma = -0,01742$
erit reuera $\frac{1}{2}\delta - \gamma = -0,00560$. Tum vero inueniemus:
$\gamma = 1,40673$, vnde erit $\frac{1}{2}\delta = 1,40113$, et $\delta = 2,80226$
hincque $2\gamma - \frac{3}{2}\delta = -1,38993$ et $\frac{2\gamma - \frac{3}{2}\delta}{nn} = -0,00794$.
Verum ex cognita ratione motus medii ad motum anomaliae est $O = 1,0085272$, vnde $\varkappa = 1,0089562$
Verum esse debet $\varkappa = 1 + \frac{3 + 4\mu + \delta}{4nn}$; vnde foret
$0,0089562 = 0,008289 + \frac{\mu}{nn}$; ideoque $\frac{\mu}{nn} = 0,000667$
qui valor cum sit tam exiguus, merito dubitamus, num μ non prorsus sit $= 0$.

CAPUT

CAPUT VII.

CORRECTIO INAEQUALITATUM LUNAE, ANTE INUENTARUM.

§. 100.

Quoniam nunc quidem valores litterarum γ et δ ita inuenimus, vt eos pro proxime veris habere queamus, ex iis coefficientes terminorum, quibus inaequalitates lunae continentur, accuratius definire poterimus. Cum enim sit $\gamma = 1,40673$ et $\delta = 2,80226$, colligamus hic in vnum omnes formulas, quas hactenus pro inueniendis coefficientibus assumtis elicuimus. Posueramus autem:

$$\begin{aligned}\int R\,dr = {} & \mathfrak{A}\cos 2\eta + a\,kk\cos 2\eta + \mathfrak{B}\cos 4\eta + b\,kk\cos 4\eta \\ & + \mathfrak{C}\,k\cos r + \mathfrak{D}\,k\cos(2\eta-r) + \mathfrak{F}\,k\cos(4\eta-r) \\ & \qquad + \mathfrak{E}\,k\cos(2\eta+r) + \mathfrak{G}\,k\cos(4\eta+r) \\ & + \mathfrak{H}\,kk\cos 2r + \mathfrak{I}\,k^2\cos(2\eta-2r) + \mathfrak{L}\,k^2\cos(4\eta-2r) \\ & \qquad + \mathfrak{K}\,k^2\cos(2\eta+2r) + \mathfrak{M}\,k^2\cos(4\eta+2r)\end{aligned}$$

$$\begin{aligned}0 = {} & A\cos 2\eta + a\,kk\cos 2\eta + B\cos 4\eta + b\,kk\cos 4\eta \\ & \qquad + D\,k\cos(2\eta-r) + F\,k\cos(4\eta-r) \\ & \qquad + E\,k\cos(2\eta+r) + G\,k\cos(4\eta+r) \\ & + H\,kk\cos 2r + J\,kk\cos(2\eta-2r) + L\,kk\cos(4\eta-2r) \\ & \qquad + K\,kk\cos(2\eta+2r) + M\,kk\cos(4\eta+2r)\end{aligned}$$

§. 101.

§. 101. Hinc posito $\varkappa = \sqrt{\left(1 + \frac{3+4\mu+\delta}{2nn}\right)}$ collegimus fore

$$\begin{aligned}
\frac{d\Phi}{dr} = {} & \varkappa + \frac{A(3\varkappa A+2\mathfrak{A})}{2n^4} + \frac{D(3\varkappa D+2\mathfrak{D})}{2n^4}kk + \frac{A(3\varkappa a+\mathfrak{a})}{2n^4}kk \\
& + \frac{a(3\varkappa A+2\mathfrak{A})}{2n^4}kk - \frac{(2\varkappa A+\mathfrak{A})}{nn}\cos 2\eta - \frac{(2\varkappa B+\mathfrak{B})}{nn}\cos 4\eta \\
& + \frac{A(3\varkappa A+2\mathfrak{A})}{2n^4}\cos 4\eta - \frac{\mathfrak{C}}{nn}k\cos r - \frac{(2\varkappa a+\mathfrak{a})}{nn}k^2\cos 2\eta \\
& - \frac{(2\varkappa b+\mathfrak{b})}{nn}k^2\cos 4\eta + \frac{D(3\varkappa A+2\mathfrak{A})+A(3\varkappa D+2\mathfrak{D})}{2n^4}k\cos r \\
& - \frac{(2\varkappa D+\mathfrak{D})}{nn}k\cos(2\eta-r) - \frac{(2\varkappa F+\mathfrak{F})}{nn}k\cos(4\eta-r) \\
& + \frac{D(3\varkappa A+2\mathfrak{A})+A(3\varkappa D+2\mathfrak{D})}{2n^4}k\cos(4\eta-r) \\
& - \frac{(2\varkappa E+\mathfrak{E})}{nn}k\cos(2\eta+r) - \frac{(2\varkappa G+\mathfrak{G})}{nn}k\cos(4\eta+r) \\
& - \frac{(2\varkappa J+\mathfrak{J})}{nn}kk\cos(2\eta-2r) - \frac{(2\varkappa H+\mathfrak{H})}{nn}kk\cos 2r \\
& + \frac{J(3\varkappa A+2\mathfrak{A})+A(3\varkappa J+2\mathfrak{J})}{2n^4}k^2\cos 2r \\
& - \frac{(2\varkappa K+\mathfrak{K})}{nn}kk\cos(2\eta+2r) - \frac{(2\varkappa L+\mathfrak{L})}{nn}k^2\cos(4\eta-2r) \\
& - \frac{D(3\varkappa D+2\mathfrak{D})+J(3\varkappa A+2\mathfrak{A})+A(3\varkappa J+2\mathfrak{J})}{2n^4}k^2\cos(4\eta-2r) \\
& - \frac{(2\varkappa M+\mathfrak{M})}{nn}k^2\cos(4\eta+2r)
\end{aligned}$$

§. 102.

§. 102. Si iam ponamus

$$n + \frac{A(3nA+2\mathfrak{A})}{2n^4} + \frac{D(3nD+2\mathfrak{D})}{2n^4}kk$$
$$+ \frac{A(3na+2\mathfrak{a})}{2n^4}kk + \frac{a(3nA+2\mathfrak{A})}{2n^4}kk$$

$- \frac{1}{n} = \alpha$; vt fit neglectis terminis admodum exiguis

$$\frac{d\eta}{dr} = \alpha - \frac{(2nA+\mathfrak{A})}{nn}\operatorname{cof} 2\eta - \frac{(2n+\mathfrak{C})}{nn}k \operatorname{cof} r$$
$$- \frac{(2nD+\mathfrak{D})}{nn}k\operatorname{cof}(2\eta-r) - \frac{(2nE+\mathfrak{E})}{nn}k^2\operatorname{cof}(2\eta+r)$$

ex fuperioribus capitibus repetimus has determinationes:

$$2\alpha\mathfrak{A} = -\tfrac{3}{2}$$
$$4\alpha\mathfrak{B} = -\frac{3A}{2nn} + \frac{\mathfrak{A}(3nA+\mathfrak{A})}{nn}$$
$$\mathfrak{C} = \frac{\mathfrak{A}(2nD+\mathfrak{D})}{nn} - \frac{(\mathfrak{D}-\mathfrak{E})(2nA+\mathfrak{A})}{nn} - \frac{\mathfrak{A}(2nE+\mathfrak{E})}{nn} - \frac{3(D-E)}{2nn}$$
$$(2\alpha-1)\mathfrak{D} = -3 + \frac{(2n+\mathfrak{C})}{nn}\mathfrak{A}$$
$$(2\alpha+1)\mathfrak{E} = -3 + \frac{(2n+\mathfrak{C})}{nn}\mathfrak{A}$$
$$(4\alpha-1)\mathfrak{F} = \frac{2(2n+\mathfrak{C})}{nn}\mathfrak{B} + \frac{\mathfrak{A}(2nD+\mathfrak{D})}{nn} + \frac{\mathfrak{D}(2nA+\mathfrak{A})}{nn} - \frac{3(2A+D)}{2nn}$$
$$(4\alpha+1)\mathfrak{G} = \frac{2(2n+\mathfrak{C})}{nn}\mathfrak{B} + \frac{\mathfrak{A}(2nE+\mathfrak{E})}{nn} + \frac{\mathfrak{E}(2nA+\mathfrak{A})}{nn} - \frac{3(2A+E)}{2nn}$$
$$2\alpha\mathfrak{a} = -\tfrac{15}{4} + \frac{(\mathfrak{D}+\mathfrak{E})(2n+\mathfrak{C})}{nn} + \frac{2\mathfrak{b}(2nA+\mathfrak{A})}{nn}$$
$$+ \frac{2\mathfrak{F}(2nD+\mathfrak{D})}{nn} + \frac{\mathfrak{D}(2nF+\mathfrak{F})}{nn}$$

$$4\alpha\mathfrak{h} = -\frac{3D}{nn} + \frac{2(\mathfrak{F}+\mathfrak{G})(2n+\mathfrak{E})}{nn} + \frac{\mathfrak{E}(2\kappa D+\mathfrak{D})}{nn}$$

$$+ \frac{\mathfrak{a}(2\kappa A+\mathfrak{A})}{nn} + \frac{\mathfrak{D}(2\kappa E+\mathfrak{E})}{nn}$$

$$2\mathfrak{H} = -\frac{3(2D+J)}{2nn} + \frac{\mathfrak{E}(2\kappa D+\mathfrak{D})}{nn} + \frac{\mathfrak{A}(2\kappa J+\mathfrak{J})}{nn} - \frac{(\mathfrak{J}-\mathfrak{K})(2\kappa A+\mathfrak{A})}{nn}$$

$$2(\alpha-1)\mathfrak{J} = -\frac{15}{4} - \frac{3(H-L)}{2nn} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn}$$

$$+ \frac{\mathfrak{D}(2n+\mathfrak{E})}{nn} + \frac{2\mathfrak{L}(2\kappa A+\mathfrak{A})}{nn}$$

$$2(\alpha+1)\mathfrak{K} = -\frac{15}{4} - \frac{3H}{2nn} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn}$$

$$+ \frac{\mathfrak{E}(2n+\mathfrak{E})}{nn} + \frac{2\mathfrak{G}(2\kappa D+\mathfrak{D})}{nn}$$

$$2(2\alpha-1)\mathfrak{L} = -\frac{3(2D+J)}{2nn} + \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\kappa H+\mathfrak{H})}{nn} + \frac{2\mathfrak{F}(2n+\mathfrak{E})}{nn}$$

$$+ \frac{\mathfrak{A}(2\kappa J+\mathfrak{J})}{nn} + \frac{\mathfrak{D}(2\kappa D+\mathfrak{D})}{nn} + \frac{\mathfrak{J}(2\kappa A+\mathfrak{A})}{nn}$$

$$2(2\alpha+1)\mathfrak{M} = \ldots \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\kappa H+\mathfrak{H})}{nn} + \frac{2\mathfrak{G}(2n+\mathfrak{E})}{nn} + \frac{\mathfrak{K}(2\kappa A+\mathfrak{A})}{nn}$$

§. 103. Antequam vlterius progrediamur, sequentes notandae sunt nouae denominationes

$$A' = 2\alpha A + \frac{A(2\kappa B+\mathfrak{B})}{nn} - \frac{B(2\kappa A+\mathfrak{A})}{nn}$$

$$B' = 4\alpha B - \frac{A(2\kappa A+\mathfrak{A})}{nn}$$

$$C' = \frac{\mathfrak{A}(D-E) - A(\mathfrak{D}-\mathfrak{E})}{nn}$$

$$D' = (2\alpha-1)D - \frac{(2n+\mathfrak{E})}{nn}A$$

E' =

$$E' = (2a+1)\,E - \frac{(2n+\mathfrak{E})}{nn}\,A$$

$$F' = (4a-1)\,F - \frac{4B}{n} - \frac{A(2nD+\mathfrak{D}) - D(2nA+\mathfrak{A})}{nn}$$

$$G' = (4a+1)\,G - \frac{4B}{n} - \frac{A(2nE+\mathfrak{E}) - E(2nA+\mathfrak{A})}{nn}$$

$$a' = 2na - \frac{2b(2nA+\mathfrak{A})}{nn} - \frac{(D+E)(2n+\mathfrak{E})}{nn}$$
$$- \frac{2F(2nD+\mathfrak{D})}{nn} + \frac{D(2nF+\mathfrak{F})}{nn}$$

$$b' = 4ab - \frac{a(2nA+\mathfrak{A})}{nn} - \frac{A(2na+\mathfrak{a})}{nn} - \frac{D(2nE+\mathfrak{E})}{nn}$$
$$- \frac{E(2nD+\mathfrak{D})}{nn} - \frac{2(F+G)(2n+\mathfrak{E})}{nn}$$

$$H' = 2H + \frac{D\mathfrak{E} - \mathfrak{D}E - A(\mathfrak{J}-\mathfrak{K}) + \mathfrak{A}(J-K)}{nn}$$

$$J' = 2(a-1)\,J + \frac{3A}{2n} - \frac{A(2nH+\mathfrak{H})}{nn} - \frac{D(2n+\mathfrak{E})}{nn}$$
$$- \frac{2\mathfrak{L}(2nA+\mathfrak{A})}{nn} + \frac{A(2nL+\mathfrak{L})}{nn}$$

$$K' = 2(a+1)K - \frac{3A}{n} - \frac{A(2nH+\mathfrak{H})}{nn} - \frac{E(2n+\mathfrak{E})}{nn} - \frac{2G(2nD+\mathfrak{D})}{nn}$$

$$L' = (2a-1)\,L - \frac{3B}{n} - \frac{2B(2nH+\mathfrak{H})}{nn} - \frac{2F(2n+\mathfrak{E})}{nn}$$
$$- \frac{D(2nD+\mathfrak{D})}{nn} - \frac{A(2nJ+\mathfrak{J})}{nn} - \frac{J(2nA+\mathfrak{A})}{nn}$$

$$M' = 2(2a+1)M - \frac{3B}{n} - \frac{2B(2nH+\mathfrak{H})}{nn} - \frac{2G(2n+\mathfrak{E})}{nn} - \frac{E(2nE+\mathfrak{E})}{nn}$$

§. 104. Nunc vt terminos completos obtineamus, ſaltem eos qui angulos 2η et r inuoluunt, notandum est in nostris aequationibus sin 2η et cos 2η non per $\frac{3}{2}(1+2kk)$ sed per $\frac{3}{2}(1+2kk+\frac{9}{2}ee)$ esse multiplicatos. Hinc cum sit fere $\frac{9}{4}ee = \frac{1}{4}kk$, loco kk hic scribi oportebit $\frac{5}{4}kk$, vnde in valore ipsius a pro 3 scripsi $3\cdot\frac{5}{4}$ seu $\frac{15}{4}$. Deinde vt in his terminis quoque rationem habeamus inclinationis orbitae, cuius medius valor sit $= e$, ponamus $\frac{3}{4}(nn+2+3\mu+\gamma)\,\text{tang}\, e^2 = f = \frac{3}{4}(\frac{3}{2}\varkappa\varkappa nn - \frac{1}{2}nn - \frac{5}{4} + \gamma - \frac{3}{4}\delta)\,\text{tang}\, e^2$ ob $\mu = \frac{1}{2}(\varkappa\varkappa - 1)nn - \frac{3}{4} - \frac{1}{4}\delta$, eritque nostra aequatio:

$$\frac{ddv}{dr^2} = \tfrac{1}{2}\delta - \gamma + \tfrac{1}{4}kk - \gamma k\cos r + \tfrac{3}{2}kk\cos 2r + \tfrac{3}{2}\cos 2\eta + \tfrac{15}{4}kk\cos 2\eta$$
$$+ f + \tfrac{1}{2}fkk + fk\cos r + \tfrac{1}{2}fkk\cos 2r$$
$$+ 3k\,\text{cs}(2\eta - r) + 3k\,\text{cs}(2\eta + r) + \tfrac{15}{4}kk\,\text{cs}(2\eta - 2r) + \tfrac{15}{4}kk\,\text{cs}(2\eta + 2r)$$
$$- 2\varkappa\int R\,dr - v\left(1 - \tfrac{3}{2}kk + \frac{2f}{nn} + \frac{fkk}{nn} - \frac{(2\gamma - \frac{3}{2}\delta)}{nn}\right)$$
$$+ v\left(3\varkappa\varkappa + \frac{3}{2nn} - \frac{2f}{nn} + \frac{(2\gamma - \frac{3}{2}\delta)}{nn}\right)k\cos r$$
$$+ v\left(\tfrac{3}{2} - \frac{f}{nn}\right)k^2\cos 2r$$
$$+ v\left(\frac{3}{2nn}\cos 2\eta + \frac{3k}{nn}\cos(2\eta - r) + \frac{3k}{nn}\cos(2\eta + r)\right)$$
$$+ \frac{1}{nn}\left(\int R\,dr\right)^2 + \frac{6v}{nn}\int R\,dr + \frac{3vv}{nn} - \frac{3vv}{nn}k\cos r$$

§. 105. Sit breuitatis gratia:

$$1 + \frac{2f}{nn} - \frac{(2\gamma - \frac{3}{2}\delta)}{nn} = g;\quad 3\varkappa\varkappa + \frac{3}{2nn} - \frac{2f}{nn} + \frac{(2\gamma - \frac{3}{2}\delta)}{nn} = b$$

et $1 - \frac{3}{2}kk + \frac{2f}{nn} + \frac{fkk}{nn} - \frac{(2\gamma - \frac{3}{2}\delta)}{nn} = \mathfrak{C}$, quo

termino

termino in angulis ex 2η et r compositis vtemur: eritque $\frac{ddv}{dr^2} =$

$$\tfrac{1}{2}\delta - \gamma + \tfrac{1}{4}kk + f + \tfrac{1}{2}fkk + \frac{3A}{4nn} + \frac{3akk}{4nn}$$

$$+ \frac{3Dkk}{2nn} + \frac{3Ekk}{2nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{\mathfrak{C}\mathfrak{C}kk}{2nn} + \frac{\mathfrak{D}\mathfrak{D}kk}{2nn} + \frac{\mathfrak{E}\mathfrak{E}kk}{2nn} - \frac{3ADkk}{2nn}$$

$$+ \frac{3A\mathfrak{A}}{nn} + \frac{3D\mathfrak{D}kk}{nn} + \frac{3AA}{2nn} + \frac{3DDkk}{2nn} + \frac{3EEkk}{2nn}$$

$$+ \cos 2\eta \left\{ \tfrac{3}{2} - 2\kappa\mathfrak{A} - \beta A \right)$$

$$+ kk\cos 2\eta \left\{ \tfrac{15}{4} - 2\kappa\mathfrak{a} - \beta\alpha + \tfrac{1}{2}bD + \tfrac{1}{2}bE + \frac{\mathfrak{C}D}{nn} + \frac{3\mathfrak{C}E}{nn} \right.$$

$$+ \cos 4\eta \left\{ -2\kappa\mathfrak{B} - \beta B + \frac{3A}{4nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A\mathfrak{A}}{nn} + \frac{3AA}{2nn} \right.$$

$$+ kk\cos 4\eta \left\{ \begin{array}{l} -2\kappa\mathfrak{b} - \beta b + \tfrac{1}{2}bF + \tfrac{1}{2}bG + \frac{3E}{2nn} + \frac{3D}{2nn} \\ + \frac{\mathfrak{A}a}{nn} + \frac{\mathfrak{D}\mathfrak{E}}{nn} + \frac{3DE}{nn} + \frac{3Aa}{nn} + \frac{3A\alpha}{nn} + \frac{3\mathfrak{A}a}{nn} \end{array} \right.$$

$$+ k\cos r \left\{ \begin{array}{l} -\gamma + f - 2\kappa\mathfrak{C} + \frac{3D}{4nn} + \frac{3E}{4nn} + \frac{3A}{nn} + \frac{\mathfrak{A}\mathfrak{D}}{nn} + \frac{\mathfrak{A}\mathfrak{E}}{nn} \\ + \frac{3A\mathfrak{D}}{nn} + \frac{3A\mathfrak{E}}{nn} + \frac{3\mathfrak{A}D}{nn} + \frac{3\mathfrak{A}E}{nn} + \frac{3AD}{nn} + \frac{3AE}{nn} - \frac{3AA}{2nn} \end{array} \right.$$

$$+ k\cos(2\eta - r) \left\{ 3 - 2\kappa\mathfrak{D} - \beta D + \tfrac{1}{2}bA + \frac{3F}{4nn} + \frac{\mathfrak{A}\mathfrak{C}}{nn} + \frac{3A\mathfrak{C}}{nn} \right.$$

$$+ k\cos(2\eta + r) \left\{ 3 - 2\kappa\mathfrak{E} - \beta E + \tfrac{1}{2}bA + \frac{3G}{4nn} + \frac{\mathfrak{A}\mathfrak{C}}{nn} + \frac{3A\mathfrak{C}}{nn} \right.$$

$$+kk\cos 2r\left\{\begin{array}{l}\frac{3}{2}+\frac{1}{2}f-2\varkappa\mathfrak{H}-\beta H+\frac{3J}{4nn}+\frac{3K}{4nn}+\frac{3E}{2nn}+\frac{3D}{2nn}\\ +\frac{\mathfrak{E}\mathfrak{E}}{nn}+\frac{\mathfrak{D}\mathfrak{E}}{nn}+\frac{3D\mathfrak{E}}{nn}+\frac{3\mathfrak{D}E}{nn}+\frac{3DE}{nn}-\frac{3AD}{2nn}\end{array}\right.$$

$$+kk\cos(2\eta-2r)\left\{\begin{array}{l}\frac{15}{4}-2\varkappa\mathfrak{J}-\beta J+\frac{1}{2}bD+\left(\frac{3}{4}-\frac{f}{2nn}\right)A\\ +\frac{3H}{4nn}+\frac{\mathfrak{A}\mathfrak{H}}{nn}+\frac{3A\mathfrak{H}}{nn}+\frac{3\mathfrak{C}D}{nn}\end{array}\right.$$

$$+kk\cos(2\eta+2r)\left\{\begin{array}{l}\frac{15}{4}-2\varkappa\mathfrak{K}-\beta K+\frac{1}{2}bE+\left(\frac{3}{4}-\frac{f}{2nn}\right)A\\ +\frac{3H}{4nn}+\frac{\mathfrak{A}\mathfrak{H}}{nn}+\frac{3A\mathfrak{H}}{nn}+\frac{3\mathfrak{C}E}{nn}\end{array}\right.$$

$$+kk\cos(4\eta-2r)\left\{\begin{array}{l}-2\varkappa\mathfrak{L}-\beta L+\left(\frac{3}{4}-\frac{f}{2nn}\right)B\\ +\frac{3J}{4nn}+\frac{3D}{2nn}+\frac{\mathfrak{D}\mathfrak{D}}{2nn}+\frac{3DD}{2nn}+\frac{1}{2}bF\end{array}\right.$$

$$+kk\cos(\eta 4+2r)\left\{\begin{array}{l}-2\varkappa\mathfrak{M}-\beta M+\left(\frac{3}{4}-\frac{f}{2nn}\right)B\\ +\frac{3K}{4nn}+\frac{3E}{2nn}+\frac{\mathfrak{E}\mathfrak{E}}{2nn}+\frac{3EE}{2nn}+\frac{1}{2}bG\end{array}\right.$$

$$+k\cos(4\eta-r)\left\{\begin{array}{l}-2\varkappa\mathfrak{F}-\beta F+\frac{1}{2}hB+\frac{3D}{4nn}+\frac{3A}{2nn}\\ +\frac{\mathfrak{A}\mathfrak{D}}{nn}+\frac{3A\mathfrak{D}}{nn}+\frac{3\mathfrak{A}D}{nn}+\frac{3AD}{nn}-\frac{3AA}{4nn}\end{array}\right.$$

$$+k\cos(4\eta+r)\left\{\begin{array}{l}-2\varkappa\mathfrak{G}-\beta G+\frac{1}{2}bB+\frac{3E}{4nn}+\frac{3A}{2nn}\\ +\frac{\mathfrak{A}\mathfrak{E}}{nn}+\frac{3A\mathfrak{E}}{nn}+\frac{3\mathfrak{A}E}{nn}+\frac{3AE}{nn}-\frac{3AA}{4nn}\end{array}\right.$$

§. 106.

§. 106. Hinc denique nascentur sequentes aequalitates.

I. $$\tfrac{1}{2}\delta - \gamma + \tfrac{1}{4}kk + \tfrac{1}{2}fkk + \frac{3A}{4nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A\mathfrak{A}}{nn} + \frac{3AA}{2nn}$$
$$+ \frac{3(D+E)}{nn}kk - \frac{3ADkk}{2nn} + \frac{\mathfrak{C}\mathfrak{C}kk}{2nn} + \frac{(\mathfrak{D}\mathfrak{D}+\mathfrak{E}\mathfrak{E})}{2nn}kk$$
$$+ \frac{3D\mathfrak{D}}{nn}kk + \frac{3(DD+EE)}{2nn}kk + \frac{\mathfrak{A}\mathfrak{a}}{nn}kk$$
$$+ \frac{3Aa}{nn}kk + \frac{3\mathfrak{A}a}{nn}kk + \frac{3A\mathfrak{a}}{nn}kk = \frac{A'(2\kappa A+\mathfrak{A})}{nn} + \frac{a'(2\kappa A+\mathfrak{A})}{nn}kk$$
$$+ \frac{D'(2\kappa D+\mathfrak{D})}{nn}kk + \frac{A'(2\kappa a+\mathfrak{a})}{nn}kk$$

II. $$\tfrac{1}{2} - 2\kappa\mathfrak{A} - \epsilon A = -2\alpha A' + \frac{A'(2\kappa B+\mathfrak{B})}{nn} + \frac{2B'(2\kappa A+\mathfrak{A})}{nn}$$

III. $$-2\kappa\mathfrak{B} - \epsilon B + \frac{3A}{4nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A\mathfrak{A}}{nn} + \frac{3AA}{2nn}$$
$$= -4\alpha B' + \frac{A'(2\kappa A+\mathfrak{A})}{nn}$$

IV. $$\tfrac{1}{4}\zeta - 2\kappa\mathfrak{a} - \epsilon a + \tfrac{1}{2}b(D+E) + \frac{3\mathfrak{C}(D+E)}{nn} = -2\alpha a'$$
$$+ \frac{(D'+E')(2n+\mathfrak{C})}{nn} + \frac{2b'(2\kappa A+\mathfrak{A})}{nn} + \frac{2F'(2\kappa D+\mathfrak{D})}{nn}$$

V. $$-2\kappa\mathfrak{b} - \epsilon b + \tfrac{1}{2}b(F+G) + \frac{3(D+E)}{nn} + \frac{3D(E+J)}{nn} + \frac{3Aa}{nn} = -4\alpha b'$$
$$+ \frac{a'(2\kappa A+\mathfrak{A})}{nn} + \frac{E'(2\kappa D+\mathfrak{D})}{nn} + \frac{2(F'+G')(2n+\mathfrak{C})}{nn} + \frac{D'(2\kappa E+\mathfrak{E})}{nn}$$

VI.

VI. $$-\gamma + f - 2\varkappa\mathfrak{E} + \frac{3(D+E)}{4nn} + \frac{3A}{nn} - \frac{3AA}{2nn} + \frac{\mathfrak{A}(\mathfrak{D}+\mathfrak{E})}{nn}$$

$$+ \frac{3A(\mathfrak{D}+\mathfrak{E})}{nn} + \frac{3A(D+E)}{nn} + \frac{3\mathfrak{A}(D+E)}{nn} =$$

$$-C' + \frac{A'(2\varkappa D+\mathfrak{D})}{nn} + \frac{A'(2\varkappa E+\mathfrak{E})}{nn} + \frac{(D'+E')(2\varkappa A+\mathfrak{A})}{nn}$$

VII. $$\mathfrak{z} - 2\varkappa\mathfrak{D} - \beta D + \tfrac{1}{2}bA + \frac{3F}{4nn} + \frac{\mathfrak{E}(\mathfrak{A}+3A)}{nn} =$$

$$-(2\alpha-1)D' + \frac{A'(2\varkappa+\mathfrak{E})}{nn}$$

VIII. $$\mathfrak{z} - 2\varkappa\mathfrak{E} - \beta E + \tfrac{1}{2}bA + \frac{3G}{4nn} + \frac{\mathfrak{E}(\mathfrak{A}+3A)}{nn} =$$

$$-(2\alpha+1)E' + \frac{A'(2\varkappa+\mathfrak{E})}{nn}$$

IX. $$-3\varkappa\mathfrak{F} - \beta F + \tfrac{1}{2}bB + \frac{3A}{2nn} + \frac{3D}{4nn} + \frac{\mathfrak{A}\mathfrak{D}}{nn} + \frac{3A\mathfrak{D}}{nn}$$

$$+ \frac{3\mathfrak{A}D}{nn} + \frac{3AD}{nn} - \frac{3AA}{4nn} = -(4\alpha-1)F + \frac{4B'}{n}$$

$$+ \frac{A'(2\varkappa D+\mathfrak{D})}{nn} + \frac{D'(2\varkappa A+\mathfrak{A})}{nn}.$$

X. $$-2\varkappa\mathfrak{G} - \beta G + \tfrac{1}{2}bB + \frac{3A}{2nn} + \frac{3E}{4nn} + \frac{\mathfrak{A}\mathfrak{E}}{nn}$$

$$+ \frac{3A\mathfrak{E}}{nn} + \frac{3\mathfrak{A}E}{nn} + \frac{3AE}{nn} - \frac{3AA}{4nn} = -(4\alpha+1)G' + \frac{4B'}{n}$$

$$+ \frac{A'(2\varkappa E+\mathfrak{E})}{nn} + \frac{E'(2\varkappa A+\mathfrak{A})}{nn}$$

XI.

XI. $$\frac{1}{4}+\frac{1}{2}f-2\kappa\mathfrak{H}-6H+\frac{3(D+E)}{2nn}+\frac{3(J+K)}{4nn}$$
$$-\frac{3AD}{2nn}+\frac{\mathfrak{C}\mathfrak{C}}{2nn}-\frac{3AE}{2nn}+\frac{\mathfrak{D}\mathfrak{E}}{nn}+\frac{3\mathfrak{D}E}{nn}+\frac{3D\mathfrak{E}}{nn}+\frac{3DE}{nn}$$
$$+\frac{\mathfrak{A}\mathfrak{J}}{nn}+\frac{3\mathfrak{A}J}{nn}+\frac{3A\mathfrak{J}}{nn}+\frac{3AJ}{nn}=-2H'+\frac{A'(2\kappa J+\mathfrak{J})}{nn}$$
$$+\frac{E'(2\kappa D+\mathfrak{D})}{nn}+\frac{D'(2\kappa E+\mathfrak{E})}{nn}+\frac{(J'+K')(2\kappa A+\mathfrak{A})}{nn}$$

XII. $$\frac{15}{4}-2\kappa\mathfrak{J}-6J+\frac{1}{2}bD+\left(\frac{1}{4}-\frac{f}{2nn}\right)A+\frac{3H}{4nn}$$
$$+\frac{\mathfrak{H}(\mathfrak{A}+3A)}{nn}+\frac{3\mathfrak{C}D}{nn}=-2(\alpha-1)J'+\frac{3A'}{2n}$$
$$+\frac{D'(2n+\mathfrak{C})}{nn}+\frac{A'(2\kappa H+\mathfrak{H})}{nn}+\frac{2L'(2\kappa A+\mathfrak{A})}{nn}$$

XIII. $$\frac{15}{4}-2\kappa\mathfrak{K}-6K+\frac{1}{2}bE+\left(\frac{1}{4}-\frac{f}{2nn}\right)A+\frac{3H}{4nn}$$
$$+\frac{\mathfrak{H}(\mathfrak{A}+3A)}{nn}+\frac{3\mathfrak{C}E}{nn}=-2(\alpha+1)K'+\frac{3A'}{2n}$$
$$+\frac{E'(2n+\mathfrak{C})}{nn}+\frac{A'(2\kappa H+\mathfrak{H})}{nn}+\frac{2G'(2\kappa D+\mathfrak{D})}{nn}$$

XIV. $$-2\kappa\mathfrak{C}-6L+\frac{1}{2}bF+\left(\frac{1}{4}-\frac{f}{2nn}\right)B-\frac{3AD}{2nn}$$
$$+\frac{3D}{2nn}+\frac{3J}{4nn}+\frac{\mathfrak{D}\mathfrak{D}}{2nn}+\frac{3\mathfrak{D}D}{nn}+\frac{3DD}{2nn}=-2(2\alpha-1)L'+\frac{3B'}{n}$$
$$+\frac{2F'(2n+\mathfrak{C})}{nn}+\frac{A'(2\kappa J+\mathfrak{J})}{nn}+\frac{J'(2\kappa A+\mathfrak{A})}{nn}+\frac{D'(2\kappa D+\mathfrak{D})}{nn}$$

$$\text{XV.}\quad -2\kappa\mathfrak{M} - 6M + \tfrac{1}{2}bG + \left(\tfrac{1}{4} - \frac{f}{2nn}\right)B - \frac{3AE}{2nn}$$
$$+ \frac{3E}{2nn} + \frac{3K}{4nn} + \frac{\mathfrak{E}\mathfrak{E}}{2nn} + \frac{3\mathfrak{E}E}{nn} + \frac{3EE}{2nn} =$$
$$-2(2\alpha+1)M' + \frac{3B'}{n} + \frac{2G'(2n+\mathfrak{E})}{nn}.$$

§. 107. Nunc antequam hos valores inuenire queamus, verus valor ipsius α inuestigari debet: quod fiet ex valore integrali ipsius Φ, qui si vti §. 98. ponatur

$$\Phi = Or + \begin{matrix}\mathfrak{A}'\sin 2\eta \\ + \mathfrak{a}'kk\sin 2\eta\end{matrix} + \text{etc. obtinebitur.}$$

$$\kappa + \frac{A(3\kappa A+2\mathfrak{A})}{2n^4} + \frac{D(3\kappa D+2\mathfrak{D})}{2n^4}kk + \frac{A(3\kappa a+\mathfrak{a})}{2n^4}kk + \frac{a(3\kappa A+2\mathfrak{A})}{2n^4}kk =$$
$$= O - (\mathfrak{A}'+\mathfrak{a}'kk)\frac{(2\kappa A+\mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2\kappa D+\mathfrak{D})}{nn}kk = \alpha + \frac{1}{n}$$

vbi ex obseruationibus constat esse $O = 1,0085272$ Proxime autem esse supra inuenimus esse:

$\mathfrak{A}' = 0,01$	$\mathfrak{A} = -0,80$	$A = -1,25$
$\mathfrak{a}' = 0,05$	$\mathfrak{a} = -2,05$	$a = -12,60$
$\mathfrak{D}' = -0,44$	$\mathfrak{D} = -3,60$	$D = 34,25$

atque $\kappa = 1,0085$; $kk = 0,003$; $nn = 175,71795$ vnde inuenimus $O + 0,000649 = \alpha + \frac{1}{n} = \kappa + 0,000285$

§. 108. Cum nunc sit $O = 1,0085272$, erit $\alpha + \frac{1}{n} = 1,009176$, et ob $\frac{1}{n} = 0,075438$, habebitur vrues valor:

$$\alpha = 0,933738 \quad \text{et} \quad l\alpha = 9,9702255$$
$$\text{atque}\quad \kappa = 1,008991 \quad \text{et} \quad l\kappa = 0,0038874$$

Hine

Hinc iam primo obtinemus:

$\mathfrak{A} = -0,80313 \qquad l\text{-}\mathfrak{A} = 9,9047898$

Deinde cum sit satis prope $\mathfrak{C} = -0,67465$, erit

$\frac{(2n+\mathfrak{C})}{nn} = 0,147037$ et $l\frac{2n+\mathfrak{C}}{nn} = 9,1674260$

$\mathfrak{D} = -3,593620 \ldots l\text{-}\mathfrak{D} = 0,5555310$

$\mathfrak{E} = -1,087320 \ldots l\text{-}\mathfrak{E} = 0,0363580$

atque porro ex valore ipsius A proxime cognito erit

$\mathfrak{B} = +0,006967 \ldots l\,\mathfrak{B} = 7,8430540$

et quia est satis prope $B = 0,0128$, erit $A' = 2\alpha A - 0,000720$ et $B' = 4\alpha B - 0,023926$, vnde fit:

$\frac{1}{2} + 1,62172 - \mathfrak{C}A = -4\alpha\alpha A + 0,00144\alpha + 0,000374\alpha A$
$\qquad - 0,152\,\alpha B + 0,00091$

$+ 0,01247 - \mathfrak{C}B = -16\alpha\alpha B + 0,09570\alpha - 0,038032\alpha A$
$\qquad + 0,000014$

§. 109. Nunc primum quaeri debent valores litterarum f, h et k: et cum sit $\varepsilon = 5°, 9'$ et $2\gamma - \frac{1}{2}\delta = -1,3899$ proxime, reperietur

$f = 1,093757$ et $lf = 0,0389208$

$h = 3,0423 \ldots lh = 0,4832020$

$k = 1,01591 \ldots lk = 0,0068560$

hincque erit

$2,4720\, A = -3,11947 - 0,142\, B$

$12,9369\, B = +0,07684 - 0,0355\, A$

unde concluditur fore:

$A = -1,262463 \ldots l\text{-}A = 0,1012186$

$B = +0,009404 \ldots l\,B = 7,9733114$

 Porro

Porro vero eſt

$$D' = 0,867676\ D + 0,185628$$

$$E' = 2,867676\ E + 0,185628$$

et $A' = -2,35859 \quad . \quad . \quad l\!-\!A' = 0,3726530$

§. 110. Ex his valoribus aequationes VII et VIII induent has formas,

$$3 + 7,25185 - 6D - 1,92040 - 0,00244 + 0,01704 = -0,75286\ D - 0,16106 - 0,34680$$

$$3 + 2,19420 - 6E - 1,92040 + 0,00006 + 0,01704 = -8,22357\ E - 0,53232 - 0,34680$$

vnde prodibit

$$D = +33,6600 \quad . \quad . \quad . \quad l\,D = 1,5271130$$

$$E = -\ 0,5785 \quad . \quad . \quad . \quad l\!-\!E = 9,7623410$$

ergo

$$D' = 29,39153$$

$$E' = -1,47347$$

Ex his nanciſcemur ſequentes formulas pro calculo ſequenti

$$\frac{2\varkappa A + \mathfrak{A}}{nn} = -0,01884 \qquad l\!-\!\frac{(2\varkappa A + \mathfrak{A})}{nn} = 8,275051$$

$$\frac{2\varkappa B + \mathfrak{B}}{nn} = +0,000148 \qquad l\,\frac{2\varkappa B + \mathfrak{B}}{nn} = 6,170262$$

$$\frac{2\varkappa D + \mathfrak{D}}{nn} = +0,36611 \qquad l\,\frac{2\varkappa D + \mathfrak{D}}{nn} = 9,563604$$

$$\frac{2\varkappa E + \mathfrak{E}}{nn} = -0,01283 \qquad l\!-\!\frac{(2\varkappa E + \mathfrak{E})}{nn} = 8,108292$$

§. 111.

§. 111. Ex his iam porro inuenitur

$$\mathfrak{C} = -0,64383 \quad \ldots \quad l\text{-}\mathfrak{C} = 9,808772$$

atque $C' = -0,13847$

Porro valores litterarum $\mathfrak{F}$ et $\mathfrak{G}$ determinabuntur per has aequationes

$$(4a-1)\,\mathfrak{F} = 0,002049 - 0,294032 + 0,067699 + 0,021554 - 0,287335$$

$$(4a+1)\,\mathfrak{G} = 0,002049 + 0,010306 + 0,020484 + 0,021554 + 0,004939$$

ex quibus reperitur

$$\mathfrak{F} = -0,17957 \quad \ldots \quad l\text{-}\mathfrak{F} = 9,254241$$

$$\mathfrak{G} = +0,01253 \quad \ldots \quad l\,\mathfrak{G} = 8,097936$$

atque

$$F' = (4a-1)\,F - 0,00283 + 0,46219 + 0,63411 = (4a-1)\,F + 1,09347$$

$$G' = (4a+1)\,G - 0,00283 - 0,01620 - 0,01090 = (4a+1)\,G - 0,02993$$

vnde aequationes IX et X prodibunt.

$$+0,36238 - 1,01591\,F + 0,60353 - 0,00680 - 1,00349 = -(4a-1)^2\,F - 2,98415 - 0,86349 + 0,00337 - 0,55370$$

$$-0,02528 - 1,01591\,G + 0,00353 - 0,00680 + 0,04634 = -(4a+1)^2\,G + 0,14473 + 0,03027 + 0,00337 + 0,02776$$

seu

$$6,43486\;F = -\;3,75359$$

$$21,40769\;G = +\;0,18534$$

§. 110. Hinc prodeunt sequentes valores correcti pro F et G,

$$F = -0,58360 \quad \ldots \quad l\text{-}F = 9,766112$$

$$G = +0,00866 \quad \ldots \quad l\,G = 7,937400$$

Ex formula autem sexta hinc leui calculo colligitur fore:

$$\gamma - f = 1,58161 \quad \text{et} \quad \gamma = 2,67537$$

Valores autem ex F et G deriuati erunt

$$F' = -0,49919 \quad \text{et} \quad G' = 0,01107$$

$$\frac{2\varkappa F + \mathfrak{F}}{nn} = -0,00772 \quad l - \frac{(2\varkappa F + \mathfrak{F})}{nn} = 7,887828$$

$$\frac{2\varkappa G + \mathfrak{G}}{nn} = 0,00017 \quad l \frac{2\varkappa G + \mathfrak{G}}{nn} = 6,232305$$

§. 113. Nunc procedamus ad valores litterarum $\mathfrak{a}$ et $\mathfrak{b}$ qui erunt

$$1,867676\,\mathfrak{a} = -3,75000 - 0,68827 - 0,13148 - 0,03764\,\mathfrak{b} + 0,02776$$

$$3,735352\,\mathfrak{b} = -0,57467 - 0,04913 - 0,39807 - 0,01884\,\mathfrak{a} + 0,04611$$

vnde reperitur:

$$\mathfrak{a} = -2,42686 \quad \ldots \quad l - \mathfrak{a} = 0,385044$$

$$\mathfrak{b} = -0,24899 \quad \ldots \quad l - \mathfrak{b} = 9,396182$$

huncque porro

$$a' = 2aa + 0,03768\,b - 4,86420 + 0,16048$$

$$b' = 4ab + 0,01884\,a + 0,16887 + 0,64373 + 0,01450\,a - 0,01744$$

seu

$$a' = 2aa + 0,03768\,b - 4,70372$$

$$b' = 4ab + 0,03334\,a + 0,79516$$

§. 114. Aequationes IV et V hinc induent sequentes formas:

IV.

$$\text{IV.} + 3{,}75000 - 1{,}01591\,a + 50{,}32200 - 0{,}36363 = -4aaa$$
$$+ 4{,}89730)$$
$$-0{,}07034\,b + 8{,}78034 + 4{,}10500 - 0{,}36551 - 0{,}00125\,a$$
$$-0{,}14067\,b \qquad -0{,}02996$$

$$\text{V.} + 0{,}50246 - 1{,}01591\,b - 0{,}87350 + 0{,}28239 - 0{,}95732$$
$$-0{,}02155\,a$$
$$= -16aab - 0{,}12447\,a - 2{,}96860 - 0{,}00142\,b$$
$$-0{,}03517\,a + 0{,}17723$$
$$-0{,}14354$$
$$-0{,}91659$$

Hinc fit

$$2{,}47355\,a = -0{,}21101\,b - 46{,}11580$$
$$12{,}93836\,b = -0{,}13809\,a - 2{,}80553$$

et $a = -18{,}64200 \;\ldots\; l{-}a = 1{,}270493$

$b = -0{,}01794 \;\ldots\; l{-}b = 8{,}253822$

ex quibus oriuntur:

$a' = -39{,}52164 \;\ldots\; b' = +0{,}10663$

et $\frac{2na+\mathfrak{a}}{nn} = -0{,}22790 \quad l{-}\frac{(2na+\mathfrak{a})}{nn} = 9{,}357744$

valor autem ipſius $\frac{2nb+\mathfrak{b}}{nn}$ nullius plane erit momenti, vnde eum praetermittimus.

§. 115. Ex prima autem aequatione §. 106. colligitur

$$\tfrac{1}{2}\,\delta = \gamma - f + 0{,}02285$$

ſupra autem inuenimus eſſe $\gamma - f = 1{,}58161$; ſicque erit $\frac{1}{2}\,\delta = 1{,}60446$ atque

$\delta = 3{,}20892 \;\ldots\; l\delta = 0{,}506358$

Nunc cum ſit proxime: $\mathfrak{H} = -0{,}123$; $H = -1{,}033$ ideoque $\frac{2nH+\mathfrak{H}}{nn} = -0{,}0126$; ob $\mathfrak{L} = -1{,}453$ et $L = +6{,}252$; habebimus

-0,

$$
\begin{aligned}
-0{,}132324\,\mathfrak{J} = -3{,}75000 &+ 0{,}00882 - 0{,}09088\\
&+ 0{,}05336 - 0{,}01012\\
&- 0{,}52839 + 0{,}05474\\
+3{,}867676\,\mathfrak{K} = -3{,}75000 &+ 0{,}00882 - 0{,}09088\\
&+ 0{,}01012\\
&- 0{,}15987 + 0{,}00917
\end{aligned}
$$

Hinc reperitur

$$\mathfrak{J} = 32{,}05945 \quad \ldots \quad l\,\mathfrak{J} = 1{,}505956$$

$$\mathfrak{K} = -1{,}02714 \quad \ldots \quad l{-}\mathfrak{K} = 0{,}011629$$

§. 116. Hinc vlterius progrediendo habebimus.

$$
\begin{aligned}
J' = 2(\alpha-1)J &+ 0{,}28571 - 4{,}94924 + 0{,}23502 =\\
&- 0{,}01591 \qquad\qquad - 0{,}08015\\
2(\alpha-1)J &- 4{,}52457\\
K' = 2(\alpha+1)K &+ 0{,}28571 + 0{,}08507 - 0{,}00317 =\\
&- 0{,}01591\\
2(\alpha+1)K &+ 0{,}35170
\end{aligned}
$$

vnde aequationes XII et XIII fiunt

$$
\begin{aligned}
+3{,}75000 - 64{,}69550 &- 1{,}01591\,J + 51{,}28180\\
-0{,}94292 + 0{,}00321 &- 0{,}36999 =\\
-\underline{0{,}00441)}&\\
-4(\alpha-1)^2 J - 0{,}59871 &- 0{,}26690 + 4{,}32164\\
&+ 0{,}02972 - 0{,}47004\\
\hline
+3{,}75000 + 2{,}07275 &- 1{,}01591\,K - 0{,}88006\\
-0{,}94292 + 0{,}00321 &+ 0{,}00636 =\\
-\underline{0{,}00441)}&\\
-4(\alpha+1)^2 K - 1{,}36026 &- 0{,}26690 - 0{,}21665\\
&+ 0{,}02972 + 0{,}00810
\end{aligned}
$$

ex quibus colligitur fore

$$J = -14{,}09600 \quad \ldots \quad l{-}J = 1{,}149096$$

$$K = -0{,}41676 \quad \ldots \quad l{-}K = 9{,}619888$$

Hinc

Hinc $J' = -2,65933$ . . . $K' = -1,26020$

atque $\frac{2nJ+\mathfrak{J}}{nn} = +0,02057$. $l\frac{2nJ+\mathfrak{J}}{nn} = 8,313172$

$\frac{2nK+\mathfrak{K}}{nn} = -0,01063$. $l-\frac{2nK+\mathfrak{K}}{nn} = 8,026598$

§. 117. Quaeramus iam valorem ipsius $\mathfrak{H}$, ex aequatione

$$2\mathfrak{H} = -0,45434 - 0,39771 - 0,01652 + 0,62330$$

erit $\mathfrak{H} = -0,12264$ . . . $l-\mathfrak{H} = 9,088632$

hincque reperitur: $H' = 2H + 0,08011$

vnde aequatio XI praebet:

$$2,04688 + 0,24748 - 1,01591\,H + 0,27805 - 3,06194$$
$$+ 0,36275 + 0,00118 - 0,00623 \quad + 0,02224 + 0,03552$$
$$- 0,62485 - 0,33247 - 0,83753 \quad + 0,49710 =$$
$$- 4\,H - 0,16022 - 0,04851 \quad - 0,53944 - 0,37802$$
$$+ 0,07384$$

seu $2,98409\,H = -2,68053$

Ergo $H = -0,89829$ . . . $l-H = 9,953417$

$H' = -1,71647$; $\frac{2nH+\mathfrak{H}}{nn} = -0,01102$

§. 118. Tandem supersunt litterae $\mathfrak{L}$ et $\mathfrak{M}$

$$2\,(2n-1)\,\mathfrak{L} = -0,45434 - 0,05280 - 0,01652 - 1,31570$$
$$+ 0,00158 \qquad - 0,60396$$
$$- 0,00015$$

$$2\,(2n+1)\,\mathfrak{M} = +0,00158 + 0,00368 + 0,01935$$
$$- 0,00015$$

Hinc $\mathfrak{L} = +1,40715$ . . $l\mathfrak{L} = 0,148340$

$\mathfrak{M} = +0,00426$ . . $l\mathfrak{M} = 7,629896$

Deinde vero habebitur:

$$L' = 2(2a-1)\,L - 0{,}00213 + 0{,}17162 - 12{,}31170 - 0{,}02596$$
$$+ 0{,}00013 \qquad\qquad - 0{,}26555$$

$$M' = 2(2a+1)\,M - 0{,}00213 - 000254 - 0{,}00742$$
$$+ 0{,}00013$$

seu

$$L' = 2\,(2a-1)\,L - 12{,}43359$$
$$M' = 2\,(2a+1)\,M - 0{,}01196$$

§. 119. Nunc denique aggrediamur aequationes XIV et XV

XIV. $-2{,}83960 - 1{,}01591\,L - 0{,}88774 + 0{,}00702 + 0{,}36275$
$+ 0{,}28733 \qquad - 0{,}06016 + 0{,}03675 + 7{,}60650 =$
$-3{,}01144\,L + 21{,}57666 + 0{,}00255 - 0{,}14680 + 10{,}75272$

XV. $+0{,}00860 - 1{,}01591\,M + 0{,}01317 + 0{,}00702 - 0{,}00623$
$- 0{,}00494 \qquad - 0{,}00176 + 0{,}00336 + 0{,}01360 =$
$-32{,}89415\,M + 0{,}06859 + 0{,}00255 + 0{,}00325$

ex quibus eruitur

$$L = +\,13{,}86720 \;\cdots\; lL = 1{,}141988$$
$$M = +\,0{,}00131 \;\cdots\; lM = 7{,}117165$$

hincque $L' = 11{,}3090$ et $M' = -\,0{,}00445$

$$\frac{2\varkappa L + \mathfrak{L}}{nn} = 0{,}15125 \;\cdots\; l\frac{2\varkappa L + \mathfrak{L}}{nn} = 9{,}179684$$

$$\frac{2\varkappa M + \mathfrak{M}}{nn} = 0{,}00004 \qquad l\frac{2\varkappa M + \mathfrak{M}}{nn} = 5{,}592770$$

Ex his valoribus nouae correctiones inueniri possent, sed differentiae prodirent tam exiguae, vt operae pretium non sit eas inuestigare.

§. 120.

§. 120. His igitur valoribus inuentis, denotante iam a distantiam Lunae mediam a Terra, et eius distantia curtata $= x$, cum sit $x = \frac{(1-kk)a}{1-k\cos r}u$, erit:

	log. coefficient:
$u = 1 - 0{,}0074991\cos 2\eta$	7,875009
$+ 0{,}0000532\cos 4\eta$	5,725912
$+ 0{,}191557k\cos(2\eta - r)$	9,282297
$- 0{,}003293k\cos(2\eta + r)$	7,517525
$- 0{,}003321k\cos(4\eta - r)$	7,521296
$+ 0{,}000049k\cos(4\eta + r)$	5,692584
$- 0{,}00511kk\cos 2r$	7,708601
$- 0{,}08022kk\cos(2\eta - 2r)$	8,904280
$- 0{,}00237kk\cos(2\eta + 2r)$	7,375072
$+ 0{,}07892kk\cos(4\eta - 2r)$	8,897172
$+ 0{,}00001kk\cos(4\eta + 2r)$	4,872349

vbi quidem in duobus primis terminis simul eos, qui per kk erant affecti, sumus complexi, posito $k = 0{,}05445$. Etiamsi enim hic valor non omnino esset iustus, tamen inde in his terminis minimis nullus error nasci poterit.

§. 121. Porro quoque hinc ex §. 116. valorem ipsius $\frac{d\Phi}{dr}$ determinabimus, quatenus a sola excentricitate orbitae lunaris pendet.

		log. coeff.
$\frac{d\Phi}{dr} =$	$1,009276$	$0,004010$
	$+ 0,0195144 \cos 2\eta$	$8,290355$
	$- 0,0000322 \cos 4\eta$	$5,507856$
	$- 0,001231k \cos r$	$7,090258$
	$- 0,366103k \cos(2\eta - r)$	$9,563604$
	$+ 0,012832k \cos(2\eta + r)$	$8,108292$
	$+ 0,002829k \cos(4\eta - r)$	$7,451633$
	$- 0,000171k \cos(4\eta + r)$	$6,232305$
	$+ 0,01182kk \cos 2r$	$8,072618$
	$- 0,02057kk \cos(2\eta - 2r)$	$8,313172$
	$+ 0,01063kk \cos(2\eta + 2r)$	$8,026598$
	$- 0,09883kk \cos(4\eta - 2r)$	$8,994889$
	$- 0,00004kk \cos(4\eta + 2r)$	$5,592770$

§. 122.

§. 122. Cum nunc sit $\frac{d\theta}{dr} = \frac{ds}{dr} = \frac{1+2ee}{n} + \frac{2}{n} k \cos r + \frac{3}{2n} kk \cos 2r$, erit

			log. coeff.
$\frac{d\eta}{dr} =$	0,933838		9,970272
	+ 0,0195144	$\cos 2\eta$	8,290355
	— 0,0000322	$\cos 4\eta$	5,507856
	— 0,152101 k	$\cos r$	9,182132
	— 0,366103 k	$\cos(2\eta - r)$	9,563604
	+ 0,012829 k	$\cos(2\eta + r)$	8,108292
	+ 0,002829 k	$\cos(4\eta - r)$	7,451633
	— 0,000171 k	$\cos(4\eta + r)$	6,232305
	— 0,10133 kk	$\cos 2r$	9,005738
	— 0,02057 kk	$\cos(2\eta - 2r)$	8,313172
	+ 0,01063 kk	$\cos(2\eta + 2r)$	8,026598
	— 0,09883 kk	$\cos(4\eta - 2r)$	8,994889
	— 0,00004 kk	$\cos(4\eta + 2r)$	5,592770

quae formulae ad motum Lunae horarium tam absolutum quam a sole adhiberi possunt, quemadmodum illa distantiam definiens diametro apparenti et parallaxi horizontali inuestigandae inseruit.

§. 23. Quaeramus nunc valorem integralem pro longitudine Lunae Φ, quatenus a sola excentricitate orbitae lunaris pendet, ac ponamus,

$$\begin{aligned}\Phi = O r &+ \mathfrak{A}' \sin 2\eta + a'kk \sin 2\eta + \mathfrak{B}' \sin 4\eta + b'kk \sin 4\eta \\ &+ \mathfrak{C}' k \sin r + \mathfrak{D}' k \sin(2\eta - r) + \mathfrak{F}' k \sin(4\eta - r) \\ &+ \mathfrak{E}' k \sin(2\eta + r) + \mathfrak{G}' k \sin(4\eta + r) \\ &+ \mathfrak{H}' kk \sin 2r + \mathfrak{J}' kk \sin(2\eta - 2r) + \mathfrak{L}' kk \sin(4\eta - 2r) \\ &+ \mathfrak{K}' kk \sin(2\eta + 2r) + \mathfrak{M}' kk \sin(4\eta + 2r)\end{aligned}$$

atque sequentes obtinebimus formulas:

$$+0{,}0188387 = 2\alpha\mathfrak{A}' - \frac{\mathfrak{A}'(2\kappa B + \mathfrak{B})}{nn} - \frac{2\mathfrak{B}'(2\kappa A + \mathfrak{A})}{nn}$$

$$-0{,}0000370 = 4\alpha\mathfrak{B}' - \frac{\mathfrak{A}'(2\kappa A + \mathfrak{A})}{nn}$$

$$-0{,}001231 = \mathfrak{C}' - \frac{\mathfrak{A}'(2\kappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{A}'(2\kappa E + \mathfrak{E})}{nn} - \frac{(\mathfrak{D}' + \mathfrak{E}')(2\kappa A + \mathfrak{A})}{nn}$$

$$-0{,}366103 = (2\alpha - 1)\mathfrak{D}' - \frac{\mathfrak{A}'(2\kappa + \mathfrak{C})}{nn}$$

$$+0{,}012832 = (2\alpha + 1)\mathfrak{E}' - \frac{\mathfrak{A}'(2\kappa + \mathfrak{C})}{nn}$$

$$+0{,}002829 = (4\alpha - 1)\mathfrak{F}' - \frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2\kappa D + \mathfrak{D})}{nn} - \frac{\mathfrak{D}'(2\kappa A + \mathfrak{A})}{nn}$$

$$-0{,}000171 = (4\alpha + 1)\mathfrak{G}' - \frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2\kappa E + \mathfrak{E})}{nn} - \frac{\mathfrak{E}'(2\kappa A + \mathfrak{A})}{nn}$$

+

$$+0{,}22790 = 2n\mathfrak{a}' - \frac{(\mathfrak{D}'+\mathfrak{E}')(2n+\mathfrak{C})}{nn} - \frac{2\mathfrak{F}'(2nD+\mathfrak{D})}{nn} - \frac{2\mathfrak{b}'(2nA+\mathfrak{A})}{nn}$$

$$+0{,}00163 = 4n\mathfrak{b}' - \frac{\mathfrak{a}'(2nA+\mathfrak{A})}{nn} - \frac{\mathfrak{E}'(2nD+\mathfrak{D})}{nn} - \frac{2(\mathfrak{F}'+\mathfrak{G}')(2n+\mathfrak{C})}{nn}$$

$$+0{,}01182 = 2\mathfrak{H}' - \frac{\mathfrak{E}'(2nD+\mathfrak{D})}{nn} - \frac{\mathfrak{A}'(2nJ+\mathfrak{J})}{nn} - \frac{(\mathfrak{J}'+\mathfrak{K}')(2nA+\mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2nE+\mathfrak{E})}{nn}$$

$$-0{,}02057 = 2(n-1)\mathfrak{J}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{D}'(2n+\mathfrak{C})}{nn} - \frac{\mathfrak{A}'(2nH+\mathfrak{H})}{nn} - \frac{2\mathfrak{L}'(2nA+\mathfrak{A})}{nn}$$

$$+0{,}01063 = 2(n+1)\mathfrak{K}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{E}'(2n+\mathfrak{C})}{nn} - \frac{\mathfrak{A}'(2nH+\mathfrak{H})}{nn} - \frac{2\mathfrak{G}'(2nD+\mathfrak{D})}{nn}$$

$$-0{,}09883 = 2(2n-1)\mathfrak{L}' - \frac{3\mathfrak{B}'}{n} + \frac{2\mathfrak{F}'(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{A}'(2nJ+\mathfrak{J})}{nn} - \frac{\mathfrak{J}'(2nA+\mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2nD+\mathfrak{D})}{nn}$$

$$-0{,}00004 = 2(2n+1)\mathfrak{M}' - \frac{3\mathfrak{B}'}{n} - \frac{2\mathfrak{G}'(2n+\mathfrak{C})}{nn}$$

§. 124.

§. 124. Ex his eliciuntur valores sequentes:

$\mathfrak{A}' = +0{,}0100887$	$l\,\mathfrak{A}' = 8{,}003837$	$a' = 0{,}09140$
$\mathfrak{B}' = -0{,}0000409$	$l{-}\mathfrak{B}' = 5{,}611723$	$l\,a' = 8{,}960934$
$\mathfrak{C}' = +0{,}010146$	$l\,\mathfrak{C}' = 8{,}006295$	$b' = 0{,}00089$
$\mathfrak{D}' = -0{,}420226$	$l{-}\mathfrak{D}' = 9{,}623483$	$l\,b' = 6{,}949340$
$\mathfrak{E}' = +0{,}004992$	$l\,\mathfrak{E}' = 7{,}698261$	$\mathfrak{A}' + a'kk = 0{,}0103597$
$\mathfrak{F}' = +0{,}005286$	$l\,\mathfrak{F}' = 7{,}723163$	$\mathfrak{B}' + b'kk = -0{,}0000382$
$\mathfrak{G}' = -0{,}000086$	$l{-}\mathfrak{G}' = 5{,}935307$	$l(\mathfrak{A}' + a'kk) = 8{,}015347$
$\mathfrak{H}' = +0{,}00420$	$l\,\mathfrak{H}' = 7{,}623250$	$l{-}(\mathfrak{B}' + b'kk) = 5{,}582063$
$\mathfrak{I}' = +0{,}57328$	$l\,\mathfrak{I}' = 9{,}758367$	
$\mathfrak{K}' = +0{,}00318$	$l\,\mathfrak{K}' = 7{,}502427$	
$\mathfrak{L}' = -0{,}15083$	$l{-}\mathfrak{L}' = 9{,}178488$	
$\mathfrak{M}' = -0{,}00002$	$l{-}\mathfrak{M}' = 5{,}301030$	

§. 125. Pro longitudine ergo Lunae habemus hactenus hanc formulam

		log. coeff.
$\phi = \text{Const.}$	$+ 1{,}0085272\, r$	0,003687
	$+ 0{,}0103597 \sin 2\eta$	8,015347
	$- 0{,}0000382 \sin 4\eta$	5,582063
	$+ 0{,}010146k \sin r$	8,006295
	$- 0{,}420226k \sin(2\eta - r)$	9,623483
	$+ 0{,}004992k \sin(2\eta + r)$	7,698261
	$+ 0{,}005286k \sin(4\eta - r)$	7,723163
	$- 0{,}000086k \sin(4\eta + r)$	5,935307
	$+ 0{,}00420kk \sin 2r$	7,623250
	$+ 0{,}57328kk \sin(2\eta - 2r)$	9,758367
	$+ 0{,}00318kk \sin(2\eta + 2r)$	7,402427
	$- 0{,}15083kk \sin(4\eta - 2r)$	9,178488
	$- 0{,}00002kk \sin(4\eta + 2r)$	5,301030

§. 126.

§. 126. Quodsi iam ponamus $k = 0{,}05445$, et hos coefficientes ad minuta secunda cum partibus decimalibus reducamus, longitudo ϕ ita exprimetur vt sit:

		log. coeff.
$\phi =$ Const.	$+ 1{,}0085272 \; r$	
	$+ 2136'',8 \sin 2\eta$	3,329772
	$- 7,\;8 \sin 4\eta$	0,895488
	$+ 113,\;9 \sin r$	2,056718
	$- 4719,\;6 \sin(2\eta - r)$	3,673906
	$+ 56,\;1 \sin(2\eta + r)$	1,748684
	$+ 59,\;4 \sin(4\eta - r)$	1,773586
	$- 1,\;0 \sin(4\eta + r)$	9,985730
	$+ 2,\;5 \sin 2r$	0,409671
	$+ 350,\;6 \sin(2\eta - 2r)$	2,544788
	$+ 1,\;5 \sin(2\eta + 2r)$	0,188848
	$- 92,\;2 \sin(4\eta - 2r)$	1,064909
	$- 0,\;0 \sin(4\eta + 2r)$	8,087451

Hisque formulis praecipuae inaequalitates, quibus motus Lunae perturbatur, continentur.

CAPUT VIII.

DE MOTU APOGEI LUNAE.

§. 127.

His inuentis iam arduam illam de motu apogei Lunae quaestionem examinare, atque adeo decidere licebit. Quanquam enim in praecedentibus calculis vbique verum apogei motum, quem obseruationes ostendunt, introduxi, ita vt id ipsum, quod in controversia est, assumsisse videar; tamen quoniam in hunc ipsum finem terrae vim, qua luna vrgetur, indefinitam sum contemplatus, dum rationi distantiarum reciprocae duplicatae terminum indefinitum adiunxi, vnde littera μ in calculum est ingressa, iudicium de eo apogei motu, qui Theoriae Neutonianae esset consentaneus, non erit difficile. Quodsi enim valor litterae μ nihilo aequalis reperiatur, hinc concludendum erit Theoriam Neutoni cum phaenomenis perfecte consentire; sin autem pro littera μ notabilis prodeat valor, Theoria ista insufficiens erit censenda.

§. 128. Motus autem apogei, quoniam huius rei in calculo nusquam mentio est facta, in ea continetur proportione, quam motus lunae medius ad motum anomaliae tenere est positus. Cum enim remotis lunae inaequalitatibus, quae regulae Keplerianae aduersantur, longitudo lunae vera obtineatur, si eius anomalia vera r ad longitudinem apogei addatur: denotet ν longitudinem apogei, eritque longitudo vera $\phi = \nu + r$, vnde

vnde fit $\varpi = \Phi - r$. Ex quo intelligitur, si $\Phi - r$ quantitatem designet constantem, apogeum in quiete relinqui, sin autem $\Phi - r$ valorem variabilem obtineat, tum apogeum quoque lunae motum esse habiturum.

§. 129. Cum autem terminos illos omnes, qui sinus angulorum implicant, ideoque inaequalitates periodicas continent, quibus apogei motus non afficitur, omittimus, per integrationem deducimur ad huiusmodi formulam $\Phi = \text{Const.} + Or$, vnde propterea habetur longitudo apogei $\varpi = \text{Const.} + (O-1)\, r$. Hinc consequimur sequentes proportiones:

I. Vt 1 ad $O-1$, ita motus anomaliae lunae ad motum apogei.

II. Vt O ad 1, ita motus lunae medius ad motum anomaliae.

III. Vt O ad $O-1$, ita motus lunae medius ad motum apogei.

§. 130. Si obseruationes consulamus, valor litterae O reperitur $= 1{,}0085272$, quem etiam in calculo vbique adhibui; propterea quod propositum erat non tam in istum valorem a priori inquirere, quam ipsam potius Theoriam ita instituere, atque si opus fuerit, emendare, vt motus inde apogei experientiae consentaneus resultaret. Vicissim autem Theoria stabilita, siue Neutoniana siue alia, quae ex determinato pro μ substituto valore oriatur, facile erit valorem ipsius O a priori eruere, quem deinceps cum valore vero 1,0085272 conferre licebit. Vel inuento valore ipsius O, apogeum lunae interuallo mensis apogistici progredietur per spatium $(O-1)\,360°$, interuallo autem mensis periodici per spatium $(1-\frac{1}{O})\,360°$. Secundum obseruationes autem apogeum promouetur

vno mense apogistico per spatium $3°, 4', 11''$.
vno mense periodico per spatium $3, 2, 38$

§. 131. Ex calculo autem §. 107. exposito valor litterae O ex elementis ante assumtis ita definitur, vt sit $O+0,000649 = \varkappa + 0,000285$ siue $O = \varkappa - 0,000364$. Etsi enim haec exigua particula 0,000364 iam ex valore ipsius α veritati consentanee assumto est orta, tamen perspicuum est, leuem differentiam nullius hic momenti futuram fuisse. Verum littera $\varkappa$ per Theoriam ita erat assumta, vt esset

$$\varkappa = \sqrt{\left(1+\frac{3+4\mu+\delta}{2nn}\right)}$$

vbi quidem valor ipsius nn ex motu medio lunae ad motum solis relato habetur, ita vt sit sine respectu ad motum apogei habito, $nn = 175,71795$. Ergo pro Theoria Neutoniana est

$$\varkappa = \sqrt{\left(1+\frac{3+\delta}{2nn}\right)} \quad \text{et} \quad O = \sqrt{\left(1+\frac{3+\delta}{2nn}\right)} - 0,000364.$$

§. 132. Hic igitur patet totam hanc inuestigationem ad inuentionem litterae δ reduci, cuius valor, vti ex superiori calculo manifestum est, a pluribus litteris et coefficientibus terminorum, quos ante eruere oportebat, pendet, ita vt neglecta hac littera δ motus apogei nullo modo recte definiri queat. Initio quidem vbi hanc litteram

teram in calculum induximus, quod factum est §. 44. haec res leuis momenti est visa; cum enim pro CC, quae erat constans per integrationem in calculum ingressa, valorem vero proximum inuenissemus $1+\frac{3+4\mu}{2nn}$, quoniam facile erat praeuidere, reliquis adhibitis elementis ad motum lunae pertinentibus, hunc valorem aliquantum immutari posse, pro vero valore ipsius $\frac{CC}{1-kk}$ posuimus $1+\frac{3+4\mu+\delta}{2nn}$. Deinde autem valor ipsius δ potissimum pendet a valore litterae γ, qua vsi sumus ad verum valorem constantis $\frac{m}{nn}=1+\frac{2+3\mu+\gamma}{nn}$ obtinendum, cum proxime verus esset inuentus $=1+\frac{2+3\mu}{nn}$.

§. 133. Ab his ergo litteris γ et δ, quae initio nullius fere vsus esse videbantur, determinatio motus apogei potissimum pendet, quae cum ex pluribus atque adeo omnibus inaequalitatibus lunae ab excentricitate orbitis determinari debeant, mirum sane non est, quod legitima motus apogei designatio, cum tantis implicata sit difficultatibus, tam dudum fuerit abscondita. Plerique enim, qui motum apogei ex sola Theoria concludere sunt annisi, ad omnes has inaequalitates non respexerunt, atque calculum perinde administrauerunt, ac si hic litteras γ et δ neglexissemus. Ac si non defuere, qui sibi persuaserunt, motum apogei cum Theoria Neutoniana consentire, ii plerumque per errorem calculi seducti ad veritatem peruenisse sibi sunt visi. Quin etiam Ipse Neutonus Theoriae suae in motu apogei determinando parum tribuisse videtur.

§. 134. Hinc ex neglectu harum litterarum γ et δ, seu ex alia omissione eodem recidente, factum est, vt Theoria Neutoni obseruationibus circa motum apogei lunae institutis plane non satisfacere sit putata; quae opinio etiam ita inualuit, vt perspicacissimus quisque hanc Theoriam insufficientem pronunciaret. Atque sagacissimus Clairaltius huic opinioni vehementissime erat addictus, antequam publice in contrarias partes discesserat. Eadem scilicet ratione ob neglectum minutarum illarum particularum erat deceptus, qua et ego fateri cogor, me per complures annos constanter esse opinatum, ex Theoria Neutoni pro motu apogei Lunae non vltra semissem prodire, ita vt error vltra semissem exsurgens committeretur.

§. 135. Fons itaque huius erroris, qui nisi summa circumspectio adhibeatur, vix euitatur, in eo latet, quod in calculo debita illa constantium determinatio, pro qua equidem hic litteras γ et δ adhibui, negligatur. Quemadmodum per hanc omissionem dimidius tantum apogei motus eliciatur, ostendisse iuuabit. Sit igitur $\delta = o$, atque littera illius O secundum Theoriam Neutonianam, qua est $\mu = o$, valor erit $O = V\,(1 + \frac{3}{2nn}) - 0,000364$; qui euolutus fit: $O = 1,0042592 - 0,000364$. Quare etiamsi particula 0,000364 vtpote ex profundiori indagine nata praetermittatur, tamen iste valor pro $O = 1,0042592$, si cum vero per obseruationes cognito $O = 1,0085272$ comparetur, exacte fere dimidium motum apogei praebet;

bet; atque adeo haec tam accurata medietas non parum digna videtur.

§. 136. Jam videamus, quam prope valorem litterae δ adhibendo ad veritatem perducamur. Inuenimus autem (115) $\delta = 3{,}20892$, vnde prodit

$$\sqrt{\left(1 + \frac{3+\delta}{2nn}\right)} = 1{,}0087947$$

qui valor iam maior eſt quam verus 1,0085272, ſed recordandum eſt inde ſubtrahi debere 0,000364, ſicque relinquetur O = 1,0084307, ex quo motus progreſſiuus apogei pro interuallo menſis apogiſtici prodibit = 3° 2′ 9″ et pro interuallo menſis periodici = 3° 0′ 37″, qui numeri duobus tantum minutis a vero deficiunt. Ad hunc defectum ſupplendum litterae μ tribui poterit valor conueniens ex formula $\mu = \frac{1}{4}(nn-1)\,nn - \frac{3}{4} - \frac{1}{4}\delta$, vnde reperitur $\mu = 0{,}03782$, qui valor tantillus eſt, ut niſi de motu apogei ſit quaeſtio, ſemper pro nihilo haberi poſſit.

§. 137. Verum nullo modo affirmare poſſumus, valores illos pro γ et δ inuentos ita eſſe abſolutos, vt nulla amplius correctione indigeant. Quin potius, ſi formulas ſupra exhibitas attentius perpendamus, tantum abeſt vt eas pro completis habere poſſimus, vt potius manifeſtum ſit, omnes reliquas inaequalitates motus lunae perinde ac eas quas iam definiuimus, terminos quoque in eas ſuppeditare. Qui etſi admodum erunt parvi, tamen omnino ſufficere poterunt ad exiguum iſtud ſupplementum, quo adhuc a vero diſtamus, conficiendum.

Cum

Cum enim sola fere inaequalitas ab angulo $2\eta - r$ pendens motum apogei a dimidio tantopere auxisset, vt valor ipsius O ab 1,0042592 vsque ad 1,0084307 increuisset, nullum fere est dubium, quin leuis defectus huius numeri a vero valore 1,0085272 a reliquis inaequalitatibus proficiscatur.

§. 138. Hinc igitur concludere debemus, Theoriam Neutonianam cum motu apogei obseruato tam exacte conuenire, vt aberratio, si quidem vlla locum habeat, tam sit exigua, vt merito pro nihilo reputari possit: neque etiam calculi ope ob summam paruitatem eam certo definire licebit. Cum itaque hoc pacto Theoria Neutoniana a fortissima obiectione sit vindicata, gloria huius insignis inuenti cum industriae tum candori excellentissimi Clairalti debetur, qui primus egregium hunc Theoriae consensum cum veritate detexit et publice est professus: cui ea re eo maiores debemus gratias, quod sine eius studio summo, quod in hac inuestigatione consumsit, Theoria Neutoniana fortasse vix vnquam ab hac suspicione insufficientiae esset liberata. Atque nunc demum pleno lumine veritas istius Theoriae, cui vni Astronomiae Theoria vniuersa innititur, fulgere est censenda, cum antea non mediocribus tenebris fuisset inuoluta.

CAPUT

CAPUT IX.

INUESTIGATIO INAEQUALITATUM LUNAE A SOLA EXCENTRICITATE ORBITAE SOLIS PENDENTIUM.

§. 139.

Quoniam in hac inuestigatione excentricitas orbitae lunaris non in censum venit, inaequalitates quas scrutamur partim ab anomalia vera solis s partim ab angulo 2η pendebunt. Cum igitur sit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} + \frac{2}{n}k\cos r - \frac{2}{n}e\cos s - \frac{2}{n}ek\cos(r-s)$$

$$+ \frac{3}{2n}kk\cos 2r + \frac{1}{2n}ee\cos 2s - \frac{2}{n}ek\cos(r+s)$$

hinc differentiale ds ad differentiale dr reducitur. Atque hoc quidem capite, quia ad excentricitatem Lunae non attendimus, erit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} - \frac{2}{n}e\cos s + \frac{1}{2n}ee\cos 2s$$

§. 140. Incipiamus ergo a formulis $\int \mathfrak{R}\,dr$ et v, quas omissis terminis ab angulo r pendentibus ponamus

$$\int \mathfrak{R}\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta - s) + \mathfrak{R}e\cos(2\eta + s)$$
$$+ \mathfrak{S}ee\cos 2s + \mathfrak{T}ee\cos(2\eta - 2s) + \mathfrak{V}ee\cos(2\eta + 2s)$$

$$v = A\cos 2\eta + Pe\cos s + Qe\cos(2\eta - s) + Re\cos(2\eta + s)$$
$$+ See\cos 2s + Tee\cos(2\eta - 2s) + Vee\cos(2\eta + 2s)$$

vbi quidem pro $\mathfrak{A}$ et A valores supra inuentos completos accipi oportet, ita vt in iis termini akk et αkk sint comprehensi; erit ergo

$$\mathfrak{A} = -0{,}81033 \qquad l-\mathfrak{A} = 9{,}908662$$
$$A = -1{,}31773 \qquad l-A = 0{,}119826$$

Valores autem hinc deriuati erunt:

$$\frac{2\varkappa A+\mathfrak{A}}{nn} = -0{,}019744, \; l\frac{-(2\varkappa A+\mathfrak{A})}{nn} = 8{,}295442$$
$$A' = -2{,}47576 \qquad l-A' = 0{,}393708$$
$$\mathfrak{A}' = +0{,}01036 \qquad l\,\mathfrak{A}' = 8{,}015347$$

Terminos autem angulum quadruplum 4η inuoluentes hic ob summam paruitatem omisi, quoniam in combinatione cum angulo s plane fierent imperceptibiles.

§. 141. Hinc iam primo colligitur:

$$\frac{d\Phi}{dr} = \varkappa - \frac{(2\varkappa A+\mathfrak{A})}{nn}\cos 2\eta - \frac{(2\varkappa P+\mathfrak{P})}{nn} e\cos s$$
$$- \frac{(2\varkappa Q+\mathfrak{Q})}{nn} e\cos(2\eta-s) - \frac{(2\varkappa R+\mathfrak{R})}{nn} e\cos(2\eta+s)$$
$$- \frac{(2\varkappa S+\mathfrak{S})}{nn} ee\cos 2s$$
$$- \frac{(2\varkappa T+\mathfrak{T})}{nn} ee\cos(2\eta-2s) - \frac{(2\varkappa V+\mathfrak{V})}{nn} ee\cos(2\eta+2s)$$

atque porro

$$\frac{d\eta}{dr} = \alpha - \frac{(2\varkappa A+\mathfrak{A})}{nn}\cos 2\eta - \left(\frac{2\varkappa P+\mathfrak{P}}{nn} - \frac{2}{n}\right) e\cos s$$
$$- \frac{(2\varkappa Q+\mathfrak{Q})}{nn} e\cos(2\eta-s) + \frac{(2\varkappa R+\mathfrak{R})}{nn} e\cos(2\eta+s)$$
$$- \left(\frac{(2\varkappa S+\mathfrak{S})}{nn} - \frac{1}{2n}\right) ee\cos 2s$$

Deinde

Deinde quia est proxime $kk = 9ee$, erit

$$R = \tfrac{3}{8}(1 + \tfrac{27}{8}ee)\sin 2\eta + \left(\frac{3Q}{2nn} - \frac{3R}{2nn}\right)e\sin s$$

$$- \left(\tfrac{9}{4} - \frac{3P}{2nn}\right)e\sin(2\eta - s) - \left(\tfrac{9}{4} - \frac{3P}{2nn}\right)e\sin(2\eta + s)$$

$$+ \left(\frac{3T}{2nn} - \frac{3V}{2nn}\right)ee\sin 2s$$

$$+ \left(\tfrac{9}{8} + \frac{3S}{2nn}\right)ee\sin(2\eta - 2s) + \left(\tfrac{9}{8} + \frac{3S}{2nn}\right)ee\sin(2\eta + 2s)$$

atque omissis terminis, quibus non est opus

$$\frac{ddv}{dr^2} = e\cos s\left\{\begin{aligned}&-\tfrac{3}{2} - 2x\mathfrak{P} - \mathfrak{C}P - \frac{9A}{4nn} + \frac{\mathfrak{A}\mathfrak{Q}}{nn} + \frac{\mathfrak{A}\mathfrak{R}}{nn}\\ &+ \frac{3AQ}{nn} + \frac{3AR}{nn} + \frac{3\mathfrak{A}(Q+R)}{nn} + \frac{3A(\mathfrak{Q}+\mathfrak{R})}{nn}\end{aligned}\right.$$

$$+ e\cos(2\eta - s)\left\{\begin{aligned}&-\tfrac{9}{4} - 2x\mathfrak{Q} - \mathfrak{C}Q - \frac{3A}{4nn} + \frac{3P}{4nn}\\ &+ \frac{\mathfrak{A}\mathfrak{P}}{nn} + \frac{3AP}{nn} + \frac{3\mathfrak{A}P}{nn} + \frac{3A\mathfrak{P}}{nn}\end{aligned}\right.$$

$$+ e\cos(2\eta + s)\left\{\begin{aligned}&-\tfrac{9}{4} - 2x\mathfrak{R} - \mathfrak{C}R - \frac{3A}{4nn} + \frac{3P}{4nn}\\ &+ \frac{\mathfrak{A}\mathfrak{P}}{nn} + \frac{3AP}{nn} + \frac{3\mathfrak{A}P}{nn} + \frac{3A\mathfrak{P}}{nn}\end{aligned}\right.$$

$$+ ee\cos 2s\left\{\begin{aligned}&+\tfrac{3}{4} - 2x\mathfrak{S} - \mathfrak{C}S - \frac{3P}{4nn} + \frac{\mathfrak{P}\mathfrak{P}}{2nn}\\ &+ \frac{3PP}{2nn} + \frac{3P\mathfrak{P}}{nn}\end{aligned}\right.$$

$$+ ee\cos(2\eta-2s)\left\{\begin{array}{l}+\frac{8}{8}-2\varkappa\mathfrak{T}-6T-\frac{9P}{8nn}+\frac{\mathfrak{A}\mathfrak{S}}{nn}\\+\frac{3AS}{nn}+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}\end{array}\right.$$

$$+ ee\cos(2\eta+2s)\left\{\begin{array}{l}+\frac{8}{8}-2\varkappa\mathfrak{V}-6V-\frac{9P}{8nn}+\frac{\mathfrak{A}\mathfrak{S}}{nn}\\+\frac{3AS}{nn}+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}\end{array}\right.$$

§. 142. Quodsi iam forma pro $\int R\,dr$ assumta differentietur, orietur:

$$R = -2\alpha\mathfrak{A}\sin 2\eta$$
$$+ e\sin s\left(-\frac{1}{n}\mathfrak{P}-\frac{(\mathfrak{Q}-\mathfrak{R})(2\varkappa A+\mathfrak{A})}{nn}\right)$$
$$+ e\sin(2\eta-s)\left(-\frac{2\mathfrak{A}}{n}+\frac{\mathfrak{A}(2\varkappa P+\mathfrak{P})}{nn}-(2\alpha-\frac{1}{n})\mathfrak{Q}\right)$$
$$+ e\sin(2\eta+s)\left(-\frac{2\mathfrak{A}}{n}+\frac{\mathfrak{A}(2\varkappa P+\mathfrak{P})}{nn}-(2\alpha+\frac{1}{n})\mathfrak{R}\right)$$
$$+ ee\sin 2s\left(\frac{1}{n}\mathfrak{P}-\frac{2}{n}\mathfrak{S}-\frac{(\mathfrak{T}-\mathfrak{V})(2\varkappa A+\mathfrak{A})}{nn}\right)$$
$$+ ee\sin(2\eta-2s)\left(\frac{1}{2n}\mathfrak{A}-\frac{3}{n}\mathfrak{Q}-(2\alpha-\frac{2}{n})\mathfrak{T}\right)$$
$$+ ee\sin(2\eta+2s)\left(\frac{1}{2n}\mathfrak{A}-\frac{1}{n}\mathfrak{R}-(2\alpha+\frac{2}{n})\mathfrak{V}\right)$$

§. 143. Comparatione ergo instituta habebitur

$$-\frac{1}{n}\mathfrak{P}-\frac{(\mathfrak{Q}-\mathfrak{R})(2\varkappa A+\mathfrak{A})}{nn}=\frac{3(Q-R)}{2nn}$$

—

$$-\frac{2}{n}\mathfrak{A}+\frac{\mathfrak{A}(2\varkappa P+\mathfrak{P})}{nn}-\left(2a-\frac{1}{n}\right)\mathfrak{Q}=-\frac{9}{4}+\frac{3P}{2nn}$$

$$-\frac{2}{n}\mathfrak{A}+\frac{\mathfrak{A}(2\varkappa P+\mathfrak{P})}{nn}-\left(2a+\frac{1}{n}\right)\mathfrak{R}=-\frac{9}{4}+\frac{3P}{2nn}$$

$$\frac{1}{n}\mathfrak{P}-\frac{2}{n}\mathfrak{S}-\frac{(\mathfrak{T}-\mathfrak{V})(2\varkappa A+\mathfrak{A})}{nn}=\frac{3(T-V)}{2nn}$$

$$\frac{1}{2n}\mathfrak{A}-\frac{3}{n}\mathfrak{Q}-\left(2a-\frac{2}{n}\right)\mathfrak{T}=\frac{9}{8}+\frac{3S}{2nn}$$

$$\frac{1}{2n}\mathfrak{A}-\frac{1}{n}R-\left(2a+\frac{2}{n}\right)\mathfrak{V}=\frac{9}{8}+\frac{3S}{2nn}$$

vnde deinceps valores litterarum germanicarum $\mathfrak{P}$, $\mathfrak{Q}$, $\mathfrak{R}$, $\mathfrak{S}$, $\mathfrak{T}$, $\mathfrak{V}$ sumus inuestigaturi.

§. 144. Differentietur simili modo quantitas v, ac ponatur:

$$\frac{dv}{dr}=-A'\sin 2\eta-P'e\sin s-Q'e\sin(2\eta-s)-S'ee\sin 2s-T'ee\sin(2\eta-2s)$$
$$-R'e\sin(2\eta+s)\qquad\qquad -V'ee\sin(2\eta+2s)$$

eritque

$A'=2a\mathfrak{A}$, cuius quidem valor iam supra habetur

$$P'=\frac{1}{n}P+\frac{(Q-R)(2\varkappa A+\mathfrak{A})}{nn}$$

$$Q'=\left(2a-\frac{1}{n}\right)Q+\frac{2}{n}A-\frac{A(2\varkappa P+\mathfrak{P})}{nn}$$

$$R'=\left(2a+\frac{1}{n}\right)R+\frac{2}{n}A-\frac{A(2\varkappa P+\mathfrak{P})}{nn}$$

$$S'=\frac{2}{n}S-\frac{1}{n}P+\frac{(T-V)(2\varkappa A+\mathfrak{A})}{nn}$$

$$T' = (2\alpha - \frac{2}{n})T + \frac{3}{n}Q - \frac{1}{2n}A$$

$$V' = (2\alpha + \frac{2}{n})V + \frac{1}{n}R - \frac{1}{2n}A$$

vnde denuo differentiando eruitur.

$$\frac{ddv}{dr^2} = e\cos s\,(-\frac{1}{n}P' + \frac{(Q'+R')(2\varkappa A + \mathfrak{A})}{nn}$$

$$e\cos(2\eta - s)\,(-(2\alpha - \frac{1}{n})Q' - \frac{2}{n}A' + \frac{A'(2\varkappa P + \mathfrak{P})}{nn}$$

$$e\cos(2\eta + s)\,(-(2\alpha + \frac{1}{n})R' - \frac{2}{n}A' + \frac{A'(2\varkappa P + \mathfrak{P})}{nn}$$

$$ee\cos 2s\,(-\frac{2}{n}S' + \frac{1}{n}P' + \frac{(T'+V')(2\varkappa A + \mathfrak{A})}{nn}$$

$$ee\cos(2\eta - 2s)\,(-(2\alpha - \frac{2}{n})T' + \frac{1}{2n}A' - \frac{3}{n}Q'$$

$$ee\cos(2\eta + 2s)\,(-(2\alpha + \frac{2}{n})V' + \frac{1}{2n}A' - \frac{1}{n}R'$$

§. 145. Sequentes ergo aequationes resoluendae occurrent

$$-\tfrac{3}{4} - 2\varkappa\mathfrak{P} - 6P - \frac{9A}{4nn} + \frac{(\mathfrak{A}+3A)(Q+R)}{nn} + \frac{(3\mathfrak{A}+3A)(Q+R)}{nn} =$$

$$-\frac{1}{n}P' + \frac{(Q'+R')(2\varkappa A + \mathfrak{A})}{nn}$$

$$-\tfrac{2}{4} - 2\varkappa\mathfrak{Q} - 6Q - \frac{3A}{4nn} + \frac{3P}{4nn} + \frac{(\mathfrak{A}+3A)\mathfrak{P}}{nn} + \frac{(3\mathfrak{A}+3A)P}{nn} =$$

$$-(2\alpha - \frac{1}{n})Q' - \frac{2}{n}A' + \frac{A'(2\varkappa P + \mathfrak{P})}{nn}$$

$$-\tfrac{3}{4}-2\kappa\mathfrak{R}-6R-\frac{3A}{4nn}+\frac{3P}{4nn}+\frac{(\mathfrak{A}+3A)\mathfrak{P}}{nn}+\frac{(3\mathfrak{A}+3A)P}{nn}=$$

$$-\left(2\alpha+\frac{1}{n}\right)R'-\frac{2}{n}A'+\frac{A'(2\kappa P+\mathfrak{P})}{nn}$$

$$+\tfrac{3}{4}-2\kappa\mathfrak{S}-6S-\frac{3P}{4nn}+\frac{\mathfrak{P}\mathfrak{P}+6P\mathfrak{P}+3PP}{2nn}=$$

$$-\frac{2}{n}S'+\frac{1}{n}P'+\frac{(T'+V')(2\kappa A+\mathfrak{A})}{nn}$$

$$+\tfrac{9}{8}-2\kappa\mathfrak{T}-6T-\frac{9P}{8nn}+\frac{(\mathfrak{A}+3A)\mathfrak{S}}{nn}+\frac{(3\mathfrak{A}+3A)S}{nn}=$$

$$-\left(2\alpha-\frac{2}{n}\right)T'+\frac{1}{2n}A'-\frac{3}{n}Q'$$

$$+\tfrac{9}{8}-2\kappa\mathfrak{V}-6V-\frac{9P}{8nn}+\frac{(\mathfrak{A}+3A)\mathfrak{S}}{nn}+\frac{(3\mathfrak{A}+3A)S}{nn}=$$

$$-\left(2\alpha+\frac{2}{n}\right)V'+\frac{1}{2n}A'-\frac{1}{n}R'$$

Neglectis primo terminis minimis, qui adhuc sunt incogniti, reperitur:

$\mathfrak{Q}=+1{,}3238$	$Q=+2{,}5714$	$Q'=4{,}40924$
$\mathfrak{R}=+1{,}2210$	$R=+2{,}0571$	$R'=3{,}79793$
$\mathfrak{P}=-0{,}0313$	$P=-1{,}4807$	$P'=-0{,}12185$

§. 146. Ex his autem accuratius ita definientur vt sit:

$\mathfrak{Q}=+1{,}33859$ . . . $l\mathfrak{Q}=0{,}126649$

$\mathfrak{R}=+1{,}23468$ . . . $l\mathfrak{R}=0{,}091545$

$Q=+2{,}60087$ . . . $lQ=0{,}415119$

$R=+2{,}00590$ . . . $lR=0{,}302308$

$Q'=4{,}44801$; $R'=3{,}67581$

et

et $\mathfrak{P} = -0,04010$ . . . $l-\mathfrak{P} = 8,603133$
$P = -1,46488$ . . . $l-P = 0,165801$
$P' = -0,12222$.

Deinde reperitur

$\mathfrak{S} = +0,03911$ . . . $l\mathfrak{S} = 8,592230$
$S = +0,60882$ . . . $lS = 9,784486$
$\mathfrak{T} = -0,85267$ . . . $l-\mathfrak{T} = 9,930827$
$\mathfrak{V} = -0,62125$ . . . $l-\mathfrak{V} = 9,793266$
$T = -2,60380$ . . . $l-T = 0,465615$
$V = -1,02720$ . . . $l-V = 0,011672$

Pro sequentibus vero calculis est

$$\frac{2\varkappa P+\mathfrak{P}}{nn} = -0,017051 \ . \ . \ . \ l-\frac{(2\varkappa P+\mathfrak{P})}{nn} = 8,231755$$

$$\frac{2\varkappa Q+\mathfrak{Q}}{nn} = +0,037487 \ . \ . \ . \ l+\frac{2\varkappa Q+\mathfrak{Q}}{nn} = 8,573878$$

$$\frac{2\varkappa R+\mathfrak{R}}{nn} = +0,030062 \ . \ . \ . \ l+\frac{2\varkappa R+\mathfrak{R}}{nn} = 8,478023$$

§. 147. Hinc igitur pro distantia lunae a terra curtata $x = \frac{(1-kk)au}{1-k\cos r}$, pars quantitatis u ab excentricitate orbitae solaris tantum pendens erit

			log. coeff.
$u =$ Praeced.	$-\ 0,008336e$	$\cos s$	7,920985
	$+\ 0,014801e$	$\cos(2\eta - s)$	8,170303
	$+\ 0,011415e$	$\cos(2\eta + s)$	8,057492
	$+\ 0,00364ee$	$\cos 2s$	7,539670
	$-\ 0,01482ee$	$\cos(2\eta - 2s)$	8,170799
	$-\ 0,00584ee$	$\cos(2\eta + 2s)$	7,766856

Deinde

Deinde vero erit

	log. coeff.
$\frac{d\Phi}{dr} = \text{Praec.} + 0{,}017641\, e \cos s$	8,231755
$- 0{,}037487\, e \cos(2\eta - s)$	8,573878
$- 0{,}030062\, e \cos(2\eta + s)$	8,478023
$- 0{,}00722\, ee \cos 2s$	7,858166
$+ 0{,}03470\, ee \cos(2\eta - 2s)$	8,540319
$+ 0{,}01533\, ee \cos(2\eta + 2s)$	8,185614

§. 148. Ponatur nunc integrale

$$\begin{aligned}\Phi = \text{Praec.} &+ \mathfrak{A}' \sin 2\eta + \mathfrak{P}' e \sin s \\ &+ \mathfrak{Q}' e \sin(2\eta - s) + \mathfrak{R}' e \sin(2\eta + r) \\ &+ \mathfrak{S}' ee \sin 2s + \mathfrak{T}' ee \sin(2\eta - 2s) \\ &+ \mathfrak{V}' ee \sin(2\eta + 2s)\end{aligned}$$

erit differentiatione peracta:

$$+ 0{,}017051 = \frac{1}{n}\mathfrak{P}' - \frac{(\mathfrak{Q}' + \mathfrak{R}')(2nA + \mathfrak{A})}{nn}$$

$$- 0{,}037489 = \left(2n - \frac{1}{n}\right)\mathfrak{Q}' + \frac{2}{n}\mathfrak{A}' - \frac{\mathfrak{A}'(2nP + \mathfrak{P})}{nn}$$

$$- 0{,}030062 = \left(2n + \frac{1}{n}\right)\mathfrak{R}' + \frac{2}{n}\mathfrak{A}' - \frac{\mathfrak{A}'(2nP + \mathfrak{P})}{nn}$$

$$- 0{,}00722 = \frac{2}{n}\mathfrak{S}' - \frac{1}{n}\mathfrak{P}' - \frac{(\mathfrak{T}' + \mathfrak{V}')(2nA + \mathfrak{A})}{nn}$$

$$+ 0{,}03470 = \left(2n - \frac{2}{n}\right)\mathfrak{T}' - \frac{1}{2n}\mathfrak{A}' + \frac{3}{n}\mathfrak{Q}'$$

$$+ 0{,}01533 = \left(2n + \frac{2}{n}\right)\mathfrak{V}' - \frac{1}{2n}\mathfrak{A}' + \frac{1}{n}\mathfrak{R}'$$

fietque his valoribus determinatis

			log. coeff.
$\varphi =$ Praec.	$+ 0,236034\, e$	$\sin s$	9,372974
	$- 0,021889\, e$	$\sin(2\eta - s)$	8,340237
	$- 0,016368\, e$	$\sin(2\eta + s)$	8,214002
	$+ 0,06615\, ee$	$\sin 2s$	8,820508
	$+ 0,02332\, ee$	$\sin(2\eta - 2s)$	8,367825
	$+ 0,00840\, ee$	$\sin(2\eta + 2s)$	7,924429

§. 149. Reducamus has inaequalitates etiam ad minuta secunda, ponendo excentricitatem orbitae solaris $e = 0,01680$, atque habebimus

			log. coeff.
$\varphi =$ Praec.	$+ 817'', 9$	$\sin s$	2,912708
	$- 75, 8$	$\sin(2\eta - s)$	1,879971
	$- 56, 7$	$\sin(2\eta + s)$	1,753736
	$+ 3, 8$	$\sin 2s$	0,585551
	$+ 1, 4$	$\sin(2\eta - 2s)$	0,132868
	$+ 0, 5$	$\sin(2\eta + 2s)$	9,689472

Denotat hic s anomaliam veram solis; vnde patet eam Lunae inaequalitatem, quae sinui huius anomaliae est proportionalis, admodum esse notabilem, dum ad 13′, 38″ exsurgit. Tabulae autem Astronomicae, vbi haec inaequalitas aequatio solaris nominatur, eam multo minorem faciunt, cuius rei caussam inuestigari adhuc conueniet.

§. 150.

§. 150. Quodsi enim litteram $\mathfrak{P}'$ accuratius definire velimus, habebimus has formulas resoluendas:

$$-\frac{1}{n}\mathfrak{P}+\frac{\mathfrak{A}(2nQ+\mathfrak{Q})}{nn}-\frac{\mathfrak{A}(2nR+\mathfrak{R})}{nn}-\frac{(\mathfrak{Q}-\mathfrak{R})(2nA+\mathfrak{A})}{nn}=\frac{3(Q-R)}{nn}$$

$$P'=\frac{1}{n}P-\frac{A(2nQ+\mathfrak{Q})}{nn}+\frac{A(2nR+\mathfrak{R})}{nn}+\frac{(Q-R)(2nA+\mathfrak{A})}{nn}$$

$$-\tfrac{3}{2}-2n\mathfrak{P}-6P-\frac{9A}{4nn}+\frac{(\mathfrak{A}+3A)(\mathfrak{Q}+\mathfrak{R})}{nn}+\frac{(3\mathfrak{A}+3A)(Q+R)}{nn}=$$

$$-\frac{1}{n}P'+\frac{A'(2nQ+\mathfrak{Q})}{nn}+\frac{A'(2nR+\mathfrak{R})}{nn}+\frac{(Q'+R')(2nA+\mathfrak{A})}{nn}$$

vnde elicimus:

$$\mathfrak{P}=-0,1183;\quad P=-1,1356;\quad \frac{P}{nn}=-0,0064$$

atque $\frac{2nP+\mathfrak{P}}{nn}=-0,01376$, fierique iam oportet

$$+0{,}01376=\frac{1}{n}\mathfrak{P}'-\frac{\mathfrak{A}'(2nQ+\mathfrak{Q})}{nn}-\frac{\mathfrak{A}'(2nR+\mathfrak{R})}{nn}-\frac{(\mathfrak{Q}'+\mathfrak{R}')(2nA+\mathfrak{A})}{nn}$$

vnde oritur $\mathfrak{P}'=+0,201385$. Quare accuratius habemus

u = Praec.	—	0,006400	e cos s		7,806180
$\frac{d\Phi}{dr}$ = Praec.	+	0,013760	e cos s		8,138618
Φ = Praec.	+	0,201385	e sin s		9,304026
		seu			
Φ = Praec.	+	701″, 1	sin s		2,845780

Ergo aequatio sinui Anguli s proportionalis tantum est 11′, 41″.

CAPUT X.

INUESTIGATIO INAEQUALITATUM LUNAE AB VTRIUSQUE ORBITAE EXCENTRICITATE SIMUL PENDENTIUM.

§. 151.

Quoniam praeuidemus inaequalitates huius generis, quae altiores litterarum k et e poteſtates ſimul complectuntur, minimas eſſe futuras, alios terminos non ſcrutabimur, niſi qui producto ſimplici ek ſint affecti. Habebimus ergo

$$\frac{ds}{dr} = \frac{d\phi}{dr} = \frac{1}{n} + \frac{2}{n}k\cos r - \frac{2}{n}e\cos s - \frac{2}{n}ek\cos(r-s)$$
$$- \frac{2}{n}ek\cos(r+s)$$

Cum igitur ad hanc inueſtigationem opus non ſit illis terminis ex praecedentibus, qui vel per k^2 vel per e^2 erant affecti, quia litterae alphabethi deficere incipiunt, litteris S, T et ſequentibus denuo utemur; quare cauendum, ne iſtae litterae cum ante adhibitis confundantur.

§. 152. Aſſumtis ergo ex terminis iam ante definitis, iis qui in eos, quos iam inueſtigamus, vim exſerunt, ponamus

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{C}k\cos r + \mathfrak{D}k\cos(2\eta-r) + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta-s)$$
$$+ \mathfrak{E}k\cos(2\eta+r) \qquad + \mathfrak{R}e\cos(2\eta+s)$$
$$+ \mathfrak{S}ek\cos(r-s) + \mathfrak{V}ek\cos(2\eta-r+s) + \mathfrak{Y}ek\cos(2\eta-r-s)$$
$$+ \mathfrak{T}ek\cos(r+s) + \mathfrak{X}ek\cos(2\eta+r+s) + \mathfrak{Z}ek\cos(2\eta+r+s)$$

$v =$

$$v = A\cos\eta \ldots + Dk\cos(2\eta - r) + Pe\cos s + Qe\cos(2\eta - s)$$
$$+ Ek\cos(2\eta + r) \qquad + Re\cos(2\eta + s)$$
$$+ Sek\cos(r - s) + Vek\cos(2\eta - r + s) + Yek\cos(2\eta - r - s)$$
$$+ Tek\cos(r + s) + Xek\cos(2\eta + r - s) + Zek\cos(2\eta + r + s)$$

eritque ex praecedentibus:

$\mathfrak{A} = -0,81033$;	$l-\mathfrak{A} = 9,908662$
$\mathfrak{C} = -0,64383$;	$l-\mathfrak{C} = 9,808771$
$\mathfrak{D} = -3,59362$;	$l-\mathfrak{D} = 0,555531$
$\mathfrak{E} = -1,08732$;	$l-\mathfrak{E} = 0,036358$
$\mathfrak{P} = -0,11830$;	$l-\mathfrak{P} = 9,072985$
$\mathfrak{Q} = +1,33859$;	$l\,\mathfrak{Q} = 0,126649$
$\mathfrak{R} = +1,23468$;	$l\,\mathfrak{R} = 0,091545$
$A = -1,31773$;	$l-A = 0,119826$
$D = +33,6600$;	$l\,D = 1,527113$
$E = -0,5785$;	$l-E = 9,762341$
$P = -1,1356$;	$l-P = 0,055225$
$Q = +2,60087$;	$l-Q = 0,415119$
$R = +2,00590$;	$l\,R = 0,302308$

§. 153. Reliqui vero valores hinc deriuati, quibus opus habemus, sunt:

$A' = -2,47576$;	$l-A' = 0,393708$
$C' = -0,13847$;	$l-C' = 9,141356$
$D' = 29,39153$;	$l\,D' = 1,468222$
$E' = -1,47347$;	$l-E' = 0,168341$
$P' = -0,0260$;	$l-P' = 8,414973$
$Q' = 4,40924$;	$l\,Q' = 0,644363$
$R' = 3,79793$;	$l\,R' = 0,579548$

$$\mathfrak{A}' = +0{,}01036 \quad ; \quad l\,\mathfrak{A}' = 8{,}015347$$
$$\mathfrak{C}' = +0{,}01015 \quad ; \quad l\,\mathfrak{C}' = 8{,}006295$$
$$\mathfrak{D}' = -0{,}42023 \quad ; \quad l-\mathfrak{D}' = 9{,}623483$$
$$\mathfrak{E}' = +0{,}00499 \quad ; \quad l-\mathfrak{E}' = 7{,}698261$$
$$\mathfrak{P}' = +0{,}20138 \quad ; \quad l\,\mathfrak{P}' = 9{,}304016$$
$$\mathfrak{Q}' = -0{,}02189 \quad ; \quad l-\mathfrak{Q}' = 8{,}340237$$
$$\mathfrak{R}' = -0{,}01637 \quad ; \quad l-\mathfrak{R}' = 8{,}214002$$

Sit breuitatis gratia

$$a' = \frac{2nA+\mathfrak{A}}{nn} = -0{,}019744 \quad ; \quad l-\frac{(2nA+\mathfrak{A})}{nn} = 8{,}295442$$
$$d' = \frac{2nD+\mathfrak{D}}{nn} = +0{,}36611 \quad ; \quad l\,\frac{2nD+\mathfrak{D}}{nn} = 9{,}563604$$
$$e' = \frac{2nE+\mathfrak{E}}{nn} = -0{,}01283 \quad ; \quad l-\frac{(2nE+\mathfrak{E})}{nn} = 8{,}108292$$
$$p' = \frac{2nP+\mathfrak{P}}{nn} = -0{,}01376 \quad ; \quad l-\frac{(2nP+\mathfrak{P})}{nn} = 8{,}138618$$
$$q' = \frac{2nQ+\mathfrak{Q}}{nn} = +0{,}03749 \quad ; \quad l\,\frac{2nQ+\mathfrak{Q}}{nn} = 8{,}573878$$
$$r' = \frac{2nR+\mathfrak{R}}{nn} = +0{,}03006 \quad ; \quad l\,\frac{2nR+\mathfrak{R}}{nn} = 8{,}478023$$

§. 154. Si simili modo vlterius ponatur:

$$s' = \frac{2nS+\mathfrak{S}}{nn}; \quad t' = \frac{2nT+\mathfrak{T}}{nn}; \quad v' = \frac{2nV+\mathfrak{V}}{nn};$$
$$x' = \frac{2nX+\mathfrak{X}}{nn}; \quad y' = \frac{2nY+\mathfrak{Y}}{nn}; \quad z' = \frac{2nZ+\mathfrak{Z}}{nn}$$

habe-

habebimus:

$$\frac{d\Phi}{dr} = \text{Praec.} - a'\cos 2\eta - \frac{\mathfrak{C}}{mn}k\cos r - d'k\cos(2\eta-r) - p'e\cos s - q'e\cos(2\eta-s)$$
$$- i'k\cos(2\eta+r) \qquad - r'e\cos(2\eta+s)$$
$$- s'ek\cos(r-s) - v'ek\cos(2\eta-r+s) - y'ek\cos(2\eta-r-s)$$
$$- t'ek\cos(r+s) - x'ek\cos(2\eta+r-s) - z'ek\cos(2\eta+r+s)$$

atque posito $\frac{2}{n} + \frac{\mathfrak{C}}{mn} = c' = 0,147197$; $l\,c' = 9,167900$

$$\frac{d\eta}{dr} = a - a'\cos 2\eta - c'k\cos r - d'k\cos(2\eta-r) + \left(\frac{2}{n} - p'\right)e\cos s - q'e\cos(2\eta-s)$$
$$- i'k\cos(2\eta+r) \qquad - r'e\cos(2\eta+s)$$
$$+ \left(\frac{2}{n} - s'\right)ek\cos(r-s) - v'ek\cos(2\eta-r+s) - y'ek\cos(2\eta-r-s)$$
$$+ \left(\frac{2}{n} - t'\right)ek\cos(r+s) - x'ek\cos(2\eta+r-s) - z'ek\cos(2\eta+r+s)$$

vbi cum sit $\frac{2}{n} = 0,150876$, erit $\frac{2}{n} - p' = 0,16464$

§. 155. Nunc termini coefficiente ek affecti, qui in formis $R =$ et $\frac{ddv}{dr^2}$ insunt, colligantur: eritque

$$R = ek\sin(r-s)\left(+\frac{3}{2nn}V - \frac{3}{2nn}X - \frac{3Q}{nn} + \frac{3R}{nn} - \frac{9D}{4nn} + \frac{9E}{4nn}\right)$$
$$ek\sin(r+s)\left(+\frac{3}{2nn}Y - \frac{3}{2nn}Z - \frac{3R}{nn} + \frac{3Q}{nn} + \frac{9E}{4nn} - \frac{9D}{4nn}\right)$$
$$ek\sin(2\eta-r+s)\left(-\tfrac{1}{2} + \frac{3}{2nn}S + \frac{3P}{nn}\right)$$
$$ek\sin(2\eta+r-s)\left(-\tfrac{1}{2} + \frac{3}{2nn}S + \frac{3P}{nn}\right)$$

$$ek \sin(2\eta - r - s)\left(-\frac{1}{2} + \frac{3}{2nn}T + \frac{3P}{nn}\right)$$

$$ek \sin(2\eta + r + s)\left(-\frac{1}{2} + \frac{3}{2nn}T + \frac{3P}{nn}\right)$$

et $\frac{ddv}{dr^2} =$

$$ek\cos(r-s)\left\{\begin{aligned}&-3-2n\mathfrak{S}-bS+\tfrac{1}{2}bP+\frac{3V}{4nn}+\frac{3X}{4nn}+\frac{3Q}{2nn}+\frac{3R}{2nn}\\&-\frac{9D}{8nn}-\frac{9E}{8nn}+\frac{\mathfrak{A}\mathfrak{B}}{nn}+\frac{\mathfrak{A}\mathfrak{Y}}{nn}+\frac{\mathfrak{E}\mathfrak{P}}{nn}+\frac{\mathfrak{D}\mathfrak{Q}}{nn}+\frac{\mathfrak{E}\mathfrak{R}}{nn}\\&+\frac{3AV}{nn}+\frac{3AX}{nn}+\frac{3DQ}{nn}+\frac{3ER}{nn}-\frac{3AQ}{2nn}-\frac{3AR}{2nn}\\&+\frac{3\mathfrak{A}V}{nn}+\frac{3A\mathfrak{B}}{nn}+\frac{3\mathfrak{A}X}{nn}+\frac{3A\mathfrak{Y}}{nn}+\frac{3\mathfrak{E}P}{nn}\\&+\frac{3\mathfrak{D}Q}{nn}+\frac{3D\mathfrak{Q}}{nn}+\frac{3\mathfrak{E}R}{nn}+\frac{3E\mathfrak{R}}{nn}\end{aligned}\right.$$

$$ek\cos(r+s)\left\{\begin{aligned}&-3-2n\mathfrak{T}-bT+\tfrac{1}{2}bP+\frac{3Y}{4nn}-\frac{3Z}{4nn}+\frac{3Q}{2nn}+\frac{3R}{2nn}\\&-\frac{9D}{8nn}-\frac{9E}{8nn}+\frac{\mathfrak{A}\mathfrak{Y}}{nn}+\frac{\mathfrak{A}\mathfrak{Z}}{nn}+\frac{\mathfrak{E}\mathfrak{P}}{nn}+\frac{\mathfrak{D}\mathfrak{R}}{nn}+\frac{\mathfrak{E}\mathfrak{Q}}{nn}\\&+\frac{3AY}{nn}+\frac{3AZ}{nn}+\frac{3DR}{nn}+\frac{3EQ}{nn}-\frac{3AQ}{2nn}-\frac{3AR}{2nn}\\&+\frac{3\mathfrak{A}Y}{nn}+\frac{3A\mathfrak{Y}}{nn}+\frac{3\mathfrak{A}Z}{nn}+\frac{3A\mathfrak{Z}}{nn}+\frac{3\mathfrak{E}P}{nn}\\&+\frac{3\mathfrak{E}Q}{nn}+\frac{3E\mathfrak{Q}}{nn}+\frac{3\mathfrak{D}R}{nn}+\frac{3D\mathfrak{R}}{nn}\end{aligned}\right.$$

ek

$$ek\cos(2\eta-r+s)\left\{\begin{array}{l}-\frac{g}{2}-2n\mathfrak{V}-\beta V+\frac{1}{2}bR+\frac{3S}{4nn}-\frac{3D}{4nn}+\frac{3P}{2nn}\\+\frac{\mathfrak{A}\mathfrak{S}}{nn}+\frac{\mathfrak{C}\mathfrak{R}}{nn}+\frac{\mathfrak{D}\mathfrak{P}}{nn}+\frac{3AS}{nn}+\frac{3DP}{nn}-\frac{3AP}{2nn}\\+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}+\frac{3\mathfrak{C}R}{nn}+\frac{3\mathfrak{D}P}{nn}+\frac{3D\mathfrak{P}}{nn}\end{array}\right.$$

$$ek\cos(2\eta+r-s)\left\{\begin{array}{l}-\frac{g}{2}-2n\mathfrak{X}-\beta X+\frac{1}{2}bQ+\frac{3S}{4nn}-\frac{3E}{4nn}+\frac{3P}{2nn}\\+\frac{\mathfrak{A}\mathfrak{S}}{nn}+\frac{\mathfrak{C}\mathfrak{Q}}{nn}+\frac{\mathfrak{E}\mathfrak{P}}{nn}+\frac{3AS}{nn}+\frac{3EP}{nn}-\frac{3AP}{2nn}\\+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}+\frac{3\mathfrak{C}Q}{nn}+\frac{3\mathfrak{E}P}{nn}+\frac{3E\mathfrak{P}}{nn}\end{array}\right.$$

$$ek\cos(2\eta-r-s)\left\{\begin{array}{l}-\frac{g}{2}-2n\mathfrak{Y}-\beta Y+\frac{1}{2}bQ+\frac{3T}{4nn}-\frac{3D}{4nn}+\frac{3P}{2nn}\\+\frac{\mathfrak{A}\mathfrak{T}}{nn}+\frac{\mathfrak{C}\mathfrak{Q}}{nn}+\frac{\mathfrak{D}\mathfrak{P}}{nn}+\frac{3AT}{nn}+\frac{3DP}{nn}-\frac{3AP}{2nn}\\+\frac{3\mathfrak{A}T}{nn}+\frac{3A\mathfrak{T}}{nn}+\frac{3\mathfrak{C}Q}{nn}+\frac{3\mathfrak{D}P}{nn}+\frac{3D\mathfrak{P}}{nn}\end{array}\right.$$

$$ek\cos(2\eta+r+s)\left\{\begin{array}{l}-\frac{g}{2}-2n\mathfrak{Z}-\beta Z+\frac{1}{2}bR+\frac{3T}{4nn}-\frac{3E}{4nn}+\frac{3P}{2nn}\\+\frac{\mathfrak{A}\mathfrak{T}}{nn}+\frac{\mathfrak{C}\mathfrak{R}}{nn}+\frac{\mathfrak{E}\mathfrak{P}}{nn}+\frac{3AT}{nn}+\frac{3EP}{nn}-\frac{3AP}{2nn}\\+\frac{3\mathfrak{A}T}{nn}+\frac{3A\mathfrak{T}}{nn}+\frac{3\mathfrak{C}R}{nn}+\frac{3\mathfrak{E}P}{nn}+\frac{3E\mathfrak{P}}{nn}\end{array}\right.$$

§. 156. Quaeramus ergo quoque ex formula assumta $\int R\,dr$ differentiale, quod erit

$$R =$$

$$ek\sin(r-s)\left(+\mathfrak{A}v'-\mathfrak{A}x'-\mathfrak{D}q'+\mathfrak{E}r'+\frac{1}{n}\mathfrak{P}+\mathfrak{Q}d'-\mathfrak{R}e'-\left(1-\frac{1}{n}\right)\mathfrak{S}-\mathfrak{V}a'+\mathfrak{X}a'\right)$$

$$ek\sin(r+s)\left(+\mathfrak{A}y'-\mathfrak{A}z'-\mathfrak{D}r'+\mathfrak{E}q'-\frac{1}{n}\mathfrak{P}-\mathfrak{Q}e'+\mathfrak{R}d'-\left(1+\frac{1}{n}\right)\mathfrak{T}-\mathfrak{Y}a'+\mathfrak{Z}a'\right)$$

$$ek\sin(2\eta-r+s)\left(-\mathfrak{A}\left(\frac{2}{n}-s'\right)-\mathfrak{D}\left(\frac{2}{n}-p'\right)+\mathfrak{R}c'-\frac{1}{n}\mathfrak{R}-\left(2\alpha-1+\frac{1}{n}\right)\mathfrak{V}\right)$$

$$ek\sin(2\eta+r-s)\left(-\mathfrak{A}\left(\frac{2}{n}-s'\right)-\mathfrak{E}\left(\frac{2}{n}-p'\right)+\mathfrak{Q}c'+\frac{1}{n}\mathfrak{Q}-\left(2\alpha+1-\frac{1}{n}\right)\mathfrak{X}\right)$$

$$ek\sin(2\eta-r-s)\left(-\mathfrak{A}\left(\frac{2}{n}-t'\right)-\mathfrak{D}\left(\frac{2}{n}-p'\right)+\mathfrak{Q}c'+\frac{1}{n}\mathfrak{Q}-\left(2\alpha-1-\frac{1}{n}\right)\mathfrak{Y}\right)$$

$$ek\sin(2\eta+r+s)\left(-\mathfrak{A}\left(\frac{2}{n}-t'\right)-\mathfrak{E}\left(\frac{2}{n}-p'\right)+\mathfrak{R}c'-\frac{1}{n}\mathfrak{R}-\left(2\alpha+1+\frac{1}{n}\right)\mathfrak{Z}\right)$$

§. 157. Ponatur ex differentiatione formae v;

$$S'=\left(1-\frac{1}{n}\right)S-A(v'-x')+Dq'-Er'-\frac{1}{n}P-Qd'+Re'+(V-X)a'$$

$$T'=\left(1+\frac{1}{n}\right)T-A(y'-z')+Dr'-Eq'+\frac{1}{n}P+Qe'-Rd'+(Y-Z)a'$$

$$V'=\left(2\alpha-1+\frac{1}{n}\right)V+A\left(\frac{2}{n}-s'\right)+D\left(\frac{2}{n}-p'\right)-Rc'+\frac{1}{n}R$$

$$X'=\left(2\alpha+1-\frac{1}{n}\right)X+A\left(\frac{2}{n}-s'\right)+E\left(\frac{2}{n}-p'\right)-Qc'-\frac{1}{n}Q$$

$$Y'=\left(2\alpha-1-\frac{1}{n}\right)Y+A\left(\frac{2}{n}-t'\right)+D\left(\frac{2}{n}-p'\right)-Qc'-\frac{1}{n}Q$$

$$Z'=\left(2\alpha+1+\frac{1}{n}\right)Z+A\left(\frac{2}{n}-t'\right)+E\left(\frac{2}{n}-p'\right)-Rc'+\frac{1}{n}R$$

vt

vt habeatur

$$\frac{dv}{dr} = -A'\sin 2\eta - C'k\sin r - D'k\sin(2\eta - r) - E'k\sin(2\eta + r)$$
$$- P'e\sin s - Q'e\sin(2\eta - s) - R'e\sin(2\eta + s)$$
$$- S'ek\sin(r - s) - V'ek\sin(2\eta - r + s) - Y'ek\sin(2\eta - r - s)$$
$$- T'ek\sin(r + s) - X'ek\sin(2\eta + r - s) - Z'ek\sin(2\eta + r + s)$$

§. 158. Haec iam forma denuo differentiata dabit

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ ek\cos(r - s)\left(+A'(v' + x') + D'q' + E'r' - \frac{1}{n}P' + Q'd' + R'c' - \left(1 - \frac{1}{n}\right)S' + (V' + X')a'\right)$$
$$+ ek\cos(r + s)\left(+A'(y' + z') + D'r' + E'q' - \frac{1}{n}P' + Q'c' + R'd' - \left(1 + \frac{1}{n}\right)T' + (Y + Z')a'\right)$$
$$+ ek\cos(2\eta - r + s)\left(-A'\left(\frac{2}{n} - s'\right) - D'\left(\frac{2}{n} - p'\right) + R'c' - \frac{1}{n}R' - \left(2\alpha - 1 + \frac{1}{n}\right)V'\right)$$
$$+ ek\cos(2\eta + r - s)\left(-A'\left(\frac{2}{n} - s'\right) - E'\left(\frac{2}{n} - p'\right) + Q'c' + \frac{1}{n}Q' - \left(2\alpha + 1 - \frac{1}{n}\right)X'\right)$$
$$+ ek\cos(2\eta - r - s)\left(-A'\left(\frac{2}{n} - s'\right) - D'\left(\frac{2}{n} - p'\right) + Q'c' + \frac{1}{n}Q' - \left(2\alpha - 1 - \frac{1}{n}\right)Y'\right)$$
$$+ ek\cos(2\eta + r + s)\left(-A'\left(\frac{2}{n} - s'\right) - E'\left(\frac{2}{n} - p'\right) + R'c' - \frac{1}{n}R' - \left(2\alpha + 1 + \frac{1}{n}\right)Z'\right)$$

§. 159. Priores autem expressiones, si litterarum cognitarum valores substituantur, sequenti modo prodibunt.

$$R = ek\sin(r - s)(-0{,}44856 + 0{,}00854(V - X))$$
$$ek\sin(r + s)(-0{,}42826 + 0{,}00854(Y - Z))$$
$$ek\sin(2\eta - r + s)(-4{,}51938 + 9{,}00854\ S).$$
$$ek\sin(2\eta + r - s)(-4{,}51938 + 0{,}00854\ S)$$
$$ek\sin(2\eta - r - s)(-4{,}51939 + 0{,}00854\ T)$$
$$ek\sin(2\eta + r + s)(-4{,}51938 + 0{,}00854\ T)$$

§. 160. Altera vero forma pro $\frac{ddv}{dr^2}$ fit

$$\frac{ddv}{dr^2} =$$

$$ekcf(r-s)(-2,83505-2k\mathfrak{S}-\mathfrak{C}S-0,02711(\mathfrak{V}+\mathfrak{X})-0,03207(V+X))$$
$$ekcf(r+s)(-3,21669-2\varkappa\mathfrak{T}-\mathfrak{C}T-2,02711(\mathfrak{Y}+\mathfrak{Z})-0,03207(Y+Z))$$
$$ekcf(2\eta-r+s)(-2,26927-2\varkappa\mathfrak{V}-\mathfrak{C}V-0,02711\mathfrak{S}-0,03207S)$$
$$ekcf(2\eta+r-s)(-0,53441-2\varkappa\mathfrak{X}-\mathfrak{C}X-0,02711\mathfrak{S}-0,03207S)$$
$$ekcf(2\eta-r-s)(-1,36456-2\varkappa\mathfrak{Y}-\mathfrak{C}Y-0,02711\mathfrak{T}-0,03207T)$$
$$ekcf(2\eta+r+s)(-1,43912-2\varkappa\mathfrak{Z}-\mathfrak{C}Z-0,02711\mathfrak{T}-0,03207T)$$

§. 161. Deinde fimili modo alterae formulae per differentiationem erutae, fubftitutis valoribus cognitis ita fe habebunt.

$$R =$$

$$ek\sin(r-s)(+0,59883+\mathfrak{A}(v'-x')-(1-\frac{1}{n})\mathfrak{S})+0,01974(\mathfrak{V}-\mathfrak{X})$$
$$ek\sin(r+s)(+0,54556+\mathfrak{A}(y'-z')-(1+\frac{1}{n})\mathfrak{T})+0,01974(\mathfrak{Y}-\mathfrak{Z})$$
$$ek\sin(2\eta-r+s)(+0,80251+\mathfrak{A}s'-(2\alpha-1+\frac{1}{n})\mathfrak{V})$$
$$ek\sin(2\eta+r-s)(+0,59936+\mathfrak{A}s'-(2\alpha+1-\frac{1}{n})\mathfrak{X})$$
$$ek\sin(2\eta-r-s)(+1,01199+\mathfrak{A}t'-(2\alpha-1-\frac{1}{n})\mathfrak{Y})$$
$$ek\sin(2\eta+r+s)(+0,38988+\mathfrak{A}t'-(\alpha+1+\frac{1}{n})\mathfrak{Z})$$

Porro

Porro reperiemus sequentes valores

$$S' = (1-\frac{1}{n})\,S - A\,(v'-x') + 0{,}38693 - 0{,}01974\,(V-X)$$

$$T' = (1+\frac{1}{n})\,T - A\,(y'-z') + 0{,}18018 - 0{,}01974\,(Y-Z)$$

$$V' = (2a-1+\frac{1}{n})\;V - A\,s' + 5{,}19902$$

$$X' = (2a+1-\frac{1}{n})\;X - A\,s' - 0{,}87311$$

$$Y' = (2a-1-\frac{1}{n})\;Y - A\,s' + 4{,}76391$$

$$Z' = (2a+1+\frac{1}{n})\;Z - A\,s' - 0{,}43800$$

ac denique $\frac{ddv}{dr^2} =$ Praec.

$$+\,ek\cos(r-s)(-(1-\frac{1}{n})S' + A'(v'+x') + 2{,}62592 - 0{,}01974\,(V'+X'))$$

$$+\,ek\cos(r+s)(-(1+\frac{1}{n})T' + A'(y'+z') + 2{,}16417 - 0{,}01974\,(Y'+Z'))$$

$$+\,ek\cos(2\eta-r+s)(-(2a-1+\frac{1}{n})\;V' + A's' - 4{,}19296)$$

$$+\,ek\cos(2\eta+r-s)(-(2a+1-\frac{1}{n})\;X' + A's' + 1{,}59777)$$

$$+\,ek\cos(2\eta-r-s)(-(2a-1-\frac{1}{n})\;Y' + A's' - 3{,}48384)$$

$$+\,ek\cos(2\eta+r+s)(-(2a+1+\frac{1}{n})\;Z' + A's' + 0{,}88865)$$

§. 162. Hinc ergo pro determinandis coefficientibus sequentes obtinemus aequationes

$$(1-\tfrac{1}{n})\mathfrak{S}=1,04739-0,00854(V-X)+\mathfrak{A}(v'-x')+0,01974(\mathfrak{V}-\mathfrak{X})$$

$$(1+\tfrac{1}{n})\mathfrak{T}=0,97382-0,00858(Y-Z)+\mathfrak{A}(y'-z')+0,01974(\mathfrak{Y}-\mathfrak{Z})$$

$$(2\alpha-1+\tfrac{1}{n})\ \mathfrak{V}=5,32189-0,00854\,S+\mathfrak{A}s'$$

$$(2\alpha+1-\tfrac{1}{n})\ \mathfrak{X}=5,11874-0,00854\,S+\mathfrak{A}s'$$

$$(2\alpha-1-\tfrac{1}{n})\ \mathfrak{Y}=5,53137-0,00854\,T+\mathfrak{A}s'$$

$$(2\alpha+1+\tfrac{1}{n})\ \mathfrak{Z}=4,90926-0,00854\,T+\mathfrak{A}s'$$

Deinde

$$+5,46097=(1-\tfrac{1}{n})S'-2n\mathfrak{S}-\mathfrak{C}S-0,02711(\mathfrak{V}+\mathfrak{X})-0,03207(V+X)$$
$$-A'(v'+x')+0,01974(V'+X')$$

$$+5,38086=(1+\tfrac{1}{n})T'-2n\mathfrak{T}-\mathfrak{C}T-0,02711(\mathfrak{Y}+\mathfrak{Z})-0,03207(Y+Z)$$
$$-A'(y'+z')+0,01974(Y'+Z')$$

$$-1,92371=(2\alpha-1+\tfrac{1}{n})V'-2n\mathfrak{V}-\mathfrak{C}V-0,02711\mathfrak{S}-0,03207\,S-A's'$$

$$+2,13218=(2\alpha+1-\tfrac{1}{n})X'-2n\mathfrak{X}-\mathfrak{C}X-0,02711\mathfrak{S}-0,03207\,S-A's'$$

$$-2,11928=(2\alpha-1-\tfrac{1}{n})Y'-2n\mathfrak{Y}-\mathfrak{C}Y-0,02711\mathfrak{T}-0,03207\,T-A's'$$

$$+2,32777=(2\alpha+1+\tfrac{1}{n})Z'-2n\mathfrak{Z}-\mathfrak{C}Z-0,02711\mathfrak{T}-0,03207\,T-A's'$$

§. 163.

§. 163. Pro vlteriori calculo est

$1-\frac{1}{n}=0,924562 \quad \ldots \quad l(1-\frac{1}{n})=9,965935$

$1+\frac{1}{n}=1,075438 \quad \ldots \quad l(1+\frac{1}{n})=0,031570$

$2\alpha-1+\frac{1}{n}=0,942914 \quad \ldots \quad l(2\alpha-1+\frac{1}{n})=9,965935$

$2\alpha+1-\frac{1}{n}=2,792038 \quad \ldots \quad l(2\alpha+1-\frac{1}{n})=0,445915$

$2\alpha-1-\frac{1}{n}=0,792038 \quad \ldots \quad l(2\alpha-1-\frac{1}{n})=9,898747$

$2\alpha+1+\frac{1}{n}=2,942914 \quad \ldots \quad l(2\alpha+1+\frac{1}{n})=0,468775$

Hinc in aequationibus posterioribus valores litterarum S′, T′, V′ substituantur, et ob $\mathfrak{C}=1,01591$, erit

$$0,16110S=-5,10323-2\kappa\mathfrak{S}-A'(v'+x')-0,02711(\mathfrak{V}+\mathfrak{X}) +0,01974(V'+X') +1,21835(v'-x')-0,03207(V+X) -0,01825(V-X)$$

$$0,14059T=+5,18709+2\kappa\mathfrak{T}+A'(y'+z')+0,02711(\mathfrak{Y}+\mathfrak{Z}) -0,01974(Y'+Z') -1,41710(y'-z')+0,03207(Y+Z) +0,02124(Y-Z)$$

$$0,12683V=+6,82591-2\kappa\mathfrak{V}+3,718s'-0,02711\mathfrak{S} -0,03207S$$

$$6,77934X=+4,56988+2\kappa\mathfrak{X}-6,1548s'+0,02711\mathfrak{S} +0,03207S$$

$$0,38858Y=+5,89253-2\kappa\mathfrak{Y}+3,5194t'-0,02711\mathfrak{T} -0,03207T$$

$$7,64474Z=+3,61676+2\kappa\mathfrak{Z}-6,3536t'+0,02711\mathfrak{T} +0,03207T$$

§. 164.

§. 164. Commodissime hi coefficientes inueniri videntur, si primo $\mathfrak{V}$, $\mathfrak{X}$, $\mathfrak{Y}$, $\mathfrak{Z}$ et V, X, Y, Z proxime quaerantur, quod fiet terminos minimos negligendo:

$$\mathfrak{V} = +5,7560 \quad \ldots \quad l\,\mathfrak{V} = 0,760125$$
$$\mathfrak{X} = +1,8334 \quad \ldots \quad l\,\mathfrak{X} = 0,263245$$
$$\mathfrak{Y} = +6,9837 \quad \ldots \quad l\,\mathfrak{Y} = 0,844088$$
$$\mathfrak{Z} = +1,6643 \quad \ldots \quad l\,\mathfrak{Z} = 0,221240$$
$$V = -37,7650 \quad \ldots \quad l{-V} = 1,577086$$
$$X = +1,2198 \quad \ldots \quad l\,X = 0,086293$$
$$Y = -21,1040 \quad \ldots \quad l{-Y} = 1,324360$$
$$Z = +0,9125 \quad \ldots \quad l\,Z = 9,960209$$
$$V' = -29,7170 \quad \ldots \quad v' = -0,398$$
$$X' = +2,5326 \quad \ldots \quad x' = +0,024$$
$$Y' = -11,9510 \quad \ldots \quad y' = -0,201$$
$$Z' = +2,2472 \quad \ldots \quad z' = +0,020$$

§. 165. Hic autem valores pro V′ et Y′ tam fiunt magni, vt vicissim post inuentas litteras S, T nimium valores modo erutos afficiant, vnde necesse erit in resolutionem harum aequationum ordinario modo instruere. Reperitur ergo

$$\mathfrak{V} = 5,7560 - 0,0193\,S - 0,0050\,\mathfrak{S}$$
$$\mathfrak{X} = 1,8333 - 0,0064\,S - 0,0016\,\mathfrak{S}$$
$$\mathfrak{Y} = 6,9837 - 0,0225\,T - 0,0058\,\mathfrak{T}$$
$$\mathfrak{Z} = 1,6643 - 0,0061\,T - 0,0016\,\mathfrak{T}$$
$$2x\,\mathfrak{V} = 11,6155 - 0,0390\,S - 0,0100\,\mathfrak{S}$$
$$2x\,\mathfrak{X} = 3,6996 - 0,0129\,S - 0,0033\,\mathfrak{S}$$
$$2x\,\mathfrak{Y} = 14,0930 - 0,0455\,T - 0,0118\,\mathfrak{T}$$
$$2x\,\mathfrak{Z} = 3,3586 - 0,0122\,T - 0,0032\,\mathfrak{T}$$

qui

qui valores fubftituti dant:

$$V = -37,7647 + 0,3912\,S + 0,0031\,\mathfrak{S}$$
$$X = +\ 1,2198 - 0,0076\,S - 0,0016\,\mathfrak{S}$$
$$Y = -21,1038 + 0,1385\,T + 0,0121\,\mathfrak{T}$$
$$Z = +\ 0,9125 - 0,0069\,T - 0,0016\,\mathfrak{T}$$

$$2\kappa V = -76,2085 + 0,7893\,S + 0,0064\,\mathfrak{S}$$
$$2\kappa X = +\ 2,4616 - 0,0153\,S - 0,0033\,\mathfrak{S}$$
$$2\kappa Y = -\ 42,5870 + 0,2794\,T + 0,0244\,\mathfrak{T}$$
$$2\kappa Z = +\ 1,8413 - 0,0140\,T - 0,0033\,\mathfrak{T}$$

§. 166. Hinc porro valores deriuati erunt

$$V' = -\ 29,7167 + 0,3767\,S + 0,0094\,\mathfrak{S}$$
$$X' = +\ 2,5326 - 0,0061\,S + 0,0029\,\mathfrak{S}$$
$$Y' = -\ 11,9511 + 0,1248\,T + 0,0161\,\mathfrak{T}$$
$$Z' = +\ 2,2472 - 0,0054\,T + 0,0028\,\mathfrak{T}$$

$$v' = -\ 0,4009 + 0,0044\,S + 0,0001\,\mathfrak{S}$$
$$x' = +\ 0,0244$$
$$y' = -\ 0,2026 + 0,0015\,T + 0,0001\,\mathfrak{T}$$
$$z' = +\ 0,0200$$

ac porro

$$\mathfrak{V} - \mathfrak{X} = 3,9227 - 0,0129\,S - 0,0034\,\mathfrak{S}$$
$$\mathfrak{Y} - \mathfrak{Z} = 5,3194 - 0,0164\,T - 0,0042\,\mathfrak{T}$$
$$V - X = -38,9845 + 0,3988\,S + 0,0047\,\mathfrak{S}$$
$$Y - Z = -22,0163 + 0,1454\,T + 0,0137\,\mathfrak{T}$$

$$\mathfrak{V} + \mathfrak{X} = 7,5893 - 0,0257\,S - 0,0066\,\mathfrak{S}$$
$$\mathfrak{Y} + \mathfrak{Z} = 8,6480 - 0,0286\,T - 0,0074\,\mathfrak{T}$$
$$V + X = -36,5449 + 0,3836\,S + 0,0015\,\mathfrak{S}$$
$$Y + Z = -20,1913 + 0,1316\,T + 0,0105\,\mathfrak{T}$$

$V'+X' = -27,1841 + 0,3706\,S + 0,0123\,\mathfrak{S}$
$Y'+Z' = -\ 9,7039 + 0,1194\,T + 0,0189\,\mathfrak{T}$
$v'-x' = -0,4253 + 0,0044\,S + 0,0001\,\mathfrak{S}$
$y'-z' = -0,2226 + 0,0015\,T + 0,0001\,\mathfrak{T}$
$v'+x' = -0,3765 + 0,0044\,S + 0,0001\,\mathfrak{S}$
$y'+z' = -0,1826 + 0,0015\,T + 0,0001\,\mathfrak{T}$

§. 167. His valoribus substitutis reperitur

$$\left(1-\frac{1}{n}\right)\mathfrak{S} = +\ 1,8024 - 0,0070\,S$$

$$\left(1+\frac{1}{n}\right)\mathfrak{T} = +\ 1,4453 - 0,0027\,T$$

vnde concluditur

$\mathfrak{S} = 1,9495 - 0,0075\,S$	$2n\mathfrak{S} = 3,9340 - 0,0160\,S$
$\mathfrak{T} = 1,3440 - 0,0025\,T$	$2n\mathfrak{T} = 2,7121 - 0,0053\,T$
$0,1401\,S = -9,2444$	$0,1479\,T = +7,9767$

§. 168. Nunc igitur habebimus

$S = -66,6980$	. . .	$l-S = 1,824113$
$T = +53,9330$	. . .	$l\,T = 1,731855$
$\mathfrak{S} = +\ 2,4497$	. . .	$l\,\mathfrak{S} = 0,389113$
$\mathfrak{T} = +\ 1,2092$	. . .	$l\,\mathfrak{T} = 0,082498$
$s' = -0,75204$	. . .	$l-s' = 9,876241$
$t' = +0,62626$	. . .	$l\,t' = 9,796755$
$v' = -\ 0,6942$	. . .	$l-v' = 9,841484$
$x' = +\ 0,0244$	. . .	$l\,x' = 8,387390$
$y' = -\ 0,1216$	. . .	$l-y' = 9,084933$
$z' = +\ 0,0200$	. . .	$l\,z' = 9,301030$

$\mathfrak{V} =$

$\mathfrak{W} = +7,0311 \quad \ldots \quad l\,\mathfrak{W} = 0,847029$

$\mathfrak{X} = +2,2561 \quad \ldots \quad l\,\mathfrak{X} = 0,353358$

$\mathfrak{Y} = +5,7659 \quad \ldots \quad l\,\mathfrak{Y} = 0,760867$

$\mathfrak{Z} = +1,3339 \quad \ldots \quad l\,\mathfrak{Z} = 0,125123$

$V = -63,8498 \quad \ldots \quad l-V = 1,805169$

$X = +1,7451 \quad \ldots \quad l\,X = 0,241820$

$Y = -13,6222 \quad \ldots \quad l-Y = 1,134241$

$Z = +0,5356 \quad \ldots \quad l\,Z = 9,728840$

§. 169. His iam valoribus inuentis pro distantia lunae $x = \frac{(1-kk)\,au}{1-k\cos r}$ erit valoris ipsius u portio ab his terminis pendens:

	Log. coeff.
$u = \text{Praec.} - 0,3796\, ek \cos(r-s)$	9,579297
$+ 0,3069\, ek \cos(r+s)$	9,487093
$- 0,3634\, ek \cos(2\eta-r+s)$	9,560344
$+ 0,0099\, ek \cos(2\eta+r-s)$	7,997004
$- 0,0775\, ek \cos(2\eta-r-s)$	8,889425
$+ 0,0030\, ek \cos(2\eta+r+s)$	7,484024

et pro longitudine lunae

$$\frac{d\Phi}{dr} = \text{Praec.} + 0,7520\, ek \cos(r-s)$$
$$- 0,6263\, ek \cos(r+s)$$
$$+ 0,6942\, ek \cos(2\eta-r+s)$$
$$- 0,0244\, ek \cos(2\eta+r-s)$$
$$+ 0,1216\, ek \cos(2\eta-r-s)$$
$$- 0,0200\, ek \cos(2\eta+r+s)$$

 cuius

cuius integrale si ponatur:

$$\Phi = \text{Praec.} + \mathfrak{S}'ek\,\text{sn}(r-s) + \mathfrak{V}'ek\,\text{sn}(2\eta-r+s) + \mathfrak{Y}'ek\,\text{sn}(2\eta-r-s)$$
$$+ \mathfrak{T}'ek\,\text{sn}(r+s) + \mathfrak{X}'ek\,\text{sn}(2\eta+r-s) + \mathfrak{Z}ek\,\text{sn}(2\eta+r+s)$$

erit

$$+0,7520 = (1-\tfrac{1}{n})\mathfrak{S}' - \mathfrak{A}'(v'+x') - \mathfrak{D}'q' - \mathfrak{E}'r' + \tfrac{1}{n}\mathfrak{P}' - \mathfrak{Q}'d' - \mathfrak{R}'c' - (\mathfrak{V}'+\mathfrak{X}')d'$$

$$-0,6263 = (1+\tfrac{1}{n})\mathfrak{T}' - \mathfrak{A}'(y'+z') - \mathfrak{D}'r' - \mathfrak{E}'q' + \tfrac{1}{n}\mathfrak{P}' - \mathfrak{Q}'c' - \mathfrak{R}'d' - (\mathfrak{Y}'+\mathfrak{Z}')d'$$

$$+0,6942 = (2a-1+\tfrac{1}{n})\mathfrak{V}' + \mathfrak{A}'(\tfrac{2}{n}-s') + \mathfrak{D}'(\tfrac{2}{n}-p') - \mathfrak{R}'(c'-\tfrac{1}{n})$$

$$-0,0244 = (2a+1-\tfrac{1}{n})\mathfrak{X}' + \mathfrak{A}'(\tfrac{2}{n}-s') + \mathfrak{E}'(\tfrac{2}{n}-p') - \mathfrak{Q}'(c'+\tfrac{1}{n})$$

$$+0,1216 = (2a-1-\tfrac{1}{n})\mathfrak{Y} + \mathfrak{A}'(\tfrac{2}{n}-t') + \mathfrak{D}'(\tfrac{2}{n}-p') - \mathfrak{Q}'(c'+\tfrac{1}{n})$$

$$-0,0200 = (2a+1+\tfrac{1}{n})\mathfrak{Z} + \mathfrak{A}'(\tfrac{2}{n}-t') + \mathfrak{E}'(\tfrac{2}{n}-p') - \mathfrak{R}'(c'-\tfrac{1}{n})$$

§. 170. Hinc autem reperitur

$\mathfrak{S}' = +0,7467 \quad \ldots \quad l\,\mathfrak{S}' = 9,873165$
$\mathfrak{T}' = -0,6185 \quad \ldots \quad l-\mathfrak{T}' = 9,791317$
$\mathfrak{V}' = +0,8143 \quad \ldots \quad l\,\mathfrak{V}' = 9,910800$
$\mathfrak{X}' = -0,0142 \quad \ldots \quad l-\mathfrak{X}' = 8,150690$
$\mathfrak{Y}' = +0,2396 \quad \ldots \quad l\,\mathfrak{Y}' = 9,379550$
$\mathfrak{Z}' = -0,0061 \quad \ldots \quad l-\mathfrak{Z}' = 7,788910$

§. 171. Quatenus ergo longitudo Lunae ab excentricitate orbitae solis pendet, erit

		log. coeff.
$\varphi =$ Praec.	$+\ 0{,}201385\, e \sin s$	9,304026
	$-\ 0{,}021889\, e \sin(2\eta - s)$	8,340237
	$-\ 0{,}016368\, e \sin(2\eta + s)$	8,214002
	$+\ 0{,}06615\, ee \sin 2s$	8,820508
	$+\ 0{,}02332\, ee \sin(2\eta - 2s)$	8,367825
	$+\ 0{,}00840\, ee \sin(2\eta + 2s)$	7,924429
	$+\ 0{,}747$[illegible] $e k \sin(r - s)$	9,873165
	$-\ 0{,}6185\, e k \sin(r + s)$	9,791317
	$+\ 0{,}8143\, e k \sin(2\eta - r + s)$	9,910800
	$-\ 0{,}0142\, e k \sin(2\eta + r - s)$	8,150690
	$+\ 0{,}2396\, e k \sin(2\eta - r - s)$	9,379550
	$-\ 0{,}0061\, e k \sin(2\eta + r + s)$	7,788910

§. 172. Hae autem ſingulae inaequalitates ad numerum minutorum ſecundorum reductae dabunt;

	log. coeff.
$\phi =$ Praec. $+ 701'',1 \sin s$.	2,845780
$- 75,8 \sin(2\eta - s)$	1,879971
$- 56,7 \sin(2\eta + s)$	1,753736
$+ 3,8 \sin 2s$	0,585551
$+ 1,4 \sin(2\eta - 2s)$	0,132862
$+ 0,5 \sin(2\eta + 2s)$	9,689472
$+ 140,9 \sin(r - s)$	2,148800
$- 116,7 \sin(r + s)$	2,067000
$+ 153,7 \sin(2\eta - r + s)$	2,186530
$- 2,7 \sin(2\eta + r - s)$	0,426300
$+ 45,2 \sin(2\eta - r - s)$	1,655200
$+ 1,2 \sin(2\eta + r + s)$	0,064600

Hic ſcilicet et inaequalitates, quas in capite praecedente inuenimus, et iſtas in hoc capite erutas ſimul ſum complexus, vt coniunctim conſpectui exponerentur.

CAPUT XI.

INUESTIGATIO INAEQUALITATUM LUNAE A PARALLAXI SOLIS PENDENTIUM.

§. 173.

Jam in formulis nostris primariis ad eos quoque terminos progrediamur, qui littera ν sunt affecti, et quoniam est $1:\nu$ vt distantia Solis media ad distantiam Lunae mediam a Terra, erit $1:\nu$ vt parallaxis Lunae media ad parallaxin solis: ex quo inaequalitates Lunae, quae hinc oriuntur, a parallaxi solis pendere dicuntur. Quoniam vero valor ipsius ν est valde paruus, quippe $\frac{1}{280}$ propemodum, alios terminos non contemplabimur, nisi qui per ν ac per νk et νe sunt multiplicati, propterea quod magis compositi fiant minimi.

§. 174. Ex terminis ergo iam inuentis hic retineamus eos, qui sunt alicuius momenti, et cum iis nouos determinandos coniungamus; sit ergo:

$$\begin{aligned}\int R\,dr = \mathfrak{A}\cos 2\eta &+ \mathfrak{C}k\cos r + \mathfrak{D}k\cos(2\eta-r) + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta-s)\\ &+ \mathfrak{E}k\cos(2\eta+r) \qquad\qquad + \mathfrak{R}e\cos(2\eta+s)\\ &+ \mathfrak{F}\nu\cos\eta \quad + \mathfrak{H}\nu k\cos(\eta-r) \quad + \mathfrak{K}\nu e\cos(\eta-s)\\ &+ \mathfrak{G}\nu\cos 3\eta \quad + \mathfrak{J}\nu k\cos(\eta+r) \quad + \mathfrak{L}\nu e\cos(\eta+s)\end{aligned}$$

$$\begin{aligned}v = A\cos 2\eta\;\ldots &+ Dk\cos(2\eta-r) + Pe\cos s + Qe\cos(2\eta-s)\\ &+ Ek\cos(2\eta+r) \qquad\qquad + Re\cos(2\eta+s)\\ &+ F\nu\cos\eta \quad + H\nu k\cos(\eta-r) \quad + K\nu e\cos(\eta-s)\\ &+ G\nu\cos 3\eta \quad + J\nu k\cos(\eta+r) \quad + L\nu e\cos(\eta+s)\end{aligned}$$

Non

Non difficulter enim praeuidere licet, terminos, qui angulos r et s cum angulo 3η habeant coniunctos, fore tam exiguos, vt ſine errore praetermitti queant.

§. 175. Quodſi iam retentis litterarum §. 153. inductarum valoribus, praeterea ponamus:

$$f' = \frac{2\varkappa F + \mathfrak{F}}{nn}\,; \quad g' = \frac{2\varkappa G + \mathfrak{G}}{nn}\,; \quad h' = \frac{2\varkappa H + \mathfrak{H}}{nn}$$

$$i' = \frac{2\varkappa J + \mathfrak{J}}{nn}\,; \quad k' = \frac{2\varkappa K + \mathfrak{K}}{nn}\,; \quad l' = \frac{2\varkappa L + \mathfrak{L}}{nn}$$

habebimus:

$$\begin{aligned}\frac{d\Phi}{dr} = \text{Praec.} & - a'\cos 2\eta - \frac{\mathfrak{E}}{nn}k\cos r - d'k\cos(2\eta - r) - p'e\cos s - q'e\cos(2\eta - s) \\ & - e'k\cos(2\eta + r) - r'e\cos(2\eta + s) \\ & - f'v\cos\eta - h'vk\cos(\eta - r) - k've\cos(\eta - s) \\ & - g'v\cos 3\eta - i'vk\cos(\eta + r) - l've\cos(\eta + s)\end{aligned}$$

atque

$$\begin{aligned}\frac{d\eta}{dr} = \alpha & - a'\cos 2\eta - c'k\cos r - d'k\cos(2\eta - r) + \left(\frac{2}{n} - p'\right)e\cos s - q'e\cos(2\eta - s) \\ & - e'k\cos(2\eta + r) - r'e\cos(2\eta + s) \\ & - f'v\cos\eta - h'vk\cos(\eta - r) - k've\cos(\eta - s) \\ & - g'v\cos\eta - i'vk\cos(\eta + s) - l've\cos(\eta + s)\end{aligned}$$

§. 176. Jam vero pro his terminis ab v pendentibus ſequentes colligemus aequationes.

$$R = v\sin\eta\left(\frac{3}{8} + \frac{8F - 3G}{2nn}\right)$$

$$v\sin 3\eta\left(\frac{1}{8} + \frac{3F}{2nn}\right)$$

$$v\,k \sin(\eta - r)\left(\tfrac{15}{16} + \frac{3\,J}{2nn} + \frac{3\,F}{nn} - \frac{3\,G}{nn}\right)$$

$$v\,k \sin(\eta + r)\left(\tfrac{15}{16} + \frac{3\,H}{2nn} - \frac{3\,G}{nn} + \frac{3\,F}{nn}\right)$$

$$v\,e \sin(\eta - s)\left(-\tfrac{3}{4} + \frac{3\,L}{2nn} - \frac{9\,F}{4nn} + \frac{9\,G}{4nn}\right)$$

$$v\,e \sin(\eta + s)\left(-\tfrac{3}{4} + \frac{3\,K}{2nn} + \frac{9\,G}{4nn} - \frac{9\,F}{4nn}\right)$$

$$\frac{ddv}{dr^2} =$$

$$v \cos\eta \quad \left\{\tfrac{9}{8} - 6F + \frac{3\,F}{4nn} + \frac{3\,G}{4nn} - 2\,x\,\mathfrak{F} + \frac{\mathfrak{A}\mathfrak{F}}{nn} + \frac{\mathfrak{A}\mathfrak{G}}{nn}\right.$$

$$v \cos 3\eta \quad \left\{\tfrac{15}{8} - 6G + \frac{3\,F}{4nn} - 2\,x\,\mathfrak{G} + \frac{\mathfrak{A}\mathfrak{F}}{nn}\right.$$

$$v\,k \cos(\eta - r)\left\{\begin{array}{l}\tfrac{45}{16} - 6H + \tfrac{1}{2}bF + \frac{3\,J}{4nn} + \frac{3\,F}{2nn} + \frac{3\,G}{2nn} - 2\,x\,\mathfrak{H} \\ + \frac{\mathfrak{A}\mathfrak{J}}{nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn} + \frac{\mathfrak{D}\mathfrak{F}}{nn} + \frac{\mathfrak{C}\mathfrak{G}}{nn}\end{array}\right.$$

$$v\,k \cos(\eta + r)\left\{\begin{array}{l}\tfrac{45}{16} - 6J + \tfrac{1}{2}bF + \frac{3\,H}{4nn} + \frac{3\,G}{2nn} + \frac{3\,F}{2nn} - 2\,x\,\mathfrak{J} \\ + \frac{\mathfrak{A}\mathfrak{H}}{nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn} + \frac{\mathfrak{D}\mathfrak{G}}{nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn}\end{array}\right.$$

$$ve\cos(\eta - s)\begin{cases} -\frac{2}{4} - \mathfrak{E}K + \frac{3L}{4nn} - \frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2\varkappa\mathfrak{K} \\ +\frac{\mathfrak{A}\mathfrak{L}}{nn} + \frac{\mathfrak{F}\mathfrak{P}}{nn} + \frac{\mathfrak{F}\mathfrak{Q}}{nn} + \frac{\mathfrak{G}\mathfrak{R}}{nn} \end{cases}$$

$$ve\cos(\eta + s)\begin{cases} -\frac{2}{4} - \mathfrak{E}L + \frac{3K}{4nn} - \frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2\varkappa\mathfrak{L} \\ +\frac{\mathfrak{A}\mathfrak{K}}{nn} + \frac{\mathfrak{F}\mathfrak{P}}{nn} + \frac{\mathfrak{F}\mathfrak{R}}{nn} + \frac{\mathfrak{G}\mathfrak{Q}}{nn} \end{cases}$$

ſequentes partes ſeorſim exponamus:

$$v\cos\eta\begin{cases} +\frac{3\mathfrak{A}F}{nn} + \frac{3\mathfrak{A}G}{nn} + \frac{3A\mathfrak{F}}{nn} + \frac{3A\mathfrak{G}}{nn} + \frac{3AF}{nn} + \frac{3AG}{nn} \\ +\frac{9A}{8nn} + \frac{15A}{8nn} \end{cases}$$

$$v\cos 3\eta\ \left\{ +\frac{3\mathfrak{A}F}{nn} + \frac{3\mathfrak{A}\mathfrak{F}}{nn} + \frac{3AF}{nn} + \frac{9A}{8nn} \right.$$

$$vk\cos(\eta - r)\begin{cases} \frac{3\mathfrak{A}J}{nn} + \frac{3\mathfrak{E}F}{nn} + \frac{3\mathfrak{D}F}{nn} + \frac{3\mathfrak{E}G}{nn} + \frac{3A\mathfrak{J}}{nn} + \frac{3D\mathfrak{F}}{nn} \\ +\frac{3E\mathfrak{G}}{nn} + \frac{3AJ}{nn} + \frac{3DF}{nn} + \frac{3EG}{nn} - \frac{3AF}{nn} - \frac{3AG}{nn} \\ +\frac{9D}{8nn} + \frac{15E}{8nn} \end{cases}$$

$$vk\cos(\eta + r)\begin{cases} \frac{3\mathfrak{A}H}{nn} + \frac{3\mathfrak{E}F}{nn} + \frac{3\mathfrak{D}\mathfrak{G}}{nn} + \frac{3\mathfrak{E}F}{nn} + \frac{3A\mathfrak{H}}{nn} \\ +\frac{3D\mathfrak{G}}{nn} + \frac{3E\mathfrak{F}}{nn} + \frac{3AH}{nn} + \frac{3DG}{nn} + \frac{3EF}{nn} \\ -\frac{3AF}{2nn} - \frac{3AG}{2nn} + \frac{9E}{8nn} + \frac{15D}{8nn} \end{cases}$$

ve coſ

$$v\varepsilon\cos(\eta-s)\begin{cases}\frac{3\mathfrak{A}L}{nn}+\frac{3\mathfrak{P}F}{nn}+\frac{3\mathfrak{Q}F}{nn}+\frac{3\mathfrak{R}G}{nn}+\frac{3A\mathfrak{L}}{nn}\\+\frac{3P\mathfrak{F}}{nn}+\frac{3Q\mathfrak{F}}{nn}+\frac{3R\mathfrak{G}}{nn}+\frac{3AL}{nn}+\frac{3PF}{nn}+\frac{3QF}{nn}\\+\frac{3RG}{nn}+\frac{9P}{8nn}+\frac{9Q}{8nn}+\frac{15R}{8nn}\end{cases}$$

$$v\varepsilon\cos(\eta+s)\begin{cases}\frac{3\mathfrak{A}K}{nn}+\frac{3\mathfrak{P}F}{nn}+\frac{3\mathfrak{Q}G}{nn}+\frac{3\mathfrak{R}F}{nn}+\frac{3A\mathfrak{K}}{nn}+\frac{3PF}{nn}\\+\frac{3Q\mathfrak{G}}{nn}+\frac{3R\mathfrak{F}}{nn}+\frac{3AK}{nn}+\frac{3PF}{nn}+\frac{3QF}{nn}+\frac{3QG}{nn}\\+\frac{9P}{8nn}+\frac{9R}{8nn}+\frac{15Q}{8nn}\end{cases}$$

§. 177. Verum differentiando nanciscemur

R ═══

$$v\sin\eta \quad [\mathfrak{A}f'-\mathfrak{A}g'-a\mathfrak{F}-\tfrac{1}{2}\mathfrak{F}a'+\tfrac{1}{2}\mathfrak{G}a'$$

$$v\sin 3\eta \quad [\mathfrak{A}f'+\tfrac{1}{2}\mathfrak{F}a'-3a\mathfrak{G}$$

$$vk\sin(\eta-r) \quad [\mathfrak{A}i'+\mathfrak{D}f'-\mathfrak{E}g'+\tfrac{1}{2}\mathfrak{F}c'-\tfrac{1}{2}\mathfrak{F}d'+\tfrac{1}{2}\mathfrak{G}c'-(a-1)\mathfrak{H}-\tfrac{1}{2}\mathfrak{J}a'$$

$$vk\sin(\eta+r)\begin{cases}\mathfrak{A}h'-\mathfrak{D}g'+\mathfrak{E}f'+\tfrac{1}{2}\mathfrak{F}c'-\tfrac{1}{2}\mathfrak{F}e'+\tfrac{1}{2}\mathfrak{G}d'-\tfrac{1}{2}\mathfrak{H}a'\\-(a+1)\mathfrak{J}\end{cases}$$

$$v\varepsilon\sin(\eta-s)\begin{cases}+\mathfrak{A}l'+\mathfrak{Q}f'-\mathfrak{R}g'-\tfrac{1}{2}(\tfrac{2}{n}-p')\mathfrak{F}-\tfrac{1}{2}\mathfrak{F}q'+\tfrac{1}{2}\mathfrak{G}r'\\-(a-\tfrac{1}{n})\mathfrak{K}-\tfrac{1}{2}\mathfrak{L}a'\end{cases}$$

$$v\varepsilon\sin(\eta+s)\begin{cases}+\mathfrak{A}k'-\mathfrak{Q}g'+\mathfrak{R}f'-\tfrac{1}{2}(\tfrac{2}{n}-p')\mathfrak{F}-\tfrac{1}{2}\mathfrak{F}r'+\tfrac{1}{2}\mathfrak{G}q'\\-(a+\tfrac{1}{n})\mathfrak{L}-\tfrac{1}{2}\mathfrak{K}a'\end{cases}$$

 Deinde

Deinde posito

$$F' = \alpha F - A(f' - g') + \tfrac{1}{2} F a' - \tfrac{3}{2} G a'$$

$$G' = 3\alpha G - A f' - \tfrac{1}{2} F a'$$

$$H' = (\alpha - 1) H - A i' - D f' + E g' - \tfrac{1}{2} F c' + \tfrac{1}{2} F d' - \tfrac{3}{2} G e' + \tfrac{1}{2} J a'$$

$$J' = (\alpha + 1) J - A b' + D g' - E f' - \tfrac{1}{2} F c' + \tfrac{1}{2} F e' - \tfrac{3}{2} G d' + \tfrac{1}{2} H a'$$

$$K' = (\alpha - 1) K - A l' - Q f' + R g' + \tfrac{1}{2} F\left(\frac{2}{n} - p'\right) + \tfrac{1}{2} F q' - \tfrac{3}{2} G r' + \tfrac{1}{2} L a'$$

$$L' = (\alpha + 1) L - A k' + Q g' - R f' + \tfrac{1}{2} F\left(\frac{2}{n} - p'\right) + \tfrac{1}{2} F r' - \tfrac{3}{2} G q' + \tfrac{1}{2} K a'$$

erit

$$\begin{aligned} \frac{dv}{dr} = & - A' \sin 2\eta - D' k \sin(2\eta - r) - P' e \sin s - Q' e \sin(2\eta - s) \\ & \qquad\qquad - E' k \sin(2\eta + r) \qquad\qquad - R' e \sin(2\eta + s) \\ & - F' v \sin \eta - H' v k \sin(\eta - r) - K' v e \sin(\eta - s) \\ & - G' v \sin 3\eta - J' v k \sin(\eta + r) - L' v e \sin(\eta + s) \end{aligned}$$

§. 178. Hinc iam denuo differentiando consequemur: $\dfrac{ddv}{dr^2} =$ Praec.

$$+ v \cos \eta \; [+ A' f' + A' g' - \alpha F' + \tfrac{1}{2} F' a' + \tfrac{3}{2} G' a'$$

$$+ v \cos 3\eta \; [+ A' f' + \tfrac{1}{2} F' a' - 3\alpha G'$$

$$+ v k \cos(\eta - r) \begin{cases} + A' i' + D' f' + E' g' + \tfrac{1}{2} F' c' + \tfrac{1}{2} F' d' + \tfrac{3}{2} G' e' \\ \qquad - (\alpha - 1) H' + \tfrac{1}{2} J' a' \end{cases}$$

$$+ v k \cos(\eta + r) \begin{cases} + A' b' + E' f' + D' g' + \tfrac{1}{2} F' c' + \tfrac{1}{2} F' e' + \tfrac{3}{2} G' d' \\ \qquad + \tfrac{1}{2} H' a' - (\alpha + 1) J' \end{cases}$$

$$+ v e \cos(\eta - s) \begin{cases} + A' l' + Q' f' + R' g' - \tfrac{1}{2}\left(\frac{2}{n} - p'\right) F' + \tfrac{1}{2} F' g' + \tfrac{3}{2} G' r' \\ \qquad - \left(\alpha - \frac{1}{n}\right) K' + \tfrac{1}{2} L' a' \end{cases}$$

+

$$+ \nu e \cos(\eta + s) \left\{ \begin{array}{l} + A'k' + R'f' + Q'g' - \tfrac{1}{2}\left(\frac{2}{n} - p'\right)F' + \tfrac{1}{2}F'r' + \tfrac{3}{2}G'q' \\ \qquad + \tfrac{1}{2}K'a' - \left(\alpha + \frac{1}{n}\right)L' \end{array} \right.$$

qui valores cum antecedentibus comparati debent, vt inde valores coefficientium eliciantur.

§. 179. Sumamus primo duos valores ab initio positos, quoniam hi a sequentibus non pendent, atque habebimus,

$$0 = \alpha \mathfrak{F} - \mathfrak{A}f' + \mathfrak{A}g' + \tfrac{1}{2}\mathfrak{F}a' - \tfrac{1}{2}\mathfrak{G}a' + \tfrac{3}{8} + \frac{3F-3G}{2nn}$$

$$0 = 3\alpha\mathfrak{G} - \mathfrak{A}f' - \tfrac{1}{2}\mathfrak{F}a' + \tfrac{3}{8} + \frac{3F}{2nn}$$

$$0 = \alpha F' - A'f' - A'g' - \tfrac{1}{2}F'a' - \tfrac{1}{2}G'a' + \tfrac{3}{8} - \mathfrak{C}F - 2\kappa\mathfrak{F}$$

$$+ \frac{3(F+G)}{4nn} + \frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{F}+\mathfrak{G}) + \frac{(3A+3A)}{nn}(F+G) + \frac{3A}{nn}$$

$$0 = 3\alpha G' - A'f' - \tfrac{1}{2}F'a' + \tfrac{3}{8} - \mathfrak{C}G - 2\kappa G$$

$$+ \frac{3F}{4nn} + \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{F} + \frac{(3\mathfrak{A}+3A)}{nn}F + \frac{9A}{8nn}$$

et $F' = \alpha F - A(f' - g') + \tfrac{1}{2}Fa' - \tfrac{1}{2}Ga'$

$G' = 3\alpha G - Af' - \tfrac{1}{2}Fa'$

at est $f' = \dfrac{2\kappa F + \mathfrak{F}}{nn}$ et $g' = \dfrac{2\kappa G + \mathfrak{G}}{nn}$

Hic igitur primum litterarum, quae sunt cognitae, valores in numeris substituantur, eritque

$$F' = 0,93905\,F + 0,00753\,\mathfrak{F} + 0,01443\,G - 0,00753\,\mathfrak{G}$$
$$G' = 2,80121\,G + 0,02505\,F + 0,00753\,\mathfrak{F}$$
$$0 = 0,92847\,\mathfrak{F} + 0,01776\,F - 0,01776\,G - 0,02501\,\mathfrak{G} + 0,37500$$
$$0 = 2,80121\,\mathfrak{G} + 0,01447\,\mathfrak{F} + 0,01776\,F + 1,87500$$

hincque

$$\mathfrak{F} = -0,01930\,F + 0,01913\,G - 0,42145$$
$$\mathfrak{G} = -0,00623\,F - 0,00010\,G - 0,66717$$

§. 180. Inde porro colligemus

$$F' = 0,93895\,F + 0,01457\,G + 0,00185$$
$$G' = 0,02491\,F + 2,80135\,G - 0,00317$$
$$f' = 0,01138\,F + 0,00011\,G - 0,00240$$
$$g' = -0,00004\,F + 0,01148\,G - 0,00380$$

quibus valoribus substitutis peruenimus ad has aequationes:

$$0,09337\,F = +0,05421\,G + 1,96867$$
$$6,83135\,G = -0,08830\,F - 3,20944$$

vnde fit:

$$F = 20,65700 \quad \ldots \quad l\,F = 1,315067$$
$$G = -0,73681 \quad \ldots \quad l{-}G = 9,867356$$
$$\mathfrak{F} = -0,83418 \quad \ldots \quad l{-}\mathfrak{F} = 9,921260$$
$$\mathfrak{G} = -0,79580 \quad \ldots \quad l{-}\mathfrak{G} = 9,900804$$
$$F' = +19,38712 \quad \ldots \quad l\,F' = 1,287512$$
$$G' = -1,55266 \quad \ldots \quad l{-}G' = 0,191075$$
$$f' = +0,23259 \quad \ldots \quad l\,f' = 9,366501$$
$$g' = -0,01309 \quad \ldots \quad l{-}g' = 8,116940$$

§. 181.

§. 181. His valoribus, qui ad inaequalitates absolutas pertinent, expeditis, progrediamur ad eos, qui ab excentricitate orbitae lunaris pendent, ac his aequationibus continentur:

$$(a-1)\mathfrak{H} - \mathfrak{A}i' + \tfrac{1}{2}\mathfrak{J}a' - \mathfrak{D}f' + \mathfrak{E}g' - \tfrac{1}{2}\mathfrak{F}(c'-d') - \tfrac{3}{2}\mathfrak{G}e'$$
$$+ \tfrac{1}{2}\tfrac{5}{8} + \frac{3J}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$(a+1)\mathfrak{J} - \mathfrak{A}b' + \tfrac{1}{2}\mathfrak{H}a' + \mathfrak{D}g' - \mathfrak{E}f' - \tfrac{1}{2}\mathfrak{F}(c'-e') - \tfrac{3}{2}\mathfrak{G}d'$$
$$+ \tfrac{1}{2}\tfrac{5}{8} + \frac{3H}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$H' = (a-1)H - Ai' + \tfrac{1}{2}Ja' - Df' + Eg' - \tfrac{1}{2}F(c'-d') - \tfrac{3}{2}Ge'$$
$$J' = (a+1)J - Ab' + \tfrac{1}{2}Ha' + Dg' - Ef' - \tfrac{1}{2}F(c'-e') - \tfrac{3}{2}Gd'$$

$$(a-1)H' - A'i' - \tfrac{1}{2}J'a' - D'f' - E'g' - \tfrac{1}{2}F'(c'+d') - \tfrac{3}{2}G'e'$$
$$+ \tfrac{1}{2}\tfrac{5}{8} - 6H - 2u\mathfrak{H} + \tfrac{1}{2}bF + \frac{3J}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{E}\mathfrak{F}}{nn} + \frac{3\mathfrak{E}F}{nn}$$
$$+ \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{J} + \frac{(3\mathfrak{A}+3A)}{nn}J + \frac{(\mathfrak{D}+3D)}{nn}\mathfrak{F} + \frac{(3\mathfrak{D}+3D)}{nn}F$$
$$+ \frac{(\mathfrak{E}+3E)}{nn}\mathfrak{G} + \frac{(3\mathfrak{E}+3E)}{nn}G - \frac{3A(F+G)}{2nn} + \frac{9D+15E}{8nn} = 0$$

$$(a+1)J' - A'b' - \tfrac{1}{2}H'a' - E'f' - D'g' - \tfrac{1}{2}F'(c'+e') - \tfrac{3}{2}G'd'$$
$$+ \tfrac{1}{2}\tfrac{5}{8} - 6J - 2uJ + \tfrac{1}{2}bF + \frac{3H}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{E}\mathfrak{F}}{nn} + \frac{3\mathfrak{E}F}{nn}$$
$$+ \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{H} + \frac{(3\mathfrak{A}+3A)}{nn}H + \frac{(\mathfrak{D}+3D)}{nn}\mathfrak{G} + \frac{(3\mathfrak{D}+3D)}{nn}G$$
$$+ \frac{(\mathfrak{E}+3E)}{nn}\mathfrak{F} + \frac{(3\mathfrak{E}+3E)}{nn}F - \frac{3A(F+G)}{2nn} + \frac{9E+15D}{8nn} = 0$$

§. 182.

§. 182. In his aequationibus substituantur valores iam cogniti, atque obtinebimus,

$$-0,06626\,\mathfrak{H} + 0,81033\,i' - 0,00987\,\mathfrak{J} + 0,00854\,J \pm 2,04620 = 0$$

$$1,83374\,\mathfrak{J} + 0,81033\,b' - 0,00987\,\mathfrak{H} + 0,00854\,H + 2,10646 = 0$$

$$H' = -0,06626\,H + 1,31773\,i' - 0,00937\,J - 5,57458$$

$$J' = 1,83374\,J + 1,31773\,b' - 0,00987\,H - 1,55427$$

$$\left.\begin{array}{l} -0,06626\,H' + 2,47576\,i' + 0,00987\,J' - 11,86106 \\ -1,01591\,H + 0,00427\,J - 0,02711\,\mathfrak{J} + 2,81250 \\ -2,01798\,\mathfrak{H} - 0,03634\,J \qquad + 31,31044 \\ \qquad\qquad + 10,60834 \end{array}\right\} = 0$$

$$\left.\begin{array}{l} 1,83374\,J' + 2,47576\,b' + 0,00987\,H' + 0,27762 \\ -1,01591\,J + 0,00427\,H - 0,02711\,\mathfrak{H} + 2,81250 \\ -2,01798\,\mathfrak{J} - 0,03634\,H \qquad + 31,81380 \\ \qquad\qquad - 0,81380 \end{array}\right\} = 0$$

§. 183. Substituamus primo loco b' et i' valores, atque nostrae aequationes reducentur ad formas sequentes,

$$-0,06626\,\mathfrak{H} + 0,01785\,J - 0,00526\,\mathfrak{J} + 2,04620 = 0$$

$$+1,83374\,\mathfrak{J} + 0,01785\,H - 0,00526\,\mathfrak{H} + 2,10646 = 0$$

$$H' = -0,06626\,H + 0,00526\,J + 0,00750\,\mathfrak{J} - 5,57458$$

$$J' = +1,83374\,J + 0,00526\,H + 0,00750\,\mathfrak{H} - 1,55427$$

qui valores in sequentibus substituti dant

$$-1,01147\,H - 2,01791\,\mathfrak{H} + 0,01417\,J - 0,01352\,\mathfrak{J} + 33,22429 = 0$$

$$+2,84674\,J - 2,01791\,\mathfrak{J} + 0,00535\,H + 0,00073\,\mathfrak{H} + 30,68146 = 0$$

vnde elicimus:

$$\mathfrak{H} = \pm 0,00077\,H + 0,26935\,J + 30,96780$$

$$\mathfrak{J} = -0,00974\,H + 0,00077\,J - 1,05989$$

§. 184.

§. 184. Hi autem valores in posterioribus aequationibus substituti producent

$$1,01290\,H + 0,52937\,J + 29,25137 = 0$$

$$2,34539\,J + 0,02500\,H + 32,84282 = 0$$

hincque tandem concluditur:

$H = -21,68110$	. . .	$l-H = 1,336081$
$J = -13,77206$	. . .	$l-J = 1,138999$
$\mathfrak{H} = +27,24155$	. . .	$l\,\mathfrak{H} = 1,435232$
$\mathfrak{J} = -1,25933$	. . .	$l-\mathfrak{J} = 0,100140$
$h' = -0,09396$	. . .	$l-h' = 8,972943$
$i' = -0,16533$	. . .	$l-i' = 9,218352$

§. 185. Nunc pro excentricitate orbitae solaris hae restant aequationes,

$$(a-\frac{1}{n})\mathfrak{K} - \mathfrak{A}l'' + \tfrac{1}{2}\mathfrak{L}s' - \mathfrak{Q}f' + \mathfrak{R}g' + \tfrac{1}{2}(\frac{2}{n}-p')\mathfrak{F} + \tfrac{1}{2}\mathfrak{F}q' - \tfrac{3}{2}\mathfrak{G}r'$$

$$-\tfrac{3}{4} + \frac{3\mathfrak{L}}{2nn} - \frac{9F}{4nn} + \frac{9G}{4nn} = 0$$

$$(a+\frac{1}{n})\mathfrak{L} - \mathfrak{A}k' + \tfrac{1}{2}\mathfrak{K}s' + \mathfrak{Q}g' - \mathfrak{R}f' + \tfrac{1}{2}(\frac{2}{n}-p')\mathfrak{F} + \tfrac{1}{2}\mathfrak{F}r' - \tfrac{3}{2}\mathfrak{G}q'$$

$$-\tfrac{3}{4} + \frac{3K}{2nn} - \frac{9F}{4nn} + \frac{9G}{4nn} = 0$$

$$K' = (a-\frac{1}{n})K - Al'' + \tfrac{1}{2}Ls' - Qf' + Rg' + \tfrac{1}{2}(\frac{2}{n}-p')F + \tfrac{1}{2}Fq' - \tfrac{3}{2}Gr'$$

$$L' = (a+\frac{1}{n})L - Ak' + \tfrac{1}{2}Ks' + Qg' - Rf' + \tfrac{1}{2}(\frac{2}{n}-p')F + \tfrac{1}{2}Fr' - \tfrac{3}{2}Gq'$$

$$(a-\frac{1}{n})K' - A'l'' - \tfrac{1}{2}L's' - Q'f' - R'g' + \tfrac{1}{2}(\frac{2}{n}-p')F' - \tfrac{1}{2}F'q' - \tfrac{3}{2}G'r'$$

$$-\tfrac{3}{4} - \mathfrak{E}K - 2k\mathfrak{K} - \frac{15F}{8nn} - \frac{9G}{8nn} + \frac{3L}{4nn} + \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{L} + \frac{(3\mathfrak{A}+3A)L}{nn}$$

$$+\frac{(\mathfrak{F}+3F)}{nn}(\mathfrak{P}+\mathfrak{Q})+\frac{(3\mathfrak{F}+3F)}{nn}(P+Q)+\frac{(\mathfrak{G}+3G)}{nn}\mathfrak{R}+\frac{(3\mathfrak{G}+3G)}{nn}R$$
$$+\frac{9P}{8nn}+\frac{9Q}{8nn}+\frac{15R}{8nn}=0$$

$$(a+\frac{1}{n})L'-A'k'-\tfrac{1}{2}K'a'-R'f'-Q'g'+\tfrac{1}{2}(\frac{2}{n}-p')F'-\tfrac{1}{2}F'r-\tfrac{1}{2}G'q'$$
$$-\tfrac{1}{2}-\mathfrak{C}L-2x\mathfrak{L}+\frac{3K}{4nn}-\frac{15F}{8nn}-\frac{9G}{8nn}+\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{K}+\frac{(3\mathfrak{A}+3A)}{nn}K$$
$$+\frac{(\mathfrak{F}+3F)}{nn}(\mathfrak{P}+\mathfrak{R})+\frac{(3\mathfrak{F}+3F)}{nn}(P+R)+\frac{(\mathfrak{G}+3G)}{nn}\mathfrak{Q}+\frac{(3\mathfrak{G}+3G)}{nn}Q$$
$$+\frac{9P}{8nn}+\frac{9R}{8nn}+\frac{15Q}{8nn}=0$$

§. 186. Hic autem obſeruo, hanc determinationem maxime eſſe lubricam, cum coefficiens litterae L, quem poſtremo eſt habitura, admodum fiat paruus; vnde is a terminis, quos omiſimus, non mediocrem mutationem perpeti poſſet. Hanc ob cauſam conſultum iudico, in calculum quoque terminos $3\eta-s$ et $3\eta+s$ introducere, quia praeuideo ab iis coefficientes terminorum, quos quaerimus, non leuiter affici. Sequenti ergo modo calculum redintegro.

§. 187. In hunc finem quoque rationem habeamus angulorum $3\eta-s$ et $3\eta+s$, ſitque

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta-s)$$
$$+ \mathfrak{R}e\cos(2\eta+s)$$
$$+\mathfrak{F}v\cos\eta+\mathfrak{K}ve\cos(\eta-s)+\mathfrak{M}ve\cos(3\eta-s)$$
$$+\mathfrak{G}v\cos\eta+\mathfrak{L}ve\cos(\eta+s)+\mathfrak{N}ve\cos(3\eta+s)$$
$$v=$$

$$\Phi = A\cos 2\eta + Pe\cos s + Qe\cos(2\eta - s)$$
$$+ Re\cos(2\eta + s)$$
$$+ Fv\cos\eta + Kve\cos(\eta - s) + Mve\cos(3\eta - s)$$
$$+ Gv\cos 3\eta + Lve\cos(\eta + s) + Nve\cos(3\eta + s)$$

§. 188. Quodsi iam ponamus:

$$\frac{2\kappa K + \mathfrak{K}}{nn} = k' \;;\quad \frac{2\kappa L + \mathfrak{L}}{nn} = l'$$
$$\frac{2\kappa M + \mathfrak{M}}{nn} = m' \quad\; \frac{2\kappa N + \mathfrak{N}}{nn} = n'$$

erit

$$\frac{d\Phi}{d\tau} = \text{Praec.} - a'\cos 2\eta - p'e\cos s - q'e\cos(2\eta - s)$$
$$- r'e\cos(2\eta + s)$$
$$- f'v\cos\eta - k've\cos(\eta - s) - m've\cos(3\eta - s)$$
$$- g'v\cos 3\eta - l've\cos(\eta + s) - n've\cos(3\eta + s)$$

atque ob $\frac{ds}{d\tau} = \frac{1}{n} - \frac{2}{n}e\cos s$ erit

$$\frac{d\eta}{d\tau} = a - a'\cos 2\eta + \left(\frac{2}{n} - p'\right)e\cos s - q'e\cos(2\eta - s)$$
$$- r'e\cos(2\eta + s)$$
$$- f'v\cos\eta - k've\cos(\eta - s) - m've\cos(3\eta - s)$$
$$- g'v\cos 3\eta - l've\cos(\eta + s) - n've\cos(3\eta + s)$$

§. 189. Formulas nunc assumtas differentiemus, solosque terminos, quibus opus habemus, in calculo exprimamus ac reperiemus:

$$R = \text{Praec.}$$

$$+ ve\sin(\eta - s)\left(+\mathfrak{A}l' - \mathfrak{A}m' + \mathfrak{Q}f' - \mathfrak{R}g' - \tfrac{1}{2}\mathfrak{F}\left(\frac{2}{n} - p'\right) - \tfrac{1}{2}\mathfrak{F}q' + \tfrac{1}{2}\mathfrak{G}r' - \mathfrak{K}\left(\alpha - \frac{1}{n}\right)\right.$$
$$- \tfrac{1}{2}\mathfrak{L}a' + \tfrac{1}{2}\mathfrak{M}a'$$

+

$$+ve\sin(\eta+s)\left(+\mathfrak{A}k'-\mathfrak{A}n'+\mathfrak{R}f'-\mathfrak{Q}g'-\tfrac{1}{2}\mathfrak{F}\left(\tfrac{2}{n}-p'\right)-\tfrac{1}{2}\mathfrak{F}r'+\tfrac{3}{2}\mathfrak{G}q'-\mathfrak{L}\left(\alpha+\tfrac{1}{n}\right)\right.$$
$$\left.-\tfrac{1}{2}\mathfrak{K}a'+\tfrac{3}{2}\mathfrak{N}a'\right)$$
$$+ve\sin(3\eta-s)\left(+\mathfrak{A}k'+\mathfrak{Q}f'+\tfrac{1}{2}\mathfrak{F}q'-\tfrac{3}{2}\mathfrak{G}\left(\tfrac{2}{n}-p'\right)+\tfrac{1}{2}\mathfrak{K}a'-\mathfrak{M}\left(3\alpha-\tfrac{1}{n}\right)\right)$$
$$+ve\sin(3\eta-s)\left(+\mathfrak{A}l'+\mathfrak{R}f'+\tfrac{1}{2}\mathfrak{F}r'-\tfrac{3}{2}\mathfrak{G}\left(\tfrac{2}{n}-p'\right)+\tfrac{1}{2}\mathfrak{L}a'-\mathfrak{N}\left(3\alpha+\tfrac{1}{n}\right)\right)$$

Ac si breuitatis gratia ponamus:

$$K'=\left(\alpha-\tfrac{1}{n}\right)K-A(l'-m')-Qf'+Rg'+\tfrac{1}{2}F\left(\tfrac{2}{n}-p'\right)+\tfrac{1}{2}Fq'-\tfrac{3}{2}Gr'$$
$$+\tfrac{1}{2}La'-\tfrac{3}{2}Ma'$$
$$L'=\left(\alpha+\tfrac{1}{n}\right)L-A(k'-n')-Rf'+Qg'+\tfrac{1}{2}F\left(\tfrac{2}{n}-p'\right)+\tfrac{1}{2}Fr'-\tfrac{3}{2}Gq'$$
$$+\tfrac{1}{2}Ka'-\tfrac{3}{2}Na'$$
$$M'=\left(3\alpha-\tfrac{1}{n}\right)M-Ak'-Qf'-\tfrac{1}{2}Fq'+\tfrac{3}{2}G\left(\tfrac{2}{n}-p'\right)-\tfrac{1}{2}Ka'$$
$$N'=\left(3\alpha+\tfrac{1}{n}\right)N-Al'-Rf'-\tfrac{1}{2}Fr'+\tfrac{3}{2}G\left(\tfrac{2}{n}-p'\right)-\tfrac{1}{2}La'$$

erit

$$\frac{dv}{dr}=-A'\sin 2\eta\quad-P'e\sin s\quad-Q'e\sin(2\eta-s)$$
$$-R'e\sin(2\eta+s)$$
$$-F'v\sin\eta\quad-K've\sin(\eta-s)\quad-M've\sin(3\eta-s)$$
$$-G'v\sin 3\eta\quad-L've\sin(\eta+s)\quad-N've\sin(3\eta+s)$$

§. 190. Hinc iam denuo differentiando nanciscemur

$$\frac{ddv}{dr^2}=\text{Praec.}$$

$$+ve\cos(\eta-s)\left\{\begin{array}{l}A'l'+A'm'+Q'f'+R'g'-\tfrac{1}{2}F'\left(\tfrac{2}{n}-p'\right)+\tfrac{1}{2}F'q'\\ +\tfrac{3}{2}G'r'+\tfrac{1}{2}L'a'+\tfrac{3}{2}M'a'-\left(\alpha-\tfrac{1}{n}\right)K'\end{array}\right.$$

+

$$+ve\cos(\eta+s)\left\{\begin{array}{l} A'k' + A'n' + R'f' + Q'g' - \tfrac{1}{2}F'(\tfrac{2}{n} - p') + \tfrac{1}{2}F'r' \\ \qquad + \tfrac{3}{2}G'q' + \tfrac{1}{2}K'a' + \tfrac{3}{2}N'a' - (\alpha + \tfrac{1}{n})L' \end{array}\right.$$

$$+ve\cos(3\eta-s)\left\{\begin{array}{l} A'k' + Q'f' + \tfrac{1}{2}F'q' - \tfrac{1}{2}G'(\tfrac{2}{n} - p') + \tfrac{1}{2}K'a' \\ \qquad - (3\alpha - \tfrac{1}{n})M' \end{array}\right.$$

$$+ve\cos(3\eta+s)\left\{\begin{array}{l} A'l' + R'f' + \tfrac{1}{2}F'r' - \tfrac{1}{2}G'(\tfrac{2}{n} - p') + \tfrac{1}{2}L'a' \\ \qquad - (3\alpha + \tfrac{1}{n})N' \end{array}\right.$$

§. 191. Quodsi autem valores iam inuenti substituantur, habebitur

$$R = \text{Praec.}$$

$$+ve\sin(\eta-s)\left\{\begin{array}{l} -0{,}85830\,\mathfrak{K} + 0{,}00526\,\mathfrak{L} - 0{,}02500\,\mathfrak{M} + 0{,}37592 \\ \qquad - 0{,}00931\,L + 0{,}00931\,M \end{array}\right]$$

$$+ve\sin(\eta+s)\left\{\begin{array}{l} -1{,}00918\,\mathfrak{L} + 0{,}00526\,\mathfrak{K} - 0{,}02500\,\mathfrak{N} + 0{,}34115 \\ \qquad - 0{,}00931\,K + 0{,}00931\,N \end{array}\right]$$

$$+ve\sin(3\eta-s)\left\{\begin{array}{l} -2{,}72578\,\mathfrak{M} - 0{,}01448\,\mathfrak{K} + 0{,}49223 \\ \qquad - 0{,}00931\,K \end{array}\right]$$

$$+ve\sin(3\eta+s)\left\{\begin{array}{l} -2{,}87666\,\mathfrak{N} - 0{,}01448\,\mathfrak{L} + 0{,}47116 \\ \qquad - 0{,}00931\,L \end{array}\right]$$

$$K' = 0{,}85830K + 0{,}00527\,L + 0{,}01447M + 1{,}48970$$
$$+ 0{,}00750\,\mathfrak{L} - 0{,}00750\,\mathfrak{M}$$
$$L' = 1{,}00916\,L + 0{,}00527\,K + 0{,}01447\,N + 1{,}55183$$
$$+ 0{,}00750\,\mathfrak{K} - 0{,}00750\,\mathfrak{N}$$
$$M' = 2{,}72528M + 0{,}02501\,K - 1{,}17408$$
$$+ 0{,}00750\,\mathfrak{K}$$
$$N' = 2{,}87666\,N + 0{,}02501\,L - 0{,}95901$$
$$+ 0{,}00750\,\mathfrak{L}$$

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ve\,\mathrm{cos}(\eta - s)\left\{\begin{array}{l} -0{,}85830\,K' - 0{,}02843L - 0{,}02843M - 0{,}32674 \\ -0{,}00987\,L' - 0{,}01409\,\mathfrak{L} - 0{,}01409\,\mathfrak{M} \\ -0{,}02961\,M' \end{array}\right]$$

$$+ve\,\mathrm{cos}(\eta + s)\left\{\begin{array}{l} -1{,}00918L' - 0{,}02843K - 0{,}02843N - 0{,}56620 \\ -0{,}00987K' - 0{,}01409\,\mathfrak{K} - 0{,}01409\,\mathfrak{N} \\ -0{,}02981\,N' \end{array}\right]$$

$$+ve\,\mathrm{cos}(3\eta - s)\left\{\begin{array}{l} -2{,}72578M' - 0{,}02843K + 0{,}74683 \\ -0{,}00987\,K' - 0{,}01409\,\mathfrak{K} \end{array}\right]$$

$$+ve\,\mathrm{cos}(3\eta + s)\left\{\begin{array}{l} -2{,}87661\,N' - 0{,}02853L + 0{,}67486 \\ -0{,}00987L' - 0{,}01409\,\mathfrak{L} \end{array}\right]$$

§. 192. Valores autem litterarum commate notatarum hic substitutae dabunt

$$\frac{ddv}{dr^2} =$$

$$ve\,\mathrm{cos}(\eta - s)\left\{\begin{array}{l} -0{,}73747K - 0{,}04264L - 0{,}12156M - 0{,}00014N \\ -0{,}00029\,\mathfrak{K} - 0{,}02053\,\mathfrak{L} - 0{,}00765\,\mathfrak{M} + 0{,}00008\,\mathfrak{N} \\ \qquad -1{,}58590 \end{array}\right]$$

ve

$$ve\cos(\eta+s)\left\{\begin{array}{l}-1{,}61923L-0{,}04222K-0{,}12821N-0{,}00014M\\-0{,}00029\mathfrak{L}-0{,}02166\mathfrak{K}-0{,}00642\mathfrak{N}+0{,}00008\mathfrak{M}\\\qquad\qquad\qquad\qquad-2{,}11858\end{array}\right\}$$

$$ve\cos(3\eta-s)\left\{\begin{array}{l}-7{,}43058M-0{,}09507K-0{,}00005L+3{,}93257\\+0{,}00008\mathfrak{M}-0{,}03453\mathfrak{K}-0{,}00007\mathfrak{L}\end{array}\right\}$$

$$ve\cos(3\eta+s)\left\{\begin{array}{l}-8{,}27529N-0{,}11338L-0{,}00005K+3{,}41830\\+0{,}00008\mathfrak{N}-0{,}03566\mathfrak{L}-0{,}00007\mathfrak{K}\end{array}\right\}$$

§. 193. His expressionibus ita euolutis atque ad calculum numericum praeparatis, quaeramus easdem expressiones ex formulis supra traditis pro R et $\frac{ddv}{dr^2}$, quae continentur in §. 52 et 54. Inde autem omittendis terminis, quos iam tractauimus, consequemur.

$$R = Pr. + ve\sin(\eta-s)\left(-\tfrac{3}{4}+\frac{3L}{2nn}-\frac{3M}{2nn}-\frac{9F}{4nn}+\frac{9G}{4nn}\right)$$

$$+ve\sin(\eta+r)\left(-\tfrac{3}{4}+\frac{3K}{2nn}-\frac{3N}{2nn}+\frac{9G}{4nn}-\frac{9F}{4nn}\right)$$

$$+ve\sin(3\eta-s)\left(-\tfrac{15}{4}+\frac{3K}{2nn}-\frac{9F}{4nn}\right)$$

$$+ve\sin(3\eta+s)\left(-\tfrac{15}{4}+\frac{3L}{2nn}-\frac{9F}{4nn}\right)$$

ddv

$$\frac{ddv}{d\tau^2} = \text{Praec.}$$

$$+ve\cos(\eta-s)\begin{cases} -\frac{9}{4}+\frac{9P}{8nn}+\frac{9Q}{8nn}+\frac{15R}{8nn}-6K+\frac{3L}{4nn}+\frac{3M}{4nn} \\ -\frac{3F}{4nn}-\frac{9F}{8nn}-\frac{9G}{8nn}-2n\mathfrak{K}+\frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{L}+\mathfrak{M}) \\ +\frac{(3\mathfrak{A}+3A)}{nn}(L+M)+\frac{(\mathfrak{P}+3P)}{nn}\mathfrak{P}+\frac{(3\mathfrak{P}+3P)}{nn}F \\ +\frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{P}+\frac{(3\mathfrak{Q}+3Q)}{nn}F+\frac{(\mathfrak{R}+3R)}{nn}\mathfrak{G} \\ +\frac{(3\mathfrak{R}+3R)}{nn}G \end{cases}$$

$$+ve\cos(\eta+s)\begin{cases} -\frac{9}{4}+\frac{9P}{8nn}+\frac{9R}{8nn}+\frac{15Q}{8nn}-6L+\frac{3K}{4nn}+\frac{3N}{4nn} \\ -\frac{3F}{4nn}-\frac{9F}{8nn}-\frac{9G}{8nn}-2n\mathfrak{L}+\frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{K}+\mathfrak{N}) \\ +\frac{(3\mathfrak{A}+3A)}{nn}(K+N)+\frac{(\mathfrak{P}+3P)}{nn}\mathfrak{F}+\frac{(3\mathfrak{P}+3P)}{nn}F \\ +\frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{G}+\frac{(3\mathfrak{Q}+3Q)}{nn}G+\frac{(\mathfrak{R}+3R)}{nn}\mathfrak{F} \\ +\frac{(3\mathfrak{R}+3R)}{nn}F \end{cases}$$

$$+ve\cos(3\eta-s)\begin{cases} -\frac{15}{4}+\frac{15P}{8nn}+\frac{9Q}{8nn}-6M+\frac{3K}{4nn}-\frac{3G}{4nn} \\ -\frac{9F}{8nn}-2n\mathfrak{M}+\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{K}+\frac{(3\mathfrak{A}+3A)}{nn}K+\frac{(\mathfrak{P}+3P)}{nn}\mathfrak{G} \\ +\frac{(3\mathfrak{P}+3P)}{nn}G+\frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{F}+\frac{(3\mathfrak{Q}+3Q)}{nn}F \end{cases}$$

$+ve$

$$+ n\cos(3\eta + s)\left\{\begin{array}{l} -\frac{15}{4} + \frac{15\,P}{8nn} + \frac{9\,R}{8nn} - \mathfrak{E}N + \frac{3\,L}{4nn} - \frac{3\,G}{4nn} - \frac{9\,F}{8nn} - 2n\mathfrak{N} \\ + \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{L} + \frac{(3\mathfrak{A}+3A)}{nn}L + \frac{(\mathfrak{P}+3P)}{nn}\mathfrak{G} + \frac{(3\mathfrak{P}+3P)}{nn}G \\ + \frac{(\mathfrak{R}+3R)}{nn}\mathfrak{F} + \frac{(3\mathfrak{R}+3R)}{nn}F \end{array}\right.$$

§. 194. Introducantur hic quoque valores iam cogniti, ac prodibit

$$\begin{array}{ll} R = Pr. & + ve\sin(\eta - s)[-1{,}02394 + 0{,}00854L - 0{,}00854M] \\ & + ve\sin(\eta + s)[-1{,}02394 + 0{,}00854K - 0{,}00854N] \\ & + ve\sin(3\eta - s)[-4{,}01450 + 0{,}00854K] \\ & + ve\sin(3\eta + s)[-4{,}01450 + 0{,}00854L] \end{array}$$

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ ve\cos(\eta - s)\left\{\begin{array}{l} -1{,}01591K - 0{,}03207L - 0{,}03207M - 1{,}16867 \\ -2{,}01798\mathfrak{K} - 0{,}02711\mathfrak{L} - 0{,}02711\mathfrak{M} \end{array}\right]$$

$$+ ve\cos(\eta + s)\left\{\begin{array}{l} -1{,}01591L - 0{,}03207K - 0{,}03207N - 1{,}93904 \\ -2{,}01798\mathfrak{L} - 0{,}02711\mathfrak{K} - 0{,}02711\mathfrak{N} \end{array}\right]$$

$$+ ve\cos(3\eta - s)\left\{\begin{array}{l} -1{,}01591M - 0{,}03207K - 2{,}49545 \\ -2{,}01798\mathfrak{M} - 0{,}02711\mathfrak{K} \end{array}\right]$$

$$+ ve\cos(3\eta + s)\left\{\begin{array}{l} -1{,}01591N - 0{,}03207L - 2{,}78450 \\ -2{,}01798\mathfrak{N} - 0{,}02711\mathfrak{L} \end{array}\right]$$

§. 195. Hinc ergo octo sequentes aequationes resultabunt

$$\text{I.}\quad 0{,}85830\,\mathfrak{K} = +\,0{,}00526\,\mathfrak{L} - 0{,}02500\,\mathfrak{M} + 1{,}39986 - 0{,}01785\,L + 0{,}01785\,M$$

$$\text{II.}\quad 1{,}00918\,\mathfrak{L} = +\,0{,}00526\,\mathfrak{K} - 0{,}02500\,\mathfrak{N} + 1{,}36509 - 0{,}01785\,K + 0{,}01785\,N$$

$$\text{III.}\quad 2{,}72578\,\mathfrak{M} = -\,0{,}01448\,\mathfrak{K} + 4{,}50673 - 0{,}01785\,K$$

$$\text{IV.}\quad 2{,}87666\,\mathfrak{N} = -\,0{,}01448\,\mathfrak{L} + 4{,}48566 - 0{,}01785\,L$$

$$\text{V.}\quad +0{,}27844K - 0{,}01057L - 0{,}08949M - 0{,}00014N + 0{,}10081 = +2{,}01769\mathfrak{K} + 0{,}00658\mathfrak{L} - 0{,}01946\mathfrak{M} + 0{,}00008\mathfrak{N}$$

$$\text{VI.}\quad -0{,}00332L - 0{,}01015K - 0{,}09614N - 0{,}00014M - 0{,}17954 = +2{,}01769\mathfrak{L} + 0{,}00545\mathfrak{K} + 0{,}02069\mathfrak{N} + 0{,}00008\mathfrak{M}$$

$$\text{VII.}\quad -6{,}41487M - 0{,}06300K - 0{,}00005L + 6{,}42802 = +2{,}01806\mathfrak{M} - 0{,}00742\mathfrak{K} - 0{,}00007\mathfrak{L}$$

$$\text{VIII.}\quad -7{,}25938N - 0{,}08131L - 0{,}00005K + 6{,}20280 = +2{,}01806\mathfrak{N} - 0{,}00855\mathfrak{L} - 0{,}00007\mathfrak{K}$$

§. 196. Ex aequationibus III et IV statim eliciuntur hi valores

$$\mathfrak{M} = -\,0{,}00531\,\mathfrak{K} - 0{,}00655\,K + 1{,}65336$$
$$\mathfrak{N} = -\,0{,}00503\,\mathfrak{L} - 0{,}00621\,L + 1{,}55933$$

qui

qui in I et II substituti praebent:

$$0,85817\mathfrak{K} = 1,35853 + 0,00526\mathfrak{L} - 0,01785(L-M) + 0,00016K$$
$$1,00905\mathfrak{L} = 1,32611 + 0,00526\mathfrak{K} - 0,01785(K-N) + 0,00015L$$

vnde obtinetur:

$$\mathfrak{K} = +1,59116 - 0,02080(L-M) + 0,00008K \pm 0,00010N$$
$$\mathfrak{L} = +1,32251 - 0,01769(K-N) + 0,00005L + 0,00011M$$
$$\mathfrak{M} = +1,64491 - 0,00655K + 0,00010(L-M)$$
$$\mathfrak{N} = +1,55268 - 0,00621L + 0,00009(K-N)$$

§. 197. His valoribus substitutis caeterae aequationes abibunt in formas sequentes:

$$0,27862K - 0,05252L - 0,04754M + 0,00017N + 3,35313 = 0$$
$$-0,00346L - 0,04584K - 0,06045N + 0,00020M + 2,53002 = 0$$
$$-6,41502M - 0,07622K + 0,00030L + 9,73587 = 0$$
$$-7,25971N - 0,09384L + 0,00028K + 9,32499 = 0$$

ex quarum binis postremis statim obtinetur:

$$M = -0,01188K + 0,00005L + 1,51765$$
$$N = -0,01293L + 0,00004K + 1,28448$$

vnde colligitur:

$$+0,27918K - 0,05252L + 3,28120 = 0$$
$$-0,00268L - 0,04584K + 2,45267 = 0$$

ac denique

$K = +40,44710$ . . . . $l\,K = 1,606887$
$L = +368,40200$ . . . . $l\,L = 2,566322$
$M = +1,05555$ . . . . $l\,M = 0,023478$
$N = -3,47730$ . . . . $l{-}N = 0,541242$

§. 198. Litterarum germanicarum valores hinc erunt:

$\mathfrak{K} = -5,04677$ . . . $l\,\mathfrak{K} = 0,703013$
$\mathfrak{L} = +0,56302$ . . . $l\,\mathfrak{L} = 9,750524$
$\mathfrak{M} = +1,41672$ . . . $l\,\mathfrak{M} = 0,151283$
$\mathfrak{N} = -0,73119$ . . . $l\,\mathfrak{N} = 9,864030$

ac litterarum hinc deriuatarum:

$k' = +0,43578$ . . . $l\,k' = 9,639267$
$l' = +4,23401$ . . . $l\,l' = 0,626752$
$m' = +0,02018$ . . . $l\,m' = 8,304921$
$n' = -0,04409$ . . . $l\,n' = 8,644340$

§. 199. Nunc igitur intelligimus inaequalitates ab angulis $3\eta - s$ et $3\eta + s$ pendentes tam esse paruas, vt sine vllo errore reiici queant, etiamsi valores K et L aliquantum immutauerint. Distantia ergo lunae curtata a terra $x = \frac{(1-kk)\,aa}{1-k\cos r}$ ita ab his inaequalitatibus parallacticis pendebit, vt sit

				Log. coeff.
$x =$ Praec.	$+\,0,11756$	v	$\cos\eta$	9,070249
	$-\,0,00419$	v	$\cos 3\eta$	7,622540
	$-\,0,1234$	vk	$\cos(\eta - r)$	9,091265
	$-\,0,0784$	vk	$\cos(\eta + r)$	8,894183
	$+\,0,2302$	ve	$\cos(\eta - s)$	9,362071
	$+\,2,0965$	ve	$\cos(r + s)$	0,321506

Motus

Motus autem momentaneus ita hinc afficietur, vt fit

$\frac{d\Phi}{dr}$ = Praec.	$-\,0{,}23259\ \nu\ \cos\eta$	9,366591
	$+\,0{,}01309\ \nu\ \cos 3\eta$	8,116940
	$+\,0{,}0939\ \nu k\cos(\eta-r)$	8,972943
	$+\,0{,}1653\ \nu k\cos(\eta+r)$	9,218352
	$-\,0{,}4358\ \nu e\cos(\eta-s)$	9,639267
	$-\,4{,}2340\ \nu e\cos(\eta+s)$	0,626752

§. 200. Quodsi iam ipsam longitudinem lunae, quatenus ab his inaequalitatibus parallacticis pendet, ponamus:

$$\Phi = \text{Praec.} + \mathfrak{F}'\nu\sin\eta + \mathfrak{H}'\nu k\sin(\eta-r) + \mathfrak{K}'\nu e\sin(\eta-s)$$
$$+ \mathfrak{G}'\nu\sin 3\eta + \mathfrak{J}'\nu k\sin(\eta+r) + \mathfrak{L}'\nu e\sin(\eta+s)$$

sequentes obtinebimus aequationes pro horum coefficientium determinatione:

$$-0{,}23259 = \alpha\mathfrak{F}' - \mathfrak{A}'f' - \mathfrak{A}'g' - \tfrac{1}{2}\mathfrak{F}'a' - \tfrac{1}{2}\mathfrak{G}'a'$$

$$+0{,}01303 = 3\alpha\mathfrak{G}' - \mathfrak{A}'f' - \tfrac{1}{2}\mathfrak{F}'a'$$

$$+0{,}0939 = (\alpha-1)\mathfrak{H}' - \mathfrak{A}'i' - \tfrac{1}{2}\mathfrak{J}'a' - \mathfrak{D}'f' - \mathfrak{C}'g'$$
$$- \tfrac{1}{2}\mathfrak{F}'c' - \tfrac{1}{2}\mathfrak{F}'d' - \tfrac{1}{2}\mathfrak{G}'c'$$

$$+0{,}1653 = (\alpha+1)\mathfrak{J}' - \mathfrak{A}'h' - \tfrac{1}{2}\mathfrak{H}'a' - \mathfrak{C}'f' - \mathfrak{D}'g'$$
$$- \tfrac{1}{2}\mathfrak{F}'c' - \tfrac{1}{2}\mathfrak{F}'c' - \tfrac{1}{2}\mathfrak{G}'d'$$

$$-0{,}4358 = \left(a-\frac{1}{n}\right)\mathfrak{K}' - \mathfrak{A}'l' - \tfrac{1}{2}\mathfrak{L}'a' - \mathfrak{Q}'f' - \mathfrak{R}'g'$$
$$+ \tfrac{1}{2}\mathfrak{F}'\left(\frac{2}{n}-p'\right) - \tfrac{1}{4}\mathfrak{F}'q' - \tfrac{3}{2}\mathfrak{G}'r'$$

$$-4{,}2340 = \left(a+\frac{1}{n}\right)\mathfrak{L}' - \mathfrak{A}'k' - \tfrac{1}{2}\mathfrak{K}'a' - \mathfrak{R}'f' - \mathfrak{Q}'g'$$
$$+ \tfrac{1}{2}\mathfrak{F}'\left(\frac{2}{n}-p'\right) - \tfrac{1}{2}\mathfrak{F}'r' - \tfrac{3}{2}\mathfrak{G}'q'$$

§. 201. Valoribus autem iam cognitis hic substitutis, aequationes istae in sequentes abibunt formas;

$$-0{,}23031 = +0{,}94361\,\mathfrak{F}' + 0{,}02961\,\mathfrak{G}'$$
$$+0{,}01550 = +2{,}80122\,\mathfrak{G}' + 0{,}00987\,\mathfrak{F}'$$
$$-0{,}0056 = -0{,}06626\,\mathfrak{H}' + 0{,}00987\,\mathfrak{J}' - 0{,}25665\,\mathfrak{F}' + 0{,}01926\,\mathfrak{G}'$$
$$+0{,}1710 = +1{,}93374\,\mathfrak{J}' + 0{,}00987\,\mathfrak{H}' - 0{,}06717\,\mathfrak{F}' - 0{,}54918\,\mathfrak{G}'$$
$$-0{,}3968 = +0{,}85830\,\mathfrak{K}' + 0{,}00987\,\mathfrak{L}' + 0{,}06357\,\mathfrak{F}' - 0{,}04509\,\mathfrak{G}'$$
$$-4{,}2330 = +1{,}00918\,\mathfrak{L}' + 0{,}00987\,\mathfrak{K}' + 0{,}06729\,\mathfrak{F}' - 0{,}05625\,\mathfrak{G}'$$

vnde colligitur fore

$\mathfrak{F}' = -0{,}24427$ . . . $l{-}\mathfrak{F}' = 9{,}387868$

$\mathfrak{G}' = +0{,}00639$ . . . $l\,\mathfrak{G}' = 7{,}805991$

$\mathfrak{H}' = +1{,}1959$ . . . $l\,\mathfrak{H}' = 0{,}077694$

$\mathfrak{J}' = +0{,}0757$ . . . $l\,\mathfrak{J}' = 8{,}879096$

$\mathfrak{K}' = -0{,}3959$ . . . $l{-}\mathfrak{K}' = 9{,}597508$

$\mathfrak{L}' = -4{,}1738$ . . . $l{-}\mathfrak{L}' = 0{,}620530$

§. 201.

§. 202. Hinc ergo habebimus ſequentes partes pro longitudine Lunae, quas ſimul ope valorum proxime cognitorum pro ν, k, e ad minuta ſecunda reducamus:

			Log. coeff.	Val. coeff. in min. ſec.
Φ = Praec.	$-0,24427$	ν ſin η	9,387868	$-175''$
	$+0,00639$	ν ſin 3η	7,805991	$+4''$
	$+1,1959$	νk ſin $(\eta-r)$	0,077694	$+59''$
	$+0,0757$	νk ſin $(\eta+r)$	8,879096	$+4''$
	$-0,3959$	νe ſin $(\eta-s)$	9,597508	$-5''$
	$-4,1738$	νe ſin $(\eta+s)$	0,620530	$-49''$

Sicque omnes iam adepti ſumus motus lunae inaequalitates, quae quidem ab inclinatione eius orbitae ad eclipticam non pendent. Interim tamen non diffiteor, dari aliquas inſuper inaequalitates, quae alicuius forte ſint momenti, quas in hac inueſtigatione praeterimus, cuiusmodi ſunt eae, quae ab angulis $2\eta-3r$ et $2\eta-2r+s$ pendent, quae ad plura minuta ſecunda asſurgere poſſe videntur. Verum earum determinatio tam eſt taedioſa, vt malim eam obſeruationibus relinquere.

§. 203. Quae ergo hactenus inuenimus, in vnum colligamus ac primo pro diſtantia lunae a terra curtata $x = \frac{(1-kk)\,au}{1-k\cos r}$ erit

$x =$

	Log. coeff.	coeff. integri
$u = 1 - 0{,}0074991 \cos 2\eta$	7,875009	$-0{,}007499$
$+ 0{,}0000532 \cos 4\eta$	5,725912	$+0{,}000053$
$+ 0{,}191557 k \cos (2\eta - r)$	9,282297	$+0{,}010430$
$- 0{,}003293 k \cos (2\eta + r)$	7,517525	$-0{,}000179$
$- 0{,}003321 k \cos (4\eta - r)$	7,521296	$-0{,}000181$
$+ 0{,}000049 k \cos (4\eta + r)$	5,692584	$+0{,}000005$
$- 0{,}00511 kk \cos 2r$	7,708601	$-0{,}000015$
$- 0{,}08022 kk \cos (2\eta - 2r)$	8,904280	$-0{,}000238$
$- 0{,}00237 kk \cos (2\eta + 2r)$	7,375072	$-0{,}000007$
$+ 0{,}07892 kk \cos (4\eta - 2r)$	8,897172	$+0{,}000234$
$+ 0{,}00001 kk \cos (4\eta + 2r)$	4,872349	$+0{,}000000$
$- 0{,}006400 e \cos s$	7,806180	$-0{,}000206$
$+ 0{,}014801 e \cos (2\eta - s)$	8,170303	$+0{,}000249$
$+ 0{,}011415 e \cos (2\eta + s)$	8,057492	$+0{,}000192$
$+ 0{,}00364 ee \cos 2s$	7,539670	$+0{,}000001$
$- 0{,}01482 ee \cos (2\eta - 2s)$	8,170799	$-0{,}000004$
$- 0{,}00584 ee \cos (2\eta + 2s)$	7,766856	$-0{,}000001$
$- 0{,}37957 ek \cos (r - s)$	9,579297	$-0{,}000347$
$+ 0{,}30693 ek \cos (r + s)$	9,487039	$+0{,}000281$
$- 0{,}36337 ek \cos (2\eta - r + s)$	9,560344	$-0{,}000332$
$+ 0{,}00993 ek \cos (2\eta + r - s)$	7,997004	$+0{,}000009$
$- 0{,}07752 ek \cos (2\eta - r - s)$	8,889425	$-0{,}000071$
$+ 0{,}00305 ek \cos (2\eta + r + s)$	7,484024	$+0{,}000003$
$+ 0{,}11756 \nu \cos \eta$	9,070249	$+0{,}000408$
$- 0{,}00419 \nu \cos 3\eta$	7,622540	$-0{,}000015$
$- 0{,}1234 \nu k \cos (\eta - r)$	9,091265	$-0{,}000024$
$- 0{,}0784 \nu k \cos (\eta + r)$	8,894183	$-0{,}000015$
$+ 0{,}2302 \nu e \cos (\eta - s)$	9,362071	$+0{,}000013$
$+ 2{,}0965 \nu e \cos (\eta + s)$	0,321506	$+0{,}000122$

Hic ad latus adiunxi valores coefficientium integrorum in numeris abſolutis expreſſos, ponendo $k = 0{,}05445$, $e = 0{,}01680$ et $\nu = \frac{1}{288}$; quos proinde, ſi hi valores aliter per obſeruationes determinentur, facile erit emendare.

§. 204.

§. 204. Pro motu autem lunae momentaneo, ex quo eius motus horarius definiri poterit, habebimus:

	Log. coeff.	Coeff. integri
$\frac{d\Phi}{dr} = 1{,}009176$	0,003967	$+1{,}009176$
$+0{,}019514\cos 2\eta$	8,290355	$+0{,}019514$
$-0{,}0000322\cos 4\eta$	5,507856	$-0{,}000032$
$-0{,}001231k\cos r$	7,090258	$-0{,}000067$
$-0{,}366103k\cos(2\eta-r)$	9,563604	$-0{,}019934$
$+0{,}012832k\cos(2\eta+r)$	8,108292	$+0{,}000699$
$+0{,}002829k\cos(4\eta-r)$	7,451633	$+0{,}000154$
$-0{,}000171k\cos(4\eta+r)$	6,232305	$-0{,}000009$
$+0{,}01182kk\cos 2r$	8,072618	$+0{,}000035$
$-0{,}02057kk\cos(2\eta-2r)$	8,313172	$-0{,}000061$
$+0{,}01063kk\cos(2\eta+2r)$	8,026598	$+0{,}000032$
$-0{,}09883kk\cos(4\eta-2r)$	8,994889	$-0{,}000293$
$-0{,}00004kk\cos(4\eta+2r)$	5,592770	$-0{,}000000$
$+0{,}013760e\cos s$	8,138618	$+0{,}000231$
$-0{,}037487e\cos(2\eta-s)$	8,573878	$-0{,}000630$
$-0{,}030062e\cos(2\eta+s)$	8,478023	$-0{,}000505$
$-0{,}00722ee\cos 2s$	7,858166	$-0{,}000002$
$+0{,}03470ee\cos(2\eta-2s)$	8,540319	$+0{,}000010$
$+0{,}01533ee\cos(2\eta+2s)$	8,185614	$+0{,}000005$
$+0{,}75204ek\cos(r-s)$	9,876241	$+0{,}000688$
$-0{,}62626ek\cos(r+s)$	9,796755	$-0{,}000573$
$+0{,}69420ek\cos(2\eta-r+s)$	9,841484	$+0{,}000635$
$-0{,}02440ek\cos(2\eta+r-s)$	8,387390	$-0{,}000022$
$+0{,}12160ek\cos(2\eta-r-s)$	9,084933	$+0{,}000111$
$-0{,}02000ek\cos(2\eta+r+s)$	9,301030	$-0{,}000018$
$-0{,}23259\nu\cos\eta$	9,366591	$-0{,}000808$
$+0{,}01309\nu\cos 3\eta$	8,116940	$+0{,}000045$
$+0{,}0939\nu k\cos(\eta-r)$	8,972943	$+0{,}000018$
$+0{,}1653\nu k\cos(\eta+r)$	9,218352	$+0{,}000031$
$-0{,}4358\nu e\cos(\eta-s)$	9,639267	$-0{,}000025$
$-4{,}2340\nu e\cos(\eta+s)$	0,626753	$-0{,}000247$

§. 205. Si iam longitudo Lunae per ſolam excentricitatem ſecundum regulas Keplerianas determinata ponatur $=\zeta$, ita vt poſita eius anomalia vera $=r$, futurum ſit $\zeta = C + 1{,}0085272\, r$, erit longitudo vera per hactenus inuentas inaequalitates.

	Log. coeff.	Val. coeff. in min. ſec.
$\phi = \zeta + 0{,}0103597 \sin 2\eta$	8,015347	$+2137''$
$- 0{,}0000382 \sin 4\eta$	5,582063	$-$ 8
$+ 0{,}010146k \sin r$	8,006295	$+$ 114
$- 0{,}420226k \sin (2\eta - r)$	9,623483	$-$ 4720
$+ 0{,}004992k \sin (2\eta + r)$	7,698261	$+$ 56
$+ 0{,}005286k \sin (4\eta - r)$	7,723163	$+$ 59
$- 0{,}000086k \sin (4\eta + r)$	5,935307	$-$ 1
$+ 0{,}00420kk \sin 2r$	7,623250	$+ 2\frac{1}{2}$
$+ 0{,}57328kk \sin (2\eta - 2r)$	9,758367	$+$ 351
$+ 0{,}00318kk \sin (2\eta + 2r)$	7,402427	$+ 1\frac{1}{2}$
$- 0{,}15083kk \sin (4\eta - 2r)$	9,178488	$-$ 92
$- 0{,}00002kk \sin (4\eta + 2r)$	5,301030	$-$ 0
$+ 0{,}201385e \sin s$	9,304026	$+$ 701
$- 0{,}021889e \sin (2\eta - s)$	8,340237	$-$ 76
$- 0{,}016368e \sin (2\eta + s)$	8,214002	$-$ 57
$+ 0{,}06615ee \sin 2s$	8,820508	$+$ 4
$+ 0{,}02332ee \sin (2\eta - 2s)$	8,367825	$+$ 1
$+ 0{,}00840ee \sin (2\eta + 2s)$	7,924429	$+ \frac{1}{2}$
$+ 0{,}74760ek \sin (r - s)$	9,873165	$+$ 141
$- 0{,}61850ek \sin (r + s)$	9,791317	$-$ 118
$+ 0{,}81430ek \sin (2\eta - r + s)$	9,910800	$+$ 154
$- 0{,}01420ek \sin (2\eta + r - s)$	8,150690	$-$ 3
$+ 0{,}23960ek \sin (2\eta - r - s)$	9,379550	$+$ 45
$- 0{,}00610ek \sin (2\eta + r + s)$	7,788910	$-$ 1
$- 0{,}24427\nu \sin \eta$	9,387868	$-$ 175
$+ 0{,}006639\nu \sin 3\eta$	7,805991	$+$ 4
$+ 1{,}1959\nu k \sin (\eta - r)$	0,077694	$+$ 59
$+ 0{,}0757\nu k \sin (\eta + r)$	8,879096	$+$ 4
$- 0{,}3959\nu e \sin (\eta - s)$	9,597508	$-$ 5
$- 4{,}1738\nu e \sin (\eta + s)$	0,620530	$-$ 49

CAPUT XII.

INUESTIGATIO INAEQUALITATUM MOTUM LINEAE NODORUM AFFICIENTIUM.

§. 206.

Antequam reliquas motus Lunae inaequalitates, quae ab inclinatione eius orbitae ad eclipticam pendent, definire licet, cum variationes, quae in motu lineae nodorum Lunae, tum eas, quae in ipſa inclinatione eius orbitae ad eclipticam deprehenduntur, inueſtigari oportet. Reſidua enim pars aequationis noſtrae principalis, qua omnes motus Lunae inaequalitates continentur, litteras π et ϱ implicat, quarum illa longitudinem nodi aſcendentis, haec vero ϱ inclinationem ad eclipticam deſignat. Niſi igitur vtriusque huius quantitatis incrementa vel decrementa ad differentiale dr reduxerimus, reſiduas motus Lunae inaequalitates determinare non poterimus.

207. Aequatio autem ſupra (55) pro motu lineae nodorum tradita, cum ſit $\frac{2uv + \int R\,dr}{nn} = \alpha + \frac{1+2ee}{nn} - \frac{d\Phi}{dr}$ ideoque

$$\frac{2uv + \int R\,dr}{nn} = a'\operatorname{cof} 2\eta - \left(\frac{2}{n} - c'\right)k\operatorname{cof} r + d'k\operatorname{cof}(2\eta - r)$$
$$+ e'k\operatorname{cof}(2\eta + r)$$
$$+ p'e\operatorname{cof} s + q'e\operatorname{cof}(2\eta - s)$$
$$+ r'e\operatorname{cof}(2\eta + s)$$

ſi ponamus breuitatis gratia $\frac{3(1+2kk+\frac{3}{2}ee)}{\kappa\kappa\, nn} = i$, vt ſit $i = 0{,}016918$, induet formam ſequentem:

$$\frac{d\pi}{dr} = -i\left(\kappa + \alpha' \cos 2\eta - \left(\frac{2}{n} - c'\right)k\cos r \begin{matrix} + d'k\cos(2\eta - r) + \text{etc.} \\ + e'k\cos(2\eta + r) \end{matrix}\right)$$

$$\left(\begin{matrix} 1 + 4k\cos r + 5kk\cos 2r - 6ek\cos(r-s) \\ -3e\cos s + \frac{3}{2}ee\cos 2s - 6ek\cos(r+s) \end{matrix}\right)\left(\frac{1}{4} + \frac{1}{4}\cos 2\eta \begin{matrix} -\frac{1}{4}\cos(2\phi - 2\pi) \\ -\frac{1}{4}\cos(2\theta - 2\pi) \end{matrix}\right)$$

$$-\frac{\kappa i v}{4}\left(\frac{1}{4}\cos\eta + \frac{5}{4}\cos 3\eta - \frac{3}{2}\cos(\phi + \theta - 2\pi) \begin{matrix} -\frac{5}{4}\cos(3\phi - \theta - 2\pi) \\ -\frac{5}{4}\cos(3\theta - \phi - 2\pi) \end{matrix}\right)$$

vbi quidem plurimi termini tam fiunt parui, vt facile negligi queant.

§. 208. Productum autem ex duobus prioribus factoribus, quoniam id in formula pro inclinatione recurrit, ſeorſim exhibeamus: fiet id autem reiectis terminis, qui prae reliquis admodum ſunt parui vt ſequitur:

$$\left.\begin{matrix}
-\kappa i + 2i\left(\frac{2}{n} - c'\right)kk - \frac{3}{2}ip'ee \\
\cos 2\eta \quad (-i\alpha' - 2id'kk \\
k\cos r \quad \left(i\left(\frac{2}{n} - c'\right) - 4\kappa i\right. \\
k\cos(2\eta - r) \quad (-id' - 2i\alpha' \\
k\cos(2\eta + r) \quad (-ie' - 2i\alpha' \\
kk\cos 2r \quad \left(-5\kappa i + 2i\left(\frac{2}{n} - c'\right)\right. \\
e\cos s \quad (-ip' + 3\kappa i \\
e\cos(2\eta - s) \quad (iq' + \frac{3}{2}i\alpha' \\
e\cos(2\eta + s) \quad (-ir' + \frac{3}{2}i\alpha' \\
ee\cos 2s \quad (-\frac{3}{2}\kappa i + \frac{3}{2}ip'
\end{matrix}\right\} = \begin{matrix}
-0{,}017043 \\
+0{,}000161\ \cos 2\eta \\
-0{,}068110\ k\cos r \\
-0{,}005791\ k\cos(2\eta - r) \\
+0{,}000828\ k\cos(2\eta + r) \\
-0{,}085091\ kk\cos 2r \\
+0{,}051362\ e\cos s \\
-0{,}000929\ e\cos(2\eta - s) \\
-0{,}000804\ e\cos(2\eta + s) \\
-0{,}025914\ ee\cos 2s
\end{matrix}$$

§. 209.

§. 209. His valoribus substitutis prodibit

$\frac{d\pi}{dr} = = -\!\!-\quad 0{,}004261 + 0{,}000020$

$\cos 2\eta \quad (+0{,}000040 - 0{,}004261$

$k \cos r \quad (-0{,}017043 - 0{,}000620$

$k \cos(2\eta - r)(-0{,}001448 - 0{,}008514) - 0{,}010636\,kk\,\mathrm{cs}(2\eta - 2r)$

$k \cos(2\eta + r)(+0{,}000207 - 0{,}008514) - 0{,}010636\,kk\,\mathrm{cs}(2\eta + 2r)$

$kk \cos 2r \quad (-0{,}021273$

$e \cos s \quad (+0{,}012840 - 0{,}000217$

$e \cos(2\eta - s)(-0{,}000232 + 0{,}006420) - 0{,}003239\,ee\,\mathrm{cs}(2\eta - 2s)$

$e \cos(2\eta + s)(-0{,}000201 + 0{,}006420) - 0{,}003239\,ee\,\mathrm{cs}(2\eta + 2s)$

$ee \cos 2s \quad (-0{,}006479)$

$\cos 2(\phi - \pi)\ (+0{,}003261 - 0{,}000020) - 0{,}000041 \cos \eta$

$+ 0{,}000019 \cos(3\phi - \theta - 2\pi)$

$\cos 2(\theta - \pi)\ (+0{,}004261 - 0{,}000020) - 0{,}000019 \cos 3\eta$

$+ 0{,}000019 \cos(3\theta - \phi - 2\pi)$

$k\,\mathrm{cs}(2\phi - 2\pi - r)(+0{,}008514 + 0{,}000724)$

$+ 0{,}000022 \cos(\phi + \theta - 2\pi)$

$k\,\mathrm{cs}(2\phi - 2\pi + r)(+0{,}008514 - 0{,}000103)$

$+ 0{,}010636\,kk \cos 2(\phi - r - \pi)$

$k\,\mathrm{cs}(2\theta - 2\pi - r)(+0{,}008514 - 0{,}000103)$

$k\,\mathrm{cs}(2\theta - 2\pi + r)(+0{,}008514 + 0{,}000724)$

$e\,\mathrm{cs}(2\phi - 2\pi - s)(-0{,}006420 + 0{,}000116)$

$e\,\mathrm{cs}(2\phi - 2\pi + s)(-0{,}006420 + 0{,}000100)$

$e\,\mathrm{cs}(2\theta - 2\pi - s)(-0{,}006420 + 0{,}000100)$

$+ 0{,}003239\,ee \cos 2(\theta - s - \pi)$

$e\,\mathrm{cs}(2 - \theta 2\pi + s)(-0{,}006420 + 0{,}000116)$

 §. 110.

§. 210. Habebimus ergo

$$\begin{array}{ll}
\frac{dx}{dr} = -0,004241 & -0,000041 \cos \eta \\
-0,004221 \cos 2\eta & -0,000019 \cos 3\eta \\
-0,017663\, k \cos r & +0,000020 \cos 4\eta \\
-0,009962\, k \cos(2\eta - r) & +0,004241 \cos(2\varphi - 2\pi) \\
-0,008307\, k \cos(2\eta + r) & +0,004241 \cos(2\theta - 2\pi) \\
-0,010636\, kk \cos(2\eta - 2r) & +0,000022 \cos(\varphi + \theta - 2\pi) \\
-0,010636\, kk \cos(2\eta + 2r) & +0,000019 \cos(3\varphi - \theta - 2\pi) \\
-0,021273\, kk \cos r & +0,000019 \cos(3\theta - \varphi - 2\pi) \\
+0,012623\, e \cos s & \\
+0,006188\, e \cos(2\eta - s) & \\
+0,006219\, e \cos(2\eta + s) & \\
-0,006479\, ee \cos 2s & \\
-0,003239\, ee \cos(2\eta - 2s) & \\
-0,003239\, ee \cos(2\eta + 2s) & \\
+0,009238\, k \cos(2\varphi - 2\pi - r) & -0,006304\, e \cos(2\varphi - 2\pi - s) \\
+0,008411\, k \cos(2\varphi - 2\pi + r) & -0,006320\, e \cos(2\varphi - 2\pi + s) \\
+0,008411\, k \cos(2\theta - 2\pi - r) & -0,006320\, e \cos(2\theta - 2\pi - s) \\
+0,009238\, k \cos(2\theta - 2\pi + r) & -0,006304\, e \cos(2\theta - 2\pi + s) \\
+0,010636\, kk \cos(2\varphi - 2\pi - 2r) & +0,003239\, ee \cos(2\theta - 2\pi - 2s)
\end{array}$$

§. 211. Quanquam plurimi horum terminorum tam ſunt parui, vt in ſe ſpectati tuto reiici poſſent; tamen quidam per integrationem ad magnitudinem ſatis notabilem excreſcere poſſunt. Huius autem indolis ſunt illi termini, qui eiusmodi complectuntur angulos, quorum

rum differentialia ad dr admodum paruam tenent rationem, cuiusmodi sunt anguli s, $2s$, $2\theta-2\pi$, $2\theta-2\pi-s$, $2\theta-2\pi+s$, $2\Phi-2\pi-2s$ et $2\theta-2\pi-2s$; quorum natura differentialium ex sequentibus formulis colligi potest:

$$\frac{d\eta}{dr} = \alpha - \alpha' \cos 2\eta - c'k \cos r - d'k \cos(2\eta-r) - e'k \cos(2\eta+r)$$

$$+ \left(\frac{2}{n}-p'\right) e \cos s - q' e \cos(2\eta-s) - r' e \cos(2\eta+s)$$

$$\frac{d\Phi}{dr} = \alpha + \frac{1+2ee}{n} - \alpha' \cos 2\eta + \left(\frac{2}{n}-c'\right) k \cos r$$

$$- d'k \cos(2\eta-r) - p'e \cos s - q'e \cos(2\eta-s)$$

$$- e'k \cos(2\eta+r) \qquad - r'e \cos(2\eta+s)$$

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} + \frac{2}{n} k \cos r - \frac{2}{n} e \cos s$$

§. 212. Quaeramus primo inaequalitates motus nodorum, quae neque ab excentricitate orbitae lunaris neque solaris pendent, sitque:

$$\pi = \text{Const.} - \mathfrak{O}r + \mathfrak{A} \sin 2\eta + \mathfrak{B} \sin(2\Phi-2\pi) + \mathfrak{C} \sin(2\theta-2\pi)$$

reiectis reliquis terminis, quos praeuidemus fore minimos, ac differentiando obtinebimus:

$$\frac{d\pi}{dr} = -\mathfrak{O} - \mathfrak{A}\alpha' - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C}$$

$$+ \cos 2\eta \; (2\alpha\mathfrak{A} - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C}$$

$$+ \cos(2\Phi-2\pi) \; (2(\alpha+\tfrac{1}{n})\mathfrak{B} + 0{,}008482\,\mathfrak{B} + 0{,}004221\,\mathfrak{C}$$

$$+ \cos(2\theta-2\pi) \; (-\mathfrak{B}\alpha' + 0{,}004221\,\mathfrak{B} + \frac{2\mathfrak{C}}{n} + 0{,}008482\,\mathfrak{C}$$

vnde

vnde oritur:

$$0 - 0{,}019744\mathfrak{A} + 0{,}004241\mathfrak{B} + 0{,}004241\mathfrak{C} = 0{,}004241$$
$$1{,}867476\mathfrak{A} - 0{,}004241\mathfrak{B} - 0{,}004241\mathfrak{C} = -0{,}004221$$
$$2{,}026834\mathfrak{B} + 0{,}004221\mathfrak{C} = 0{,}004241$$
$$0{,}023965\mathfrak{B} + 0{,}159358\mathfrak{C} = 0{,}004241$$

§. 213. Valores hinc igitur prodibunt sequentes:

$$\mathfrak{O} = +0{,}004078 \quad \ldots \quad l\,\mathfrak{O} = 7{,}610447$$
$$\mathfrak{A} = -0{,}002196 \quad \ldots \quad l{-}\mathfrak{A} = 7{,}341634$$
$$\mathfrak{B} = +0{,}002037 \quad \ldots \quad l\,\mathfrak{B} = 7{,}308991$$
$$\mathfrak{C} = +0{,}026307 \quad \ldots \quad l\,\mathfrak{C} = 8{,}420081$$

Vbi primum obseruo valorem ipsius $\mathfrak{O}$ iam proxime accedere ad motum medium lunae nodorum, vti per obseruationes constat; inde enim esse deberet $\mathfrak{O} = 0{,}004053$ facile autem intelligitur, hunc exiguum defectum per reliquas inaequalitates suppleri posse. Quocirca hinc erit

	Valores in min. sec.
$\pi = \text{Const.} - 0{,}004078\,r$	
$-0{,}002196 \sin 2\eta$	$-453''$
$+0{,}002037 \sin(2\phi - 2\pi)$	$+420$
$+0{,}026307 \sin(2\theta - 2\pi)$	$+5426$

quae inaequalitates mirifice conueniunt cum obseruationibus. His addi potest terminus:

$$+0{,}000336 \sin(4\theta - 4\pi)$$

cuius in minutis secundis valor est $+69''$, qui terminus cum postremo illo facile coniungi potest.

§. 214. Quaeremus iam seorsim inaequalitates, quae ab excentricitate orbitae lunaris pendent, sitque

$\pi =$

$$\pi = \text{Const.} - Or + \mathfrak{A}\sin 2\eta + \mathfrak{B}\sin(2\varphi-2\pi) + \mathfrak{C}\sin(2\theta-2\pi)$$
$$+ \mathfrak{D}k\sin(2\eta-r) + \mathfrak{E}k\sin(2\eta+r) + \mathfrak{F}k\sin r$$
$$+ \mathfrak{G}kk\sin(2\eta-2r) + \mathfrak{H}kk\sin 2r + \mathfrak{I}k\sin(2\varphi-2\pi-r)$$
$$+ \mathfrak{K}k\sin(2\varphi-2\pi+r) + \mathfrak{L}k\sin(2\theta-2\pi-r) + \mathfrak{M}k\sin(2\theta-2\pi+r)$$
$$+ \mathfrak{N}kk\sin(2\varphi-2\pi-2r)$$

eritque differentiando

$$\frac{d\pi}{dr} = \text{Pr.} + k\cos(2\eta-r)(-\mathfrak{A}c' - 0{,}009238\,\mathfrak{B} - 0{,}009238\,\mathfrak{C} + (2a-1)\mathfrak{D}$$
$$+ k\cos(2\eta+r)(-\mathfrak{A}c' - 0{,}008411\,\mathfrak{B} - 0{,}008411\,\mathfrak{C} + (2a+1)\mathfrak{E}$$
$$+ k\cos r(-\mathfrak{A}d'' - \mathfrak{A}e' - 0{,}017649\,\mathfrak{B} - 0{,}017649\,\mathfrak{C} + \mathfrak{F} - \mathfrak{D}e' - \mathfrak{E}e'$$
$$+ kk\cos 2r \quad (-0{,}010636\,\mathfrak{B} + 2\mathfrak{H} - \mathfrak{D}e' - \mathfrak{E}d'' - \mathfrak{G}e'$$
$$+ kk\cos(2\eta-2r)(-0{,}010636\,\mathfrak{C} + 2(a-1)\mathfrak{G} - \mathfrak{D}e'$$

$$+ k\cos(2\theta-2\pi-r)\left\{\begin{array}{r} -\mathfrak{B}e' + 0{,}008307\,\mathfrak{B} + \frac{2}{n}\mathfrak{C} + 0{,}017663\,\mathfrak{C} \\ + (\frac{2}{n}-1)\,\mathfrak{L} + 0{,}008482\,\mathfrak{L} \end{array}\right.$$

$$+ k\cos(2\theta-2\pi+r)\left\{\begin{array}{r} -\mathfrak{B}d'' + 0{,}009962\,\mathfrak{B} + \frac{2}{n}\mathfrak{C} + 0{,}017663\,\mathfrak{C} \\ + (\frac{2}{n}+1)\,\mathfrak{M} + 0{,}008482\,\mathfrak{M} \end{array}\right.$$

$$+ k\cos(2\varphi-2\pi-r)\left\{\begin{array}{r} + 0{,}017663\,\mathfrak{B} + 0{,}009962\,\mathfrak{C} + 2(a+\frac{1}{n})\mathfrak{I} \\ - \mathfrak{I} + 0{,}008482\,\mathfrak{I} \end{array}\right.$$

$$+ k\cos(2\varphi-2\pi+r)\left\{\begin{array}{r} + 0{,}017663\,\mathfrak{B} + 0{,}008307\,\mathfrak{C} + 2(a+\frac{1}{n})\mathfrak{K} \\ + \mathfrak{K} + 0{,}008482\,\mathfrak{K} \end{array}\right.$$

$$+ kk\cos(2\varphi-2\pi-2r)\left\{\begin{array}{r} + 0{,}021273\,\mathfrak{B} + 0{,}010636\,\mathfrak{C} + 2(a+\frac{1}{n})\mathfrak{N} \\ - 2\mathfrak{N} + 0{,}008482\,\mathfrak{N} \end{array}\right.$$

Aa §. 215.

§. 215. Superfluum foret maiorem curam in his differentialibus adhibere, quia vero proxime tantum rem determinare sufficit; erit ergo:

$$0,867476\,\mathfrak{D} = -\,0,009962$$
$$2,867476\,\mathfrak{E} = -\,0,008307$$

Hincque	et in min sec.
$\mathfrak{D} = -\,0,011480$	$\mathfrak{D}k = -\,129''$
$\mathfrak{E} = -\,0,002900$	$\mathfrak{E}k = -\,33''$

quae inaequalitates in loco nodi vix alicuius ſunt momenti, vnde eas exactius determinare non eſt opus.

§. 216. Calculo autem euoluto erit

$\pi = \text{Pr.}$	$-\,0,011480k \sin(2\eta - r)$	8,059940	$-129''$
	$-\,0,002900k \sin(2\eta + r)$	7,462400	$-\,33$
	$-\,0,017663k \sin r$	8,247064	-198
	$+\,0,090497kk \sin(2\eta - 2r)$	8,956634	$+\,55$
	$-\,0,011978kk \sin 2r$	8,078384	$-\,7$
	$+\,0,008707k \sin(2\phi - 2\pi - r)$	7,939851	$+\,98$
	$+\,0,002701k \sin(2\phi - 2\pi + r)$	7,431516	$+\,30$
	$-\,0,004680k \sin(2\theta - 2\pi - r)$	7,670224	$-\,53$
	$+\,0,004685k \sin(2\theta - 2\pi + r)$	7,670680	$+\,53$
	$+\,0,384848kk \sin(2\phi - 2\pi - 2r)$	9,585289	$+235$

§. 217. Simili modo inueſtigemus inaequalitates motus nodorum, quae pendent ab excentricitate orbitae ſolaris ſitque:

$\pi =$

$$\pi = \text{Const.} - Or + \mathfrak{A}\sin 2\eta + \mathfrak{B}\sin(2\phi - 2\pi) + \mathfrak{C}\sin(2\theta - 2\pi)$$
$$+ \mathfrak{D}e\sin s + \mathfrak{E}e\sin(2\eta - s) + \mathfrak{F}e\sin(2\eta + s)$$
$$+ \mathfrak{G}ee\sin 2s + \mathfrak{H}e\sin(2\phi - 2\pi - s) + \mathfrak{K}e\sin(2\theta - 2\pi - s)$$
$$+ \mathfrak{J}e\sin(2\phi - 2\pi + s) + \mathfrak{L}e\sin(2\theta - 2\pi + s)$$
$$+ \mathfrak{M}ee\sin(2\theta - 2\pi - 2s)$$

vnde differentiando pro terminis quaesitis erit:

$$\frac{d\pi}{dr} = \text{Praec.}$$

$$+ e\cos s \left\{ \begin{array}{l} -\mathfrak{A}q' - \mathfrak{A}r' + 0{,}006304\mathfrak{B} + 0{,}006320\mathfrak{C} + \frac{1}{n}\mathfrak{D} - \mathfrak{E}s' - \mathfrak{F}s' \\ + 0{,}006320\mathfrak{B} + 0{,}006304\mathfrak{C} \end{array} \right.$$

$$+ e\cos(2\eta - s) \left\{ \mathfrak{A}\left(\frac{2}{n} - p'\right) + 0{,}006304\mathfrak{B} + 0{,}006304\mathfrak{C} + \left(2\alpha - \frac{1}{n}\right)\mathfrak{E} \right.$$

$$+ e\cos(2\eta + s) \left\{ \mathfrak{A}\left(\frac{2}{n} - p'\right) + 0{,}006320\mathfrak{B} + 0{,}006320\mathfrak{C} + \left(2\alpha + \frac{1}{n}\right)\mathfrak{F} \right.$$

$$+ ee\cos 2s \left\{ -0{,}003239\,\mathfrak{C} - \frac{1}{n}\mathfrak{D} - \mathfrak{E}r' - \mathfrak{F}q' + \frac{2}{n}\mathfrak{G} \right.$$

$$+ e\cos(2\phi - 2\pi - s) \left\{ \begin{array}{l} -\mathfrak{B}p' - 0{,}012623\,\mathfrak{B} - 0{,}006188\,\mathfrak{C} \\ + \left(2\alpha \pm \frac{1}{n} + 0{,}008482\right)\mathfrak{H} \end{array} \right.$$

$$+ e\cos(2\phi - 2\pi + s) \left\{ \begin{array}{l} -\mathfrak{B}p' - 0{,}012623\,\mathfrak{B} - 0{,}006219\,\mathfrak{C} \\ + \left(2\alpha + \frac{3}{n} + 0{,}008482\right)\mathfrak{J} \end{array} \right.$$

$$+ e\cos(2\theta - 2\pi - s)\left\{\begin{array}{l} -\mathfrak{B}r' - 0,006219\mathfrak{B} - \frac{2}{n}\mathfrak{E} - 0,012623\mathfrak{E} \\ +\left(\frac{1}{n} + 0,008482\right)\mathfrak{K} \end{array}\right.$$

$$+ e\cos(2\theta - 2\pi + s)\left\{\begin{array}{l} -\mathfrak{B}q' - 0,006188\mathfrak{B} - \frac{2}{n}\mathfrak{E} - 0,012623\mathfrak{E} \\ +\left(\frac{3}{n} + 0,008482\right)\mathfrak{L} \end{array}\right.$$

$$+ ee\cos(2\theta - 2\pi - 2s)\left\{\begin{array}{l} +0,003239\mathfrak{B} + \frac{1}{2n}\mathfrak{E} + 0,006479\mathfrak{E} \\ +0,008482\mathfrak{M} - \frac{1}{n}\mathfrak{K} - 0,012623\mathfrak{K} \end{array}\right.$$

§. 218. Hinc reperiuntur sequentes valores

$\mathfrak{D} = 0,159070$.	$l\,\mathfrak{D} = 9,201585$;	$\mathfrak{D}e = 551''$
$\mathfrak{E} = 0,003562$.	$l\,\mathfrak{E} = 7,551680$;	$\mathfrak{E}e = 12\frac{1}{2}$
$\mathfrak{F} = 0,003301$.	$l\,\mathfrak{F} = 7,518677$;	$\mathfrak{F}e = 11\frac{1}{4}$
$\mathfrak{G} = 0,031650$.	$l\,\mathfrak{G} = 8,587191$;	$\mathfrak{G}ee = 2''$
$\mathfrak{H} = -0,003153$.	$l-\mathfrak{H} = 7,498692$;	$\mathfrak{H}e = -11''$
$\mathfrak{I} = -0,002932$.	$l-\mathfrak{I} = 7,467118$;	$\mathfrak{I}e = -10''$
$\mathfrak{K} = -0,025750$.	$l-\mathfrak{K} = 8,410784$;	$\mathfrak{K}e = -90$
$\mathfrak{L} = -0,009076$.	$l-\mathfrak{L} = 7,957885$;	$\mathfrak{L}e = -32$

At valor ipsius $\mathfrak{M}$ tam fit paruus, vt merito pro nihilo haberi possit.

§. 219.

§. 219. Colligamus ergo has inaequalitates in vnam summam, atque obtinebimus longitudinem veram nodi ascendentis

	Valor. in minut. sec.
$\pi =$ Const. $- 0{,}004053\ r$	
$- 0{,}002196 \sin 2\eta$	$- 453''$
$+ 0{,}002037 \sin(2\phi - 2\pi)$	$+ 420$
$+ 0{,}026307 \sin(2\theta - 2\pi)$	$+ 5426$
$+ 0{,}000370 \sin(4\theta - 4\pi)$	$+ 75$
$- 0{,}01766k \sin r$	$- 198$
$- 0{,}01148k \sin(2\eta - r)$	$- 129$
$- 0{,}00290k \sin(2\eta + r)$	$- 33$
$+ 0{,}0905kk \sin(2\eta - 2r)$	$+ 55$
$- 0{,}0120kk \sin 2r$	$- 7$
$+ 0{,}00871k \sin(2\phi - 2\pi - r)$	$+ 98$
$+ 0{,}00270k \sin(2\phi - 2\pi + r)$	$+ 30$
$- 0{,}00468k \sin(2\theta - 2\pi - r)$	$- 53$
$+ 0{,}00468k \sin(2\theta - 2\pi + r)$	$+ 53$
$+ 0{,}3848kk \sin(2\phi - 2\pi - 2r)$	$+ 235$
$+ 0{,}15907e \sin s$	$+ 551$
$- 0{,}02575e \sin(2\theta - 2\pi - s)$	$- 90$
$- 0{,}00907e \sin(2\theta - 2\pi + s)$	$- 32$

omissis scilicet iis inaequalitatibus, quae non supra 30'' exsurgunt.

CAPUT XIII.

INUESTIGATIO INCLINATIONIS ORBITAE LUNARIS AD ECLIPTICAM.

§. 220.

Pro inclinatione orbitae lunaris ad eclipticam inueniendа, forma §. 208. euoluta multiplicari debet per $-\frac{1}{4}\sin 2\eta + \frac{1}{4}\sin 2(\Phi-\pi) + \frac{1}{4}\sin 2(\theta-\pi)$, ac productum erit $= \frac{d.\,l\,\mathrm{tang}\,\varrho}{dr}$: Hinc ergo habebitur:

$$\begin{array}{llll}
\frac{d.\,l\,\mathrm{tang}\,\varrho}{dr} = & +0{,}004261 & \sin 2\eta & -0{,}000020\sin 4\eta \\
& +0{,}008514k & \sin(2\eta - r) & \\
& +0{,}008514k & \sin(2\eta + r) & \\
& +0{,}000827k & \sin r & \\
& -0{,}004261 & \sin(2\Phi - 2\pi) & \text{adiice} \\
& -0{,}004261 & \sin(2\theta - 2\pi) & \text{ad coeff.} \\
& -0{,}008514k & \sin(2\Phi - 2\pi - r) & -0{,}000724 \\
& -0{,}008514k & \sin(2\Phi - 2\pi + r) & +0{,}000103 \\
& -0{,}008514k & \sin(2\theta - 2\pi - r) & +0{,}000103 \\
& -0{,}008514k & \sin(2\theta - 2\pi + r) & -0{,}000724 \\
& -0{,}010636kk & \sin(2\Phi - 2\pi - 2r) & \\
& +0{,}006420e & \sin(2\theta - 2\pi - s) & -0{,}000100 \\
& +0{,}006420e & \sin(2\theta - 2\pi + s) & -0{,}000116
\end{array}$$

§. 221.

§. 221. Quaeramus primo terminos, qui a neutra excentricitate pendent, sitque

$$l\,\frac{\text{tang}\,\varrho}{\text{tang}\,\varepsilon} =$$

$$\mathfrak{A}\cos 2\eta + \mathfrak{a}\cos 4\eta + \mathfrak{B}\cos(2\Phi - 2\pi) + \mathfrak{C}\cos(2\theta - 2\pi) + \mathfrak{c}\cos(4\theta - 4\pi)$$

eritque differentiando:

$$\frac{d\,l\,\text{tang}\,\varrho}{d\,r} = \sin 2 \;\left[-2\alpha\mathfrak{A} + 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C} \right.$$

$$\sin 4\eta \left[-4\alpha\mathfrak{a} + \mathfrak{A}\alpha' \right.$$

$$\sin(2\Phi - 2\pi)\left\{ -2\left(\alpha + \frac{1}{n}\right)\mathfrak{B} - 0{,}008482\,\mathfrak{B} - 0{,}004227\,\mathfrak{C} \right.$$

$$\sin(2\theta - 2\pi)\left\{ -\frac{2}{n}\mathfrak{C} + \mathfrak{B}\alpha' - 0{,}004221\,\mathfrak{B} - 0{,}008482\,\mathfrak{C} \right.$$

$$\sin(4\theta - 4\pi)\left\{ +\; 0{,}004241\,\mathfrak{C} - \frac{4}{n}\;\mathfrak{c} \right.$$

§. 222. Ex his iam reperitur:

$$\mathfrak{A} = -\,0{,}002630 \quad \ldots \quad l - \mathfrak{A} = 7{,}419914$$
$$\mathfrak{B} = +\,0{,}002037 \quad \ldots \quad l\,\mathfrak{B} = 7{,}308991$$
$$\mathfrak{C} = +\,0{,}026307 \quad \ldots \quad l\,\mathfrak{C} = 8{,}420081$$
$$\mathfrak{a} = +\,0{,}000019 \quad \ldots \quad l\,\mathfrak{a} = 5{,}278753$$
$$\mathfrak{c} = +\,0{,}000370 \quad \ldots \quad l\,\mathfrak{c} = 6{,}567931$$

ita vt hinc fit:

$$l\,\frac{\text{tang}\,\varrho}{\text{tang}\,\varepsilon} = -\,0{,}002630\,\cos 2\eta$$
$$+\,0{,}000019\,\cos 4\eta$$
$$+\,0{,}002037\,\cos(2\Phi - 2\pi)$$
$$+\,0{,}026307\,\cos(2\theta - 2\pi)$$
$$+\,0{,}000370\,\cos(4\theta - 4\pi)$$

§. 223.

§. 223. Quaeramus iam seorsim terminos ab excentricitate Lunae pendentes, sitque

$$l\frac{\tang\rho}{\tang\iota} = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos(2\phi-2\pi) + \mathfrak{C}\cos(2\theta-2\pi)$$
$$+\mathfrak{D}k\cos(2\eta-r) + \mathfrak{E}k\cos(2\eta+r) + \mathfrak{F}k\cos r$$
$$+\mathfrak{G}k\cos(2\phi-2\pi-r) + \mathfrak{I}k\cos(2\theta-2\pi-r)$$
$$+\mathfrak{H}k\cos(2\phi-2\pi+r) + \mathfrak{K}k\cos(2\theta-2\pi+r)$$
$$+\mathfrak{L}k^2\cos(2\phi-2\pi-2r)$$

vnde differentialibus sumendis habebitur: $\frac{d.\,l\tang\rho}{dr} =$

$$k\sin(2\eta-r)\begin{cases} +\mathfrak{A}c' + 0{,}09238\,\mathfrak{B} - 0{,}009238\,\mathfrak{C} + 0{,}004241\,\mathfrak{G} \\ \quad + (2a-1)\mathfrak{D} - 0{,}004241\,\mathfrak{K} \end{cases}$$

$$k\sin(2\eta+r)\begin{cases} +\mathfrak{A}c' + 0{,}008411\,\mathfrak{B} - 0{,}008411\,\mathfrak{C} + (2a+1)\mathfrak{E} \\ \quad + 0{,}004241\,\mathfrak{H} - 0{,}004241\,\mathfrak{I} \end{cases}$$

$$k\sin r\begin{cases} +\mathfrak{A}d' - \mathfrak{A}c' + 0{,}000827\,\mathfrak{B} - 0{,}000827\,\mathfrak{C} - \mathfrak{F} - 0{,}004241\,\mathfrak{G} \\ + 0{,}004241\,\mathfrak{H} - \mathfrak{D}c' + \mathfrak{E}c' - 0{,}004241\,\mathfrak{I} + 0{,}004241\,\mathfrak{K} \end{cases}$$

$$k\sin(2\phi-2\pi-r)\begin{cases} -0{,}017663\,\mathfrak{B} - 0{,}009962\,\mathfrak{C} - 2\left(a+\frac{1}{n}\right)\mathfrak{G} + \mathfrak{G} \\ \quad - 0{,}008482\,\mathfrak{G} - 0{,}004221\,\mathfrak{I} \end{cases}$$

$$k\sin(2\phi-2\pi+r)\begin{cases} -0{,}017663\,\mathfrak{B} - 0{,}008307\,\mathfrak{C} - 2\left(a+\frac{1}{n}\right)\mathfrak{H} - \mathfrak{H} \\ \quad - 0{,}008482\,\mathfrak{H} - 0{,}004221\,\mathfrak{K} \end{cases}$$

$$k\sin(2\theta-2\pi-r)\begin{cases} +\mathfrak{B}c' - 0{,}008307\,\mathfrak{B} - \frac{2}{n}\mathfrak{C} - 0{,}017663\,\mathfrak{C} \\ +\mathfrak{G}c' - 0{,}004221\,\mathfrak{G} - \frac{2}{n}\mathfrak{I} + \mathfrak{I} - 0{,}008482\,\mathfrak{I} \end{cases}$$

k sin

$$k\sin(2\theta-2\pi+r)\left\{\begin{array}{l}+\mathfrak{B}d'-0,009962\,\mathfrak{B}-\frac{2}{n}\mathfrak{C}-0,017663\,\mathfrak{C}\\ +\mathfrak{H}d'-0,004221\,\mathfrak{H}-\frac{2}{n}\mathfrak{K}-\mathfrak{K}-0,008482\,\mathfrak{K}\end{array}\right.$$

$$kk\sin(2\Phi-2\pi-2r)\left\{\begin{array}{l}-0,021273\,\mathfrak{B}-0,010636\,\mathfrak{C}-2(\alpha+\frac{1}{n})\,\mathfrak{L}+2\,\mathfrak{L}\\ \qquad -0,008482\,\mathfrak{L}\end{array}\right.$$

hincque reperitur:

$\mathfrak{D}=$	$0,010487$	...	$l\,\mathfrak{D}=8,020638$
$\mathfrak{E}=$	$0,003166$	...	$l\,\mathfrak{E}=7,500439$
$\mathfrak{F}=$	$-0,001600$	...	$l-\mathfrak{F}=7,204120$
$\mathfrak{G}=$	$+0,008719$	...	$l\,\mathfrak{G}=7,940484$
$\mathfrak{H}=$	$+0,002699$	...	$l\,\mathfrak{H}=7,431136$
$\mathfrak{J}=$	$-0,004460$	...	$l-\mathfrak{J}=7,649305$
$\mathfrak{K}=$	$+0,004717$	...	$l\,\mathfrak{K}=7,623628$
$\mathfrak{L}=$	$+0,384890$	...	$l\,\mathfrak{L}=9,585335$

§. 224. Nunc denique pro inaequalitatibus ab excentricitate orbitae solaris pendentibus ponatur.

$$l\,\frac{\text{tang}\,\varrho}{\text{tang}\,\varepsilon}=\mathfrak{A}\cos 2\eta+\mathfrak{B}\cos(2\Phi-2\pi)+\mathfrak{M}e\cos(2\theta-2\pi-s)$$
$$+\mathfrak{C}\cos(2\theta-2\pi)+\mathfrak{N}e\cos(2\theta-2\pi+s)$$

ac differentiando prodibit: $\frac{d.\,l\,\text{tang}\,\varrho}{dr}=$

$$e\sin(2\theta-2\pi-s)\left\{\begin{array}{l}+\mathfrak{B}r'+0,006219\,\mathfrak{B}+\frac{2}{n}\mathfrak{C}+0,012623\,\mathfrak{C}\\ \qquad -\frac{1}{n}\mathfrak{M}-0,008482\,\mathfrak{M}\end{array}\right.$$

 e sin

$$e\sin(2\theta-2\pi+s)\left\{\begin{array}{c} +\mathfrak{B}q'+0{,}06188\,\mathfrak{B}+\frac{2}{n}\mathfrak{C}+0{,}012623\,\mathfrak{C} \\ -\frac{3}{n}\mathfrak{N}-0{,}008482\,\mathfrak{N} \end{array}\right.$$

vnde reperitur

$$\mathfrak{M}=-0{,}024034 \quad . \quad . \quad l-\mathfrak{M}=8{,}380835$$

$$\mathfrak{N}=-0{,}008519 \quad . \quad . \quad l-\mathfrak{N}=7{,}930332$$

§. 225. Si ergo ε denotet inclinationem mediam orbitae lunaris ad eclipticam, et ϱ inclinationem veram, erit

			log. coeff.
$l\frac{\text{tang}\,\varrho}{\text{tang}\,\varepsilon}=$	$-0{,}002630$	$\cos 2\eta$	7,419915
	$+0{,}000019$	$\cos 4\eta$	5,278753
	$+0{,}002037$	$\cos(2\Phi-2\pi)$	7,308991
	$+0{,}026307$	$\cos(2\theta-2\pi)$	8,420081
	$+0{,}000370$	$\cos(4\theta-4\pi)$	6,567931
	$+0{,}01049k$	$\cos(2\eta-r)$	8,020638
	$+0{,}00317k$	$\cos(2\eta+r)$	7,500439
	$-0{,}00160k$	$\cos r$	7,204120
	$+0{,}00872k$	$\cos(2\Phi-2\pi-r)$	7,940484
	$+0{,}00270k$	$\cos(2\Phi-2\pi+r)$	7,431136
	$-0{,}00446k$	$\cos(2\theta-2\pi-r)$	7,649305
	$+0{,}00472k$	$\cos(2\theta-2\pi+r)$	7,673628
	$+0{,}3849kk$	$\cos(2\Phi-2\pi-2r)$	9,585335
	$-0{,}02403e$	$\cos(2\theta-2\pi-s)$	8,380835
	$-0{,}00852e$	$\cos(2\theta-2\pi+s)$	7,930332

§. 226. Quodsi iam ponatur $l\frac{\text{tang}\,\rho}{\text{tang}\,\epsilon} = S$, erit ad numeros ipsos procedendo $\frac{\text{tang}\,\rho}{\text{tang}\,\epsilon} = 1 + S + \frac{1}{2} S S$ Hinc igitur negligendo terminos minimos, consequemur:

$$
\begin{aligned}
\frac{\text{tang}\,\rho}{\text{tang}\,\epsilon} = 1 & - 0{,}002604 \quad \text{cos}\, 2\eta \\
& + 0{,}000020 \quad \text{cos}\, 4\eta \\
& + 0{,}002003 \quad \text{cos}\,(2\Phi - 2\pi) \\
& + 0{,}026307 \quad \text{cos}\,(2\theta - 2\pi) \\
& + 0{,}000490 \quad \text{cos}\,(4\theta - 4\pi) \\
& - 0{,}00160k \quad \text{cos}\, r \\
& + 0{,}01049k \quad \text{cos}\,(2\eta - r) \\
& + 0{,}00317k \quad \text{cos}\,(2\eta + r) \\
& + 0{,}00885k \quad \text{cos}\,(2\Phi - 2\pi - r) \\
& + 0{,}00274k \quad \text{cos}\,(2\Phi - 2\pi + r) \\
& - 0{,}00448k \quad \text{cos}\,(2\theta - 2\pi - r) \\
& + 0{,}00470k \quad \text{cos}\,(2\theta - 2\pi + r) \\
& + 0{,}3849kk \quad \text{cos}\,(2\Phi - 2\pi - 2r) \\
& - 0{,}02403e \quad \text{cos}\,(2\theta - 2\pi - s) \\
& - 0{,}00852e \quad \text{cos}\,(2\theta - 2\pi + s)
\end{aligned}
$$

§. 227. Cum in aequatione noſtra principali, quae motum Lunae continet, inſit terminus $\frac{\tang \rho^2}{\tang \varepsilon^2}$, huiusquoque valorem euolui conueniet: erit ergo

$$\frac{\tang \rho^2}{\tang \varepsilon^2} = 1$$
$$- 0{,}005156 \cos 2\eta \quad + 0{,}000040 \cos 4\eta$$
$$+ 0{,}003938 \cos(2\Phi - 2\pi)$$
$$+ 0{,}052614 \cos(2\theta - 2\pi) + 0{,}001320 \cos(4\theta - 4\pi)$$
$$- 0{,}00290k \cos r$$
$$+ 0{,}02098k \cos(2\eta - r)$$
$$+ 0{,}00634k \cos(2\eta + r)$$
$$+ 0{,}01796k \cos(2\Phi - 2\pi - r)$$
$$+ 0{,}00556k \cos(2\Phi - 2\pi + r)$$
$$- 0{,}00896k \cos(2\theta - 2\pi - r)$$
$$+ 0{,}00940k \cos(2\theta - 2\pi + r)$$
$$+ 0{,}7698kk \cos(2\Phi - 2\pi - 2r)$$
$$+ 0{,}04806e \cos(2\theta - 2\pi - s)$$
$$+ 0{,}01704e \cos(2\theta - 2\pi + s)$$

Hicque ergo valor in ſuperiori illa aequatione ſubſtitui poterit.

§. 228. Celeb. autem Clairaut conclusit inclinationem mediam ε ex obseruationibus exquisitissimis $5^\circ\, 8'\, 9''$, ex qua igitur ad quoduis tempus inclinationem veram elicere licebit. Sit enim $\varrho = \varepsilon + \omega$, erit tang $\varrho = \frac{\text{tg}\,\varepsilon + \omega}{1 - \omega\,\text{tg}\,\varepsilon}$ $= \text{tang}\,\varepsilon + \frac{\omega}{\text{cos}\,\varepsilon^2} = V\,\text{tang}\,\varepsilon$, ponendo V pro expressione ipsius $\frac{\text{tang}\,\varrho}{\text{tang}\,\varepsilon}$. Hinc erit $\omega = (V-1)\,\text{sin}\,\varepsilon\,\text{cos}\,\varepsilon =$ $\frac{1}{2}(V-1)\,\text{sin}\,2\varepsilon = 0{,}08915\,(V-1)$: vnde reperitur in minutis secundis

$$\begin{aligned}
\varrho = \varepsilon &- 48''\ \text{cos}\ 2\eta \\
&+ 36\ \text{cos}\ (2\varphi - 2\pi) \\
&+ 484\ \text{cos}\ (2\theta - 2\pi) \\
&+ 9\ \text{cos}\ (4\theta - 4\pi) \\
&- 2\ \text{cos}\ r \\
&+ 11\ \text{cos}\ (2\eta - r) \\
&+ 3\ \text{cos}\ (2\eta + r) \\
&+ 9\ \text{cos}\ (2\varphi - 2\pi - r) \\
&+ 3\ \text{cos}\ (2\varphi - 2\pi + r) \\
&- 5\ \text{cos}\ (2\theta - 2\pi - r) \\
&+ 5\ \text{cos}\ (2\theta - 2\pi + r) \\
&+ 23\ \text{cos}\ (2\varphi - 2\pi - 2r) \\
&- 7\ \text{cos}\ (2\theta - 2\pi - s) \\
&- 3\ \text{cos}\ (2\theta - 2\pi + s)
\end{aligned}$$

§. 229.

§. 229. Hic notandum eſt, etiamſi valor inclinationis mediae ι aliquantillum immutetur, aequationes has tamen inde vix alterari, ita vt eae ſemper eaedem ſint manſurae. Perſpicuum quoque eſt in calculo aſtronomico ſufficere tres inaequalitates primores, et reliquas omnes ſine errore ſenſibili praetermitti poſſe; niſi forte aequatio 23 coſ $(2\varphi - 2\pi - 2r)$ retinenda cenſeatur, quae inter reliquas eſt maxima. Exprimit autem angulus $2\varphi - 2\pi - 2r$ duplam diſtantiam apogei Lunae ab eius nodo, a quo angulo quoque locum nodi non mediocriter affici vidimus, cum correctio hinc oriunda pro loco nodi vsque ad $23''$ aſſurgere poſſit.

CAPUT

CAPUT XIV.

INUESTIGATIO INAEQUALITATUM MOTUS LUNAE AB EIUS INCLINATIONE AD ECLIPTICAM ORIUNDARUM.

§. 230.

Ponamus more adhuc vsitato:

$$\begin{aligned}\int \mathfrak{R}\,dr = {} & \mathfrak{A}\cos 2\eta + \mathfrak{C}k\cos r + \mathfrak{D}k\cos(2\eta - r) + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta - s) \\ & + \mathfrak{E}k\cos(2\eta + r) + \mathfrak{R}e\cos(2\eta + s) \\ & + \mathfrak{F}f\cos 2\eta + \mathfrak{G}f\cos(2\phi - 2\pi) + \mathfrak{J}fk\cos r + \mathfrak{K}fk\cos(2\eta - r) \\ & + \mathfrak{H}f\cos(2\theta - 2\pi) + \mathfrak{L}fk\cos(2\eta + r) \\ & + \mathfrak{M}fk\cos(2\phi - 2\pi - r) + \mathfrak{S}fk\cos(2\theta - 2\pi - r) \\ & + \mathfrak{N}fk\cos(2\phi - 2\pi + r) + \mathfrak{T}fk\cos(2\theta - 2\pi + r) \\ & + \mathfrak{O}fkk\cos(2\phi - 2\pi - 2r) + \mathfrak{U}fe\cos(2\theta - 2\pi - s) \\ & + \mathfrak{V}fe\cos(2\theta - 2\pi + s)\end{aligned}$$

$$\begin{aligned}\text{et } v = {} & A\cos 2\eta \;.\;.\; + Dk\cos(2\eta - r) + Pe\cos s + Qe\cos(2\eta - s) \\ & + Ek\cos(2\eta + r) + Re\cos(2\eta + s) \\ & + Ff\cos 2\eta + Gf\cos(2\phi - 2\pi) + Jfk\cos r + Kfk\cos(2\eta - r) \\ & + Hf\cos(2\theta - 2\pi) + Lfk\cos(2\eta + r) \\ & + Mfk\cos(2\phi - 2\pi - r) + Sfk\cos(2\theta - 2\pi - r) \\ & + Nfk\cos(2\phi - 2\pi + r) + Tfk\cos(2\theta - 2\pi + r) \\ & + Ofkk\cos(2\phi - 2\pi - 2r) + Ufe\cos(2\theta - 2\pi - s) \\ & + Vfe\cos(2\theta - 2\pi + s)\end{aligned}$$

§. 231.

§. 231. His valoribus substitutis in formula §. 52. orietur

$$R = f\sin 2\eta \left(\quad \ldots \quad fk\sin(2\theta-2\pi-r)\left(-\frac{3M}{2nn}-\frac{3G}{nn}\right.\right.$$

$$f\sin(2\Phi-2\pi)\left(+\frac{3H}{2nn} \quad \ldots \quad fk\sin(2\theta-2\pi+r)\left(-\frac{3N}{2nn}-\frac{3G}{nn}\right.\right.$$

$$f\sin(2\theta-2\pi)\left(-\frac{3G}{2nn} \quad \ldots \quad fe\sin(2\theta-2\pi-s)\left(+\frac{9G}{4nn}\right.\right.$$

$$fk\sin r\left(+\frac{3K}{2nn}-\frac{3L}{2nn} \quad \ldots \quad fe\sin(2\theta-2\pi+s)\left(+\frac{9G}{4nn}\right.\right.$$

$$fk\sin(2\eta-r)\left(+\frac{3I}{2nn}\right.$$

$$fk\sin(2\eta-r)\left(+\frac{3I}{2nn}\right.$$

$$fk\sin(2\Phi-2\pi-r)\left(+\frac{3S}{2nn}+\frac{3H}{nn}\right.$$

$$fk\sin(2\Phi-2\pi+r)\left(+\frac{3T}{2nn}+\frac{3H}{nn}\right.$$

$$fk\sin(2\Phi-2\pi-2r)\left(+\frac{3S}{nn}\right.$$

§. 232. Altera vero aequatio fundamentalis induet formam sequentem:

$$\frac{ddv}{dr^2} = \text{Praec.} + f\cos 2\eta \ [-\beta F - 2\kappa\mathfrak{F} - 0{,}005156 - 0{,}026307$$

$$+ f\cos(2\Phi-2\pi) \ [-\beta G - 2\kappa\mathfrak{G} + 0{,}003938 - 1$$

$$+ f\cos(2\theta-2\pi) \ [-\beta H - 2\kappa\mathfrak{H} + 0{,}052614 + 0{,}002578$$

+

$$+fk\cos r\ \left[-\mathfrak{C}J-2\varkappa\mathfrak{J}-0{,}00290-0{,}00893\right.$$
$$-0{,}00098-0{,}00278-0{,}00098$$
$$+fk\cos(2\eta-r)\ \left[-\mathfrak{C}K+\tfrac{1}{2}bF-2\varkappa\mathfrak{K}+0{,}02098-0{,}00470\right.$$
$$-0{,}00258-0{,}01315$$
$$+fk\cos(2\eta+r)\ \left[-\mathfrak{C}L+\tfrac{1}{2}bF-2\varkappa\mathfrak{L}+0{,}00634+0{,}00448\right.$$
$$-0{,}00258-0{,}01315$$
$$+fk\cos(2\varphi-2\pi-r)\ \left[-\mathfrak{C}M+\tfrac{1}{2}bG-2\varkappa\mathfrak{M}+0{,}01796+0{,}00145\right.$$
$$+0{,}00197-\tfrac{1}{2}$$
$$+fk\cos(2\varphi-2\pi+r)\ \left[-\mathfrak{C}N+\tfrac{1}{2}bG-2\varkappa\mathfrak{N}+0{,}00556+0{,}00145\right.$$
$$+0{,}00197-\tfrac{1}{2}$$
$$+fkk\cos(2\varphi-2\pi-2r)\ \left[-\mathfrak{C}O+\tfrac{1}{2}bM+\tfrac{1}{4}G-2\varkappa\mathfrak{O}+0{,}7698+0{,}0089\right.$$
$$+0{,}0010+0{,}0007-\tfrac{1}{4}$$
$$+fk\cos(2\theta-2\pi-r)\ \left[-\mathfrak{C}S+\tfrac{1}{2}bH-2\varkappa\mathfrak{S}-0{,}00896-0{,}00317\right.$$
$$+0{,}02631+0{,}00129$$
$$+fk\cos(2\theta-2\pi+r)\ \left[-\mathfrak{C}T+\tfrac{1}{2}bH-2\varkappa\mathfrak{T}+0{,}00940-0{,}01049\right.$$
$$+0{,}02631+0{,}00129$$
$$+fe\cos(2\theta-2\pi\pm s)\ \left[-\mathfrak{C}U-2\varkappa\mathfrak{U}-0{,}04806\right.$$
$$+fe\cos(2\theta-2\pi\mp s)\ \left[-\mathfrak{C}V-2\varkappa\mathfrak{V}-0{,}01704\right.$$

§. 233. Quoniam manifeſtum eſt, coefficientes F, G, H etc. admodum fore paruos; cum maxima huius generis inaequalitas aliquot minuta prima non excedat, hi iidem coefficientes per nn diuiſi tam euadent parui, vt ſine errore reiici queant. Hoc autem facto quoque litterae germanicae $\mathfrak{F}$, $\mathfrak{G}$, $\mathfrak{H}$ etc. pro nihilo eſſent habendae, ex quo ſola poſterior aequatio differentio-differentialis reſoluenda ſupererit; in qua ob eandem rationem terminos ex diuiſione coefficientium per nn oriundos omiſimus, cum in tam operoſo calculo ſufficiat correctiones inde resultantes proxime saltem determinasse; praesertim cum haec praetermissio vix ad aliquot minuta secunda sit ascensura.

§. 234. Ob eandem rationem licebit in valoribus differentialium $\frac{d\phi}{dr}$ et $\frac{d\eta}{dr}$ particulas ab inclinatione pendentes negligere, vnde erit: $\frac{dw}{dr} =$

$f \sin 2\eta \,[-2\alpha F = -F']$

$f \sin(2\phi - 2\pi) \,[-2(\alpha + \frac{1}{n}) G - 0{,}008482\, G = -G']$

$f \sin(2\theta - 2\pi) \,[+G\alpha' - \frac{2}{n}) H - 0{,}008482\, H = -H']$

$fk \sin r \,[F\delta' - F\epsilon' - J = -J']$

$fk \sin(2\eta - r) \,[+F\epsilon' - (2\alpha - 1) K = -K']$

$fk \sin(2\eta + r) \,[+F\epsilon' - (2\alpha + 1) L = -L']$

$fk \sin(2\phi - 2\pi - r) \,[-2(\alpha + \frac{1}{n}) M + M - 0{,}008482\, M = -M']$

$fk \sin(2\phi - 2\pi + r) \,[-2(\alpha + \frac{1}{n}) N - N - 0{,}008482\, N = -N']$

$fkk \sin(2\phi - 2\pi - 2r) \,[-2(\alpha + \frac{1}{n}) O + 2O - 0{,}008482\, O = -O']$

$fk \sin(2\theta - 2\pi - r) \,[+G\epsilon' - \frac{2}{n} H - \frac{2}{n} S + S - 0{,}008482\, S = -S']$

$fk \sin(2\theta - 2\pi + r) \,[+G\delta' - \frac{2}{n} H - \frac{2}{n} T - T - 0{,}008482\, T = -T']$

$fe \sin(2\theta - 2\pi - s) \,[+G\gamma' + \frac{2}{n} H - \frac{1}{n} U - 0{,}008482\, U = -U']$

$fe \sin(2\theta - 2\pi + s) \,[+G\gamma' + \frac{2}{n} H - \frac{3}{n} V - 0{,}008482\, V = -V']$

§. 235.

§. 235. Si nunc ſimili modo denuo differentiemus, prodibit: $\frac{ddv}{dr^2} =$

$f \cos 2\eta \; [-2\alpha F']$

$f \cos(2\phi - 2\pi) \; [-2(\alpha + \frac{1}{n}) G' - 0{,}008482 \, G']$

$f \cos(2\theta - 2\pi) \; [+ G' a' - \frac{2}{n} H' - 0{,}008482 \, H']$

$fk \cos r \; [F' d' + F e' - J']$

$fk \cos(2\eta - r) \; [F' e' - (2\alpha - 1) K']$

$fk \cos(2\eta + r) \; [F' c' - (2\alpha + 1) L']$

$fk \cos(2\phi - 2\pi - r) \; [-2(\alpha + \frac{1}{n}) M' + M' - 0{,}008482 \, M']$

$fk \cos(2\phi - 2\pi + r) \; [-2(\alpha + \frac{1}{n}) N' - N' - 0{,}008482 \, N']$

$fkk \cos(2\phi - 2\pi - 2r) \; [-2(\alpha + \frac{1}{n}) O' + 2O' - 0{,}008482 \, O']$

$fk \cos(2\theta - 2\pi - r) \; [G' d' - \frac{2}{n} H' - \frac{2}{n} S' + S' - 0{,}008482 \, S']$

$fk \cos(2\theta - 2\pi + r) \; [G' d' - \frac{2}{n} H' - \frac{2}{n} T' - T' - 0{,}008482 \, T']$

$fe \cos(2\theta - 2\pi - s) \; [G' p' + \frac{2}{n} H' - \frac{1}{n} U' - 0{,}008482 \, U']$

$fe \cos(2\theta - 2\pi + s) \; [G' q' + \frac{2}{n} H' - \frac{3}{n} V' - 0{,}008482 \, V']$

§. 236. Hinc autem ſequentes eliciuntur valores

$F =$	$0{,}01273$	. . . $l\,F =$	$8{,}104833$
$G =$	$0{,}32213$	. . . $l\,G =$	$9{,}508032$
$H =$	$0{,}06976$	. . . $l\,H =$	$8{,}843590$
$J =$	$-1{,}87800$	. . . $l{-}J =$	$0{,}273710$

 K =

$K = +0,05615 \quad \ldots \quad l\text{-}K = 8,749352$

$L = -0,00077 \quad \ldots \quad l\text{-}L = 6,888904$

$M = -0,29638 \quad \ldots \quad l\text{-}M = 9,471854$

$N = +0,00012 \quad \ldots \quad l\,N = 6,089109$

$O = +0,32287 \quad \ldots \quad l\,O = 9,509034$

$S = +0,39091 \quad \ldots \quad l\,S = 9,592073$

$T = +0,69579 \quad \ldots \quad l\,T = 9,842475$

$U = -0,07922 \quad \ldots \quad l\text{-}U = 8,898830$

$V = -0,05141 \quad \ldots \quad l\text{-}V = 8,711093$

§. 237. Pro diſtantia ergo lunae a ſole curtata $x = \frac{(1-kk)au}{1-k\,\mathrm{coſ}\,r}$ erit

$u =$ Praec.

		Log. coeff.	Valores coeff. integr.
$+ 0,000072f$	$\mathrm{coſ}\,2\eta$	5,860017	$+ 0,000079$
$+ 0,001833f$	$\mathrm{coſ}(2\Phi - 2\pi)$	7,263216	$+ 0,002005$
$+ 0,000397f$	$\mathrm{coſ}(2\theta - 2\pi)$	6,598774	$+ 0,000434$
$- 0,01069fk$	$\mathrm{coſ}\,r$	8,028894	$- 0,000634$
$+ 0,00032fk$	$\mathrm{coſ}(2\eta - r)$	6,504536	$+ 0,000019$
$- 0,00000fk$	$\mathrm{coſ}(2\eta + r)$	4,644088	$- 0,000000$
$- 0,00169fk$	$\mathrm{coſ}(2\Phi - 2\pi - r)$	7,227038	$- 0,000100$
$+ 0,00000fk$	$\mathrm{coſ}(2\Phi - 2\pi + r)$	3,834293	$+ 0,000000$
$+ 0,00184fk^2$	$\mathrm{coſ}(2\Phi - 2\pi - 2r)$	7,264218	$+ 0,000006$
$+ 0,00223fk$	$\mathrm{coſ}(2\theta - 2\pi - r)$	7,347257	$+ 0,000132$
$+ 0,00396fk$	$\mathrm{coſ}(2\theta - 2\pi + r)$	7,597659	$+ 0,000235$
$- 0,00045fe$	$\mathrm{coſ}(2\theta - 2\pi - s)$	6,654014	$- 0,000000$
$- 0,00029fe$	$\mathrm{coſ}(2\theta - 2\pi + s)$	6,466277	$- 0,000000$

vbi notandum eſt eſſe $f = 1,093756$, et $l\,f = 0,038921$.

§. 238.

§. 238. Deinde pro motu momentaneo habebitur

$$\frac{d\Phi}{dr} = \text{Praec.}$$

		Log. coeff.	Valores coeff. in numeris.
$-0{,}000146f$	$\cos 2\eta$	6,164934	$-0{,}000160$
$-0{,}003700f$	$\cos(2\Phi-2\pi)$	7,568133	$-0{,}004046$
$-0{,}000801f$	$\cos(2\theta-2\pi)$	6,903691	$-0{,}000876$
$+0{,}02157fk$	$\cos r$	8,333811	$+0{,}001286$
$-0{,}00065fk$	$\cos(2\eta-r)$	6,809453	$-0{,}000038$
$+0{,}00000fk$	$\cos(2\eta+r)$	4,949005	$+0{,}000001$
$+0{,}00340fk$	$\cos(2\Phi-2\pi-r)$	7,531955	$+0{,}000203$
$-0{,}00000fk$	$\cos(2\Phi-2\pi+r)$	4,139210	$-0{,}000000$
$-0{,}00371fk^2$	$\cos(2\Phi-2\pi-2r)$	7,569135	$-0{,}000012$
$-0{,}00449fk$	$\cos(2\theta-2\pi-r)$	7,652174	$-0{,}000267$
$-0{,}00799fk$	$\cos(2\theta-2\pi+r)$	7,902576	$-0{,}000476$
$+0{,}00091fe$	$\cos(2\theta-2\pi-s)$	6,958931	$+0{,}000000$
$+0{,}00059fe$	$\cos(2\theta-2\pi+s)$	6,771194	$+0{,}000000$

§. 239. Pro correctione longitudinis verae hinc oriunda ponatur,

$$\Phi = \text{Praec.}$$

$$+\mathfrak{A}'f\sin 2\eta \quad +\mathfrak{J}'fk\sin r \quad +\mathfrak{M}'fk\sin(2\Phi-2\pi-r)$$

$$+\mathfrak{G}'f\sin(2\Phi-2\pi)+\mathfrak{K}'fk\sin(2\eta-r)+\mathfrak{N}'fk\sin(2\Phi-2\pi+r)$$

$$+\mathfrak{H}'f\sin(2\theta-2\pi)+\mathfrak{L}'fk\sin(2\eta+r)+\mathfrak{O}'fkk\sin(2\Phi-2\pi-2r)$$

$$+\mathfrak{S}'fk\sin(2\theta-2\pi-r)+\mathfrak{U}'fe\sin(2\theta-2\pi-s)$$

$$+\mathfrak{T}'fk\sin(2\theta-2\pi+r)+\mathfrak{W}'fe\sin(2\theta-2\pi+s)$$

 eris

eritque:

$$2a\mathfrak{F}' = -0,000146$$
$$0,026834\,\mathfrak{G}' = -0,003700$$
$$0,159358\,\mathfrak{H}' - \mathfrak{G}'a' = -0,000801$$
$$\mathfrak{J}' - \mathfrak{F}'d' - \mathfrak{F}'c' = +0,02157$$
$$0,867476\,\mathfrak{K}' - \mathfrak{F}'c' = -0,00065$$
$$2,867476\,\mathfrak{L}' - \mathfrak{F}'c' = -0,00000$$
$$1,026834\,\mathfrak{M}' = +0,00340$$
$$3,026834\,\mathfrak{N}' = -0,00000$$
$$0,026834\,\mathfrak{O}' = -0,00371$$
$$-0,840642\,\mathfrak{S}' - \mathfrak{G}'c' + \frac{2}{n}\mathfrak{H}' = -0,00449$$
$$+1,159358\,\mathfrak{T}' - \mathfrak{G}'d' + \frac{2}{n}\mathfrak{H}' = -0,00799$$
$$+0,083920\,\mathfrak{U}' - \mathfrak{G}'r' - \frac{2}{n}\mathfrak{H}' = +0,00091$$
$$+0,234796\,\mathfrak{W}' - \mathfrak{G}'q' - \frac{2}{n}\mathfrak{H}' = +0,00059$$

§. 240. Expeditis igitur his formulis orietur:

		Log. coeff.	coeff. tot. in sec.
$\Phi = Pr.$ $-0,000078f$	$\sin 2\eta$	5,893680	$-\ 18''$
$-0,001825f$	$\sin(2\Phi - 2\pi)$	7,261316	$-\ 422$
$-0,004800f$	$\sin(2\theta - 2\pi)$	7,681286	-1083
$+0,02154fk$	$\sin r$	8,333246	$+\ 264$
$-0,00074fk$	$\sin(2\eta - r)$	6,867925	$-\ 9$
$+0,00332fk$	$\sin(2\Phi - 2\pi - r)$	7,520465	$+\ 41$
$-0,13818fk^2$	$\sin(2\Phi - 2\pi - 2r)$	9,140450	$-\ 92$
$+0,00446fk$	$\sin(2\theta - 2\pi - r)$	7,649421	$+\ 55$
$-0,00685fk$	$\sin(2\theta - 2\pi + r)$	7,835624	$-\ 84$
$+0,00310fe$	$\sin(2\theta - 2\pi - s)$	7,491107	$+\ 11$
$-0,00030fe$	$\sin(2\theta - 2\pi + s)$	6,474418	$-\ 1$

§. 242.

§. 241. Haec omnia satis conueniunt cum notis inaequalitatibus motus lunae, nisi quod inaequalitas ab angulo $2\theta - 2\pi$ pendens plane aduersari videatur, cum nullum eius vestigium in tabulis astronomicis occurrat; quod quidem eo magis est mirandum, cum correctio inde oriunda ad 18′, 3″ exsurgat. Lubens equidem agnosco, in hoc calculo non omnem curam esse adhibitam, vt hanc aequationem tanquam omnibus numeris absolutam spectare liceat, quoniam ad plurimos terminos, quos formulae nostrae suppeditant, non respexi. Interim tamen calculum repetenti mox patebit, non admodum enormiter esse aberratum, praesertim cum aequatio ab angulo $2\Phi - 2\pi$ pendens, quae pari passu procedit, veritati perquam consentanea prodierit, cum ea reductio lunae ad eclipticam contineatur. Ac si quidem haec inaequalitas ad semissem vsque diminuatur, tamen tanta remanet, vt merito dubitare debeamus, eius effectum ab Astronomis non esse animaduersum; cum eius omissio vix per aliam aequationem compensari queat. Hancobrem, siue omissio terminorum neglectorum sit in causa, siue etiam in calculo numerico error fuerit admissus, quod facile euenire potuit, istam inuestigationem in capite sequenti accuratius suscipiamus.

CAPUT

CAPUT XV.

ACCURATIOR INUESTIGATIO INAEQUALITATUM LUNAE AB INCLINATIONE EJUS ORBITAE PENDENTIUM.

§. 242.

Quoniam praecipuum dubium circa inaequalitatem ab angulo $2\theta - 2\pi$ pendentem versatur, nostram inuestigationem ab iis inaequalitatibus, quae simul ab alterutra excentricitate pendent, abstrahamus. Ponamus ergo:

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{F} f \cos 2\eta + \mathfrak{G} f \cos(2\Phi - 2\pi) + \mathfrak{H} f \cos(2\theta - 2\pi) + \mathfrak{J} f \cos(4\theta - 4\pi)$$

et

$$v = A\cos 2\eta + F f\cos 2\eta + G f\cos(2\Phi - 2\pi) + H f\cos(2\theta - 2\pi) + J f\cos(4\theta - 4\pi)$$

Positoque $\frac{2\varkappa F + \mathfrak{F}}{\varkappa\varkappa} = f'$; $\frac{2\varkappa G + \mathfrak{G}}{\varkappa\varkappa} = g'$; $\frac{2\varkappa H + \mathfrak{H}}{\varkappa\varkappa} = h'$; $\frac{2\varkappa J + \mathfrak{J}}{\varkappa\varkappa} = i'$ erit:

$$\frac{d\Phi}{dr} = \alpha + \frac{1}{n} - \alpha'\cos 2\eta - f' f\cos 2\eta - g' f\cos(2\Phi - 2\pi) - h' f\cos(2\theta - 2\pi) - i' f\cos(4\theta - 4\pi);$$

$$\frac{d\eta}{dr} = \alpha - \alpha'\cos 2\eta - f' f\cos 2\eta - g' f\cos(2\Phi - 2\pi) - h' f\cos(2\theta - 2\pi) - i' f\cos(4\theta - 4\pi)$$

$$\frac{d\theta}{dr} = \frac{1}{n} \text{ et } \frac{d\pi}{dr} = -0{,}004241 - 0{,}004221\cos 2\eta + 0{,}004241\cos(2\Phi - 2\pi) + 0{,}004241\cos(2\theta - 2\pi)$$

§. 243.

§. 243. His valoribus in formulis principalibus substitutis habebimus has aequationes:

$$R = -\frac{3G}{2nn} f \sin(2\theta - 2\pi) + \frac{3H}{2nn} f \sin(2\Phi - 2\pi)$$

$$\frac{ddv}{dr^2} = f \cos 2\eta \left\{ -\mathfrak{C}F - 2\varkappa\mathfrak{F} \right.$$

$$f \cos(2\Phi - 2\pi) \left\{ \begin{array}{l} -\mathfrak{C}G + \frac{3H}{4nn} - 2\varkappa\mathfrak{G} + \frac{\mathfrak{A}\mathfrak{H}}{nn} + \frac{3A\mathfrak{H}}{nn} \\ + \frac{3\mathfrak{A}H}{nn} + \frac{3AH}{nn} \end{array} \right.$$

$$f \cos(2\theta\,2\pi) \left\{ \begin{array}{l} -\mathfrak{C}H + \frac{3G}{4nn} - 2\varkappa\mathfrak{H} + \frac{\mathfrak{A}\mathfrak{G}}{nn} + \frac{3A\mathfrak{G}}{nn} \\ + \frac{3\mathfrak{A}G}{nn} + \frac{3AG}{nn} \end{array} \right.$$

$$f \cos(4\theta - 4\pi) [-\mathfrak{C}J - 2\varkappa\mathfrak{J}$$

$$f \cos 2\eta \left(-0{,}005156 - 0{,}026307 + 0{,}001969 \frac{A}{nn} \right)$$

$$f \cos(2\Phi - 2\pi) \left(+0{,}003938 - 1 - 0{,}052614 \frac{A}{nn} - 0{,}002578 \frac{A}{nn} \right)$$

$$f \cos(2\theta - 2\pi) \left(+0{,}052614 + 0{,}002578 - 0{,}001969 \frac{2A}{nn} + \frac{A}{nn} \right)$$

$$f \cos(4\theta - 4\pi) \left(+0{,}001320 + 0{,}026307 \frac{A}{nn} \right)$$

§. 244. Vel in numeris erit

$$R = 0{,}008540 H f \sin(2\Phi - 2\pi) - 0{,}008540 G f \sin(2\theta - 2\pi)$$

$\frac{ddv}{dr^2} =$

$f \cos 2\eta \; [-1{,}01591 F - 2\varkappa \mathfrak{F} - 0{,}031478$

$f \cos(2\phi - 2\pi) \Big\{ -1{,}01591 G - 2\varkappa \mathfrak{G} + 0{,}000636 H - 0{,}845648 - 0{,}027110 \mathfrak{H}$

$f \cos(2\theta - 2\pi) \Big\{ -1{,}01591 H - 2\varkappa \mathfrak{H} + 0{,}000636 G + 0{,}047722 - 0{,}027110 \mathfrak{G}$

$f \cos(4\theta - 4\pi) [-1{,}01591 J - 2\varkappa \mathfrak{J} + 0{,}001123$

Nunc autem ex formulis assumtis erit

$R = f \sin 2\eta \; [-2\alpha \mathfrak{F} + 0{,}004241 \mathfrak{G} - 0{,}004241 \mathfrak{H}]$
$\quad f \sin(2\phi - 2\pi)[+ \mathfrak{A} b' - 2{,}026834 \mathfrak{G} + 0{,}004221 \mathfrak{H}]$
$\quad f \sin(2\theta - 2\pi)[- \mathfrak{A} g' - 0{,}023965 \mathfrak{G} - 0{,}159358 \mathfrak{H}]$
$\quad f \sin(4\theta - 4\pi)[+ 0{,}004241 \mathfrak{H} - 0{,}318716 \mathfrak{J}]$

at est

$\mathfrak{A} b' = -0{,}00931 H - 0{,}00461 \mathfrak{H}$; et $\mathfrak{A} g' = -0{,}00931 G - 0{,}00461 \mathfrak{G}$

vnde fit: $1{,}867476 \mathfrak{F} = 0{,}004241 (\mathfrak{G} - \mathfrak{H})$
$2{,}026874 \mathfrak{G} = -0{,}01785 H - 0{,}00039 \mathfrak{H}$
$0{,}159358 \mathfrak{H} = +0{,}01785 G - 0{,}01935 \mathfrak{G}$
$0{,}318716 \mathfrak{J} = +0{,}004241 \mathfrak{H}$

§. 245. Tum simili modo differentiando valorem ipsius v ponatur

$F' = 1{,}867476 F - 0{,}004241 G + 0{,}004241 H$
$G' = 2{,}026834 G + 0{,}01091 H + 0{,}00750 \mathfrak{H}$
$H' = 0{,}159358 H + 0{,}03909 G - 0{,}00750 \mathfrak{G}$
$J' = 0{,}318716 F - 0{,}004241 H$

vt

vt fit $\frac{dv}{dr} =$

$$A' \sin 2\eta - F' f \sin 2\eta - G' f \sin(2\Phi - 2\pi)$$
$$- H' f \sin(2\theta - 2\pi) - J' f \sin(4\theta - 4\pi)$$

eritque

$$\frac{ddv}{dr^2} = f \cos 2\eta \, [-2\alpha F' + 0{,}004241\, G' + 0{,}004241\, H']$$
$$f \cos(2\Phi - 2\pi)[+A' b' - 2{,}026834\, G' - 0{,}004221\, H']$$
$$f \cos(2\Phi - 2\pi)[+A' g' - 0{,}159358\, H' - 0{,}023965\, G']$$
$$f \cos(4\theta - 4\pi)[+ 0{,}004241\, H' - 0{,}318716\, J']$$

feu $\frac{ddv}{dr^2} =$

$$f \cos 2\eta \begin{cases} -3{,}48745\, F + 0{,}01668\, G - 0{,}00721\, H - 0{,}00003\, \mathfrak{G} \\ \qquad + 0{,}00007\, \mathfrak{H} \end{cases}$$
$$f \cos(2\Phi - 2\pi)\, [-4{,}10820\, G - 0{,}05121\, H - 0{,}02929\, \mathfrak{H} + 0{,}00003\, \mathfrak{G}$$
$$f \cos(2\theta - 2\pi)\, [-0{,}02565\, H - 0{,}08323\, G - 0{,}01290\, \mathfrak{G} - 0{,}00018\, \mathfrak{H}$$
$$f \cos(4\theta - 4\pi)\, [-0{,}10158\, J + 0{,}00016\, G + 0{,}00202\, H - 0{,}00003\, \mathfrak{G}$$

§. 246. Hinc pro $\mathfrak{F}$ et $\mathfrak{J}$ fubftitutis valoribus

$$\mathfrak{F} = 0{,}00227\, (\mathfrak{G} - \mathfrak{H}) \quad \text{et} \quad \mathfrak{J} = 0{,}01331\, \mathfrak{H}$$

habebimus has aequationes

$$+ 2{,}47154\, F - 0{,}01668\, G + 0{,}00721\, H - 0{,}00456\, \mathfrak{G} + 0{,}00456\, \mathfrak{H} = 0{,}031478$$
$$+ 3{,}09229\, G + 0{,}05185\, H + 0{,}00218\, \mathfrak{H} - 2{,}01799\, \mathfrak{G} = 0{,}995648$$
$$- 0{,}99026\, H + 0{,}08387\, G - 0{,}01421\, \mathfrak{G} - 2{,}01780\, \mathfrak{H} = -0{,}047722$$
$$- 0{,}91433\, J - 0{,}00016\, G - 0{,}00202\, H + 0{,}00003\, \mathfrak{G} - 0{,}02685\, \mathfrak{H} = -0{,}001123$$

§. 247. Deinde reperitur

$$\mathfrak{G} = -0,00881\,H - 0,00002\,G$$
$$\mathfrak{H} = +0,11201\,G + 0,00107\,H$$

qui valores substituti praebent:

$$+2,47154\,F - 0,01618\,G + 0,00725\,H = 0,031478$$
$$+3,09256\,G + 0,06964\,H = 0,995648$$
$$+0,99229\,H + 0,14239\,G = 0,047722$$
$$-0,91430\,J - 0,00297\,G - 0,00205\,H = -0,001123$$

vnde tandem reperitur

$$F = +0,014830 \quad \ldots \quad l\,F = 8,171166$$
$$G = +0,321910 \quad \ldots \quad l\,G = 9,507734$$
$$H = +0,001901 \quad \ldots \quad l\,H = 7,278927$$
$$J = +0,000180 \quad \ldots \quad l\,J = 6,256381$$

atque

$$\mathfrak{F} = -0,000080 \quad \ldots \quad l{-}\mathfrak{F} = 5,903090$$
$$\mathfrak{G} = -0,000023 \quad \ldots \quad l{-}\mathfrak{G} = 5,361728$$
$$\mathfrak{H} = +0,036056 \quad \ldots \quad l\,\mathfrak{H} = 8,556977$$
$$\mathfrak{J} = +0,000480 \quad \ldots \quad l\,\mathfrak{J} = 6,681241$$

§. 248. Hinc ergo pro distantia $x = \frac{(1-kk)\,au}{1-k\cos r}$ reperitur

	log.coeff.	val.coeff.
$x = $ Pr. $+ 0,000084 f \cos 2\eta$	5,926350	+0,000092
$+ 0,001832 f \cos(2\Phi - 2\pi)$	7,262918	+0,002004
$+ 0,000011 f \cos(2\theta - 2\pi)$	5,034111	+0,000012
$+ 0,000001 f \cos(4\theta - 4\pi)$	4,011565	+0,000001

At

At pro motu momentaneo erit

	log.coeff.	val.coeff.
$\frac{d\Phi}{dt} = \text{Pr.} - 0{,}000169 f \cos 2\eta$	6,227887	−0,000185
$- 0{,}003697 f \cos(2\Phi - 2\pi)$	7,567849	−0,004043
$- 0{,}000227 f \cos(2\theta - 2\pi)$	6,356026	−0,000248
$- 0{,}000005 f \cos(4\theta - 4\pi)$	4,698970	−0,000005

vnde quidem iam patet inaequalitatem ab angulo $2\theta - 2\pi$ pendentm multo fore minorem, quam supra inueneramus, in quo non paruum veritatis criterium cernitur.

§. 249. Pro ipsa iam longitudine lunae ponamus:

$$\Phi = \text{Pr.} + \mathfrak{A}' \sin 2\eta + \mathfrak{F}' f \sin 2\eta + \mathfrak{G}' f \sin(2\Phi - 2\pi) + \mathfrak{H}' f \sin(2\theta - 2\pi) + \mathfrak{J}' f \sin(4\theta - 4\pi)$$

atque obtinebimus has aequationes:

$$2\alpha\mathfrak{F}' - 0{,}004241 (\mathfrak{G}' + \mathfrak{H}') = -0{,}000169$$

$$2{,}026834\,\mathfrak{G}' + 0{,}004221\,\mathfrak{H}' - 0{,}000227\,\mathfrak{A}' = -0{,}003697$$

$$0{,}159358\,\mathfrak{H}' + 0{,}023965\,\mathfrak{G}' - 0{,}003697\,\mathfrak{A}' = -0{,}000227$$

$$0{,}318716\,\mathfrak{J}' - 0{,}004241\,\mathfrak{H}' = -0{,}000005$$

vnde erit

	log. coeff.	val.tot.coeff. in min. sec.
$\Phi = \text{Pr.} - 0{,}000096 f \sin 2\eta$	5,984018	− 22″
$- 0{,}001823 f \sin(2\Phi - 2\pi)$	7,260310	−411
$- 0{,}000910 f \sin(2\theta - 2\pi)$	6,959131	−205
$- 0{,}000028 f \sin(4\theta - 4\pi)$	5,450835	− 6

Patet ergo reuera aequationem ab angulo $2\theta - 2\pi$ ortam multo esse minorem, quam capite praecedente inueneramus, atque nunc quidem non vltra 205″ seu 3′, 25″ ascendere. Nullum igitur est dubium, quin haec aequatio tabulas lunares ad multo maiorem perfectionem sit euectura.

§. 250. Cum igitur neglectus terminorum, minimorum tantum errorem pepererit in aequatione ab angulo $2\theta - 2\pi$ pendente, operae erit pretium, etiam aequationes insuper ab excentricitate orbitae lunaris pendentes curatius inuestigare, quae quidem alicuius videntur esse momenti. In hunc finem ponamus.

$$\int R\,dr = \mathfrak{A}\,\mathrm{cos}\,2\eta + \mathfrak{C}k\,\mathrm{cos}\,r + \mathfrak{D}k\,\mathrm{cos}(2\eta - r) + \mathfrak{E}k\,\mathrm{cos}(2\eta + r) + 32kk\,\mathrm{cos}(2\eta - 2r)$$
$$+ \mathfrak{G}f\,\mathrm{cos}(2\theta - 2\pi) \quad + \mathfrak{H}f\,\mathrm{cos}(2\theta - 2\pi)$$
$$+ \mathfrak{J}fk\,\mathrm{cos}\,r + \mathfrak{M}fk\,\mathrm{cos}(2\Phi - 2\pi - r) \quad + \mathfrak{S}fk\,\mathrm{cos}(2\theta - 2\pi - r)$$
$$+ \mathfrak{O}fkk\,\mathrm{cos}(2\Phi - 2\pi - 2r) + \mathfrak{T}fk\,\mathrm{cos}(2\theta + 2\pi + r)$$

$$\text{et } v = A\,\mathrm{cos}\,2\eta + Dk\,\mathrm{cos}(2\eta - r) + Ek\,\mathrm{cos}(2\eta + r) - 14kk\,\mathrm{cos}(2\eta - 2r)$$
$$+ Gf\,\mathrm{cos}(2\Phi - 2\pi) \quad + Hf\,\mathrm{cos}(2\theta - 2\pi)$$
$$+ Jfk\,\mathrm{cos}\,r \quad + Mfk\,\mathrm{cos}(2\Phi - 2\pi - r) \quad + Sfk\,\mathrm{cos}(2\theta - 2\pi - r)$$
$$+ Ofkk\,\mathrm{cos}(2\Phi - 2\pi - 2r) + Tfk\,\mathrm{cos}(2\theta - 2\pi + r)$$

§. 251. His valoribus in aequationibus nostris principalibus substitutis, habebimus:

$$R = fk\,\mathrm{sin}\,r \quad (0)$$
$$fk\,\mathrm{sin}(2\Phi - 2\pi - r) \quad (0{,}00854\,S + 0{,}01708\,H)$$
$$fkk\,\mathrm{sin}(2\Phi - 2\pi - 2r) \quad (0{,}01708\,S)$$
$$fk\,\mathrm{sin}(2\theta - 2\pi - r) \quad (-0{,}00854\,M - 0{,}01708\,G)$$
$$fk\,\mathrm{sin}(2\theta - 2\pi + r) \quad (-0{,}01708\,G)$$

et:

$$\frac{ddv}{dr^2} =$$

$$fk\,\mathrm{cos}\,r \begin{cases} -6J - 2\kappa\mathfrak{J} - 0{,}00290 - 0{,}00893 - 0{,}00278 \\ -0{,}00098 - 0{,}00098 - 0{,}00448\dfrac{A}{nn} + 0{,}00470\dfrac{A}{nn} \\ +0{,}01315\dfrac{A}{nn} + 0{,}01315\dfrac{A}{nn} + 0{,}0263\dfrac{D}{nn} \\ +0{,}0263\dfrac{E}{nn} \end{cases}$$

fk

$$fk\cos(2\Phi-2\pi-r)\left\{\begin{array}{l}-\mathfrak{C}M-2\varkappa\mathfrak{M}+\tfrac{1}{2}bG+0{,}00427S-0{,}00854H\\ -0{,}02711\mathfrak{S}-0{,}03634S+\dfrac{\mathfrak{C}}{nn}\mathfrak{S}+\dfrac{3\mathfrak{C}}{nn}G\\ +0{,}55422\mathfrak{H}+0{,}51332H-\dfrac{3AH}{2nn}+0{,}01796\\ +0{,}00145+0{,}001969-\tfrac{1}{2}+0{,}00896\dfrac{A}{nn}\\ +0{,}01049\dfrac{A}{nn}-0{,}02631\dfrac{A}{nn}-0{,}00197\dfrac{A}{nn}\\ -0{,}00129\dfrac{A}{nn}-0{,}05261\dfrac{D}{nn}-0{,}00258\dfrac{E}{nn}\end{array}\right.$$

$$fkk\cos(2\Phi-2\pi-2r)\left\{\begin{array}{l}-\mathfrak{C}O-2\varkappa\mathfrak{O}+\tfrac{1}{2}bM+\tfrac{3}{4}\mathfrak{G}-\dfrac{10}{nn}\mathfrak{H}+\dfrac{54}{nn}H\\ -\dfrac{3DH}{2nn}+0{,}7698+0{,}00898+0{,}00098\\ +0{,}00072-\tfrac{1}{4}+0{,}00448\dfrac{A}{nn}+0{,}0052\dfrac{A}{nn}\\ -0{,}00064\dfrac{A}{nn}+0{,}00896\dfrac{D}{nn}-0{,}01315\dfrac{A}{nn}\\ -0{,}02630\dfrac{D}{nn}+0{,}01049\dfrac{E}{nn}-0{,}00129\dfrac{E}{nn}\\ -0{,}00129\dfrac{E}{nn}-0{,}00042\end{array}\right.$$

$$fk\cos(2\theta-2\pi-r)\left\{\begin{array}{l}-\mathfrak{C}S-2\varkappa\mathfrak{S}+\tfrac{1}{4}bH+0{,}00427M-0{,}00854G\\ -0{,}02711\mathfrak{M}-0{,}03634M+\dfrac{\mathfrak{C}}{nn}\mathfrak{H}+\dfrac{3\mathfrak{C}}{nn}H\\ -0{,}01606\mathfrak{G}-0{,}02844G-\dfrac{3AG}{2nn}-0{,}00896\\ -0{,}00327+0{,}026307+0{,}00129-0{,}00179\dfrac{A}{nn}\\ -0{,}00145\dfrac{A}{nn}+\dfrac{A}{2nn}-0{,}00394\dfrac{E}{nn}+\dfrac{E}{nn}\end{array}\right.$$

fk

$$fk\cos(2\theta-2\pi+r)\left\{\begin{array}{l} -6T-2\kappa\mathfrak{T}+\frac{1}{2}bH+0{,}00854G+\frac{\mathfrak{G}}{nn}\mathfrak{H}+\frac{3\mathfrak{G}}{nn}H \\ +0{,}55422\mathfrak{G}+0{,}51332G-\frac{3AG}{2nn}+0{,}00940 \\ -0{,}01049+0{,}026307+0{,}00129-0{,}00556\frac{A}{nn} \\ -0{,}00145\frac{A}{nn}-0{,}003938\frac{A}{nn}+\frac{A}{2nn} \\ -0{,}0394\frac{D}{nn}+\frac{D}{nn} \end{array}\right.$$

§. 252. Hae autem formulae intricatae reducuntur ad sequentes

$$\frac{ddv}{dr}=\text{Pr.}+fk\cos r\,(-6J-2\kappa\mathfrak{J}-0{,}01182)$$

$$fk\cos(2\phi-2\pi-r)\left(\begin{array}{l}-6M-2\kappa\mathfrak{M}-0{,}03207S+0{,}53619\\ -0{,}02711\mathfrak{S}\end{array}\right)$$

$$fkk\cos(2\phi-2\pi-2r)\,(-6O-2\kappa\mathfrak{O}+1{,}52115M+0{,}76615)$$

$$fk\cos(2\theta-2\pi-r)\left(\begin{array}{l}-6S-2\kappa\mathfrak{S}-0{,}03207M+0{,}00286\\ -0{,}02711\mathfrak{M}\end{array}\right)$$

$$fk\cos(2\theta-2\pi+r)\,(-6T-2\kappa\mathfrak{T}+0{,}38147)$$

§. 253. Nunc eosdem valores ex formulis assumtis eruamus, ac posito more adhuc recepto $\frac{2\kappa J+\mathfrak{J}}{nn}=i'$, $\frac{2\kappa M+\mathfrak{M}}{nn}=m'$ etc. erit

$$\frac{d\phi}{dr}=\alpha+\frac{1}{n}-a'\cos 2\eta-d'k\cos(2\eta-r)-g'f\cos(2\phi-2\pi)$$
$$-e'k\cos(2\eta+r)-b'f\cos(2\theta-2\pi)$$
$$-i'fk\cos r-m'fk\cos(2\phi-2\pi-r)-s'fk\cos(2\theta-2\pi-r)$$
$$-n'fkk\cos(2\phi-2\pi-2r)-t'fk\cos(2\theta-2\pi+r)$$

$$\frac{d\eta}{dr} = n - a'\cos 2\eta - e'k\cos r - d'k\cos(2\eta - r) - g'k\cos(2\Phi - 2\pi)$$
$$- c'k\cos(2\eta + r) - b'k\cos(2\theta - 2\pi)$$
$$- i'fk\cos r - m'fk\cos(2\Phi - 2\pi - r) - s'fk\cos(2\theta - 2\pi - r)$$
$$- n'fkk\cos(2\Phi - 2\pi - 2r) - r'fk\cos(2\theta - 2\pi + r)$$

$$\frac{d\theta}{dr} = \frac{1}{n} + \frac{2}{n}k\cos r + \frac{3}{2n}kk\cos 2r \quad \text{atque}$$

$$\frac{d\pi}{dr} =$$
$$-0{,}004241 + 0{,}004241\cos(2\Phi - 2\pi)$$
$$-0{,}004221\cos 2\eta + 0{,}004241\cos(2\theta - 2\pi)$$
$$-0{,}01766k\cos r - 0{,}00996k\cos(2\eta - r) - 0{,}010636kk\cos(2\eta - 2r)$$
$$-0{,}00831k\cos(2\eta + r) - 0{,}021273kk\cos 2r$$
$$+0{,}00924k\cos(2\Phi - 2\pi - r) + 0{,}00841k\cos(2\theta - 2\pi - r)$$
$$+0{,}00841k\cos(2\Phi - 2\pi + r) + 0{,}00924k\cos(2\theta - 2\pi + r)$$
$$+0{,}01064kk\cos(2\Phi - 2\pi - 2r)$$

§. 254. His iam valoribus introducendis differentiemus formulas nostras assumtas pro $\int R\,dr$ et v; atque obtinebimus primo:

$$R = \text{Praec.}$$
$$+ fk\sin r\left\{\begin{array}{l} +0{,}00924\mathfrak{G} + 0{,}00841\mathfrak{H} - 0{,}004241\mathfrak{M} \\ -0{,}00841\mathfrak{G} - 0{,}00924\mathfrak{H} - 0{,}004241\mathfrak{S} \\ \qquad + 0{,}004241\mathfrak{T} \end{array}\right.$$
$$+ fk\sin(2\Phi - 2\pi - r)\left\{\begin{array}{l} +\mathfrak{A}s' - 0{,}01766\mathfrak{G} - 0{,}00996\mathfrak{H} \\ \qquad - 1{,}026834\mathfrak{M} - 0{,}004221\mathfrak{S} \end{array}\right.$$
$$+ fkk\sin(2\Phi - 2\pi - 2r)\left\{\begin{array}{l} +\mathfrak{D}s' + 32h' - 0{,}02127\mathfrak{G} - 0{,}01064\mathfrak{H} \\ -0{,}026834\mathfrak{O} - 0{,}01766\mathfrak{M} - 0{,}00996\mathfrak{S} \end{array}\right.$$

$$+fk\sin(2\theta-2\pi-r)\left\{\begin{array}{l}-\mathfrak{A}m'+\mathfrak{D}b'+\mathfrak{G}e'-\frac{2}{n}\mathfrak{H}+0{,}840642\mathfrak{S}+\mathfrak{M}a'\\-\mathfrak{E}g'-0{,}00831\mathfrak{G}-0{,}01766\mathfrak{H}-0{,}004221\mathfrak{M}\end{array}\right\}$$

$$+fk\sin(2\theta-2\pi+r)\left\{\begin{array}{l}-\mathfrak{D}g'+\mathfrak{E}b'+\mathfrak{G}d'-\frac{2}{n}\mathfrak{H}-1{,}159358\mathfrak{T}\\+0{,}00996\mathfrak{G}-0{,}01766\mathfrak{H}\end{array}\right\}$$

§. 255. Hinc igitur consequimur istas aequationes

$$0{,}00083(\mathfrak{G}-\mathfrak{H})-\mathfrak{J}-0{,}004241(\mathfrak{M}+\mathfrak{S}-\mathfrak{T})=0$$

$$\mathfrak{A}s'-0{,}01766\mathfrak{G}-0{,}00996\mathfrak{H}-0{,}004221\mathfrak{S}-1{,}026834\mathfrak{M}= 0{,}01708H+0{,}00854S$$

$$\mathfrak{D}s'+32b'-0{,}002127\mathfrak{G}-0{,}01064\mathfrak{H}-0{,}026834\mathfrak{O}-0{,}01766\mathfrak{M}-0{,}00996\mathfrak{S}=0{,}01708S$$

$$-\mathfrak{A}m'+\mathfrak{D}b'-\mathfrak{E}g'+\mathfrak{G}e'-0{,}00831\mathfrak{G}-\frac{2}{n}\mathfrak{H}-0{,}01766\mathfrak{H}+\mathfrak{M}a' -0{,}004221\mathfrak{M}+0{,}840642\mathfrak{S}=-0{,}01708G-0{,}00854M$$

$$-\mathfrak{D}g'+\mathfrak{E}b'+\mathfrak{G}d'-0{,}00996\mathfrak{G}-\frac{2}{n}\mathfrak{H}-0{,}01766\mathfrak{H}-1{,}159358\mathfrak{T}= -0{,}01708G$$

§. 256. Ponatur nunc vlterius:

$$J'=\mathfrak{J}-0{,}00083(G-H)+0{,}004241(M+S-T)$$

$$M'=1{,}026834M-As'+0{,}01766G+0{,}00996H+0{,}004221S$$

$$O'=0{,}026834O-Ds'+14b'+0{,}02127G+0{,}01064H+0{,}01766M+0{,}00996S$$

$$S'=-0{,}840642S+Am'-Db'+Eg'-Ge'+0{,}00831G+\frac{2}{n}H+0{,}01766H-Ma'+0{,}004221M$$

$$T'=1{,}159358T+Dg'-Eb'-Gd'+0{,}00996G+\frac{2}{n}H+0{,}01766H$$

eritque

eritque $\frac{ddv}{dr} = \text{Praec.}$

$$+fk\cos r\,[+0{,}01766(G'+H') - J' + 0{,}004241(M'+S'+T')]$$

$$+fk\cos(2\varphi-2\pi-r)\left\{\begin{array}{r} A's' - 0{,}01766\,G' - 0{,}00996\,H' \\ -0{,}004221\,S' - 1{,}026834\,M' \end{array}\right\}$$

$$+fkk\cos(2\varphi-2\pi-2r)\left\{\begin{array}{r} D's' - 2{,}6b' - 0{,}02127\,G' - 0{,}01064\,H' \\ -0{,}01766\,M' - 0{,}00996\,S' \\ -0{,}026834\,O' \end{array}\right\}$$

$$+fk\cos(2\theta-2\pi-r)\left\{\begin{array}{r} A'm' + D'b' + E'g' + G'e' + 0{,}840642\,S' \\ -0{,}00831\,G' - \frac{2}{n}H' - 0{,}01766\,H' \\ +M'a' - 0{,}004221\,M' \end{array}\right\}$$

$$+fk\cos(2\theta-2\pi+r)\left\{\begin{array}{r} D'g' + E'b' + G'd' - 0{,}00996\,G - \frac{2}{n}H' \\ -0{,}01766\,H' - 1{,}159358\,T' \end{array}\right\}$$

§. 257. Prioris ordinis aequationes huc reducuntur:

$$\mathfrak{J} = -0{,}004241\,(\mathfrak{M}+\mathfrak{S}+\mathfrak{T})$$

$$1{,}026834\,\mathfrak{M} = -0{,}01785\,S - 0{,}00883\,\mathfrak{S} - 0{,}00039$$

$$0{,}026834\,\mathfrak{O} = -0{,}05835\,S - 0{,}03041\,\mathfrak{S} - 0{,}01766\,\mathfrak{M} + 0{,}00690$$

$$-0{,}840642\,\mathfrak{S} = +0{,}01785\,M - 0{,}01935\,\mathfrak{M} + 0{,}00263$$

$$1{,}159358\,\mathfrak{T} = +0{,}01247$$

Hinc fit

$$\mathfrak{J} = +0{,}00007\,S + 0{,}00008\,M + 0{,}00006 \quad \text{seu} \quad \mathfrak{J} = 0$$

$$\mathfrak{M} = -0{,}01738\,S + 0{,}00018\,M - 0{,}00035$$

$$\mathfrak{O} = -2{,}16266\,S + 0{,}02394\,M + 0{,}26092$$

$$\mathfrak{S} = -0{,}00040\,S - 0{,}02123\,M - 0{,}00313$$

$$\mathfrak{T} = -0{,}01076$$

§. 258.

§. 258. Porro reperietur

$$J' = J + 0{,}004241\,(M+S-T) - 0{,}00026$$
$$M' = 1{,}026834\,M + 0{,}01935\,S - 0{,}00016\,M + 0{,}00568$$
$$O' = 0{,}026834\,O - 0{,}37652\,S + 0{,}01805\,M + 0{,}01063$$
$$S' = 0{,}840642\,S + 0{,}00013\,S + 0{,}00883\,M - 0{,}00167$$
$$T' = 1{,}159358\,T + 0{,}01023$$

ac succinctius habebitur $\frac{ddv}{dr^2} =$ Praec.

$$+fk\cos r\,\left[-J' + 0{,}004241(M'+S'+T') + 0{,}01175\right]$$
$$+fk\cos(2\Phi-2\pi-r)\left\{\begin{matrix}-1{,}026834\,M' - 0{,}00422\,S' - 0{,}02842\,S \\ + 0{,}00030\,M - 0{,}01160\end{matrix}\right\}$$
$$+fkk\cos(2\Phi-2\pi-2r)\left\{\begin{matrix}-0{,}026834\,O' - 0{,}01766\,M' - 0{,}00354\,M \\ -0{,}01511 - 0{,}00996\,S' + 0{,}33746\,S\end{matrix}\right\}$$
$$+fk\cos(2\theta-2\pi-r)\left\{\begin{matrix}+0{,}840642\,S' - 0{,}02396\,M' - 0{,}02843\,M \\ -0{,}01472 + 0{,}00025\,S\end{matrix}\right\}$$
$$+fk\cos(2\theta-2\pi+r)\,\left[-1{,}159358\,T' + 0{,}33856\right]$$

§. 259. Hinc tandem valores quaesiti eliciuntur

$J = -0{,}81144$ . . . $l\text{-}J = 9{,}909256$

$M = -1{,}25325$ . . . $l\text{-}M = 0{,}098046$

$O = -2{,}12630$ . . . $l\text{-}O = 0{,}327624$

$S = -0{,}13490$ . . . $l\text{-}S = 9{,}130012$

$T = -0{,}10080$ . . . $l\text{-}T = 9{,}003441$

ac $\mathfrak{J} = -0,00005$ . . . $l-\mathfrak{J} = 5,698970$
$\mathfrak{M} = +0,00177$ . . . $l\,\mathfrak{M} = 7,247973$
$\mathfrak{O} = +0,52266$ . . . $l\,\mathfrak{O} = 9,718219$
$\mathfrak{S} = +0,02355$ . . . $l\,\mathfrak{S} = 8,371991$
$\mathfrak{T} = +0,01076$ . . . $l\,\mathfrak{T} = 8,031812$

§. 260. Ex his iam pro distantia Lunae a terra erit

$u =$ Praec.

		Log. coeff.	Valor. coeff. totius
$-0,00462fk$	$\cos 2\eta$	7,664440	$-0,000275$
$-0,00773fk$	$\cos(2\Phi-2\pi-r)$	7,853230	$-0,000424$
$-0,01210fkk$	$\cos(2\Phi-2\pi-2r)$	8,082808	$-0,000039$
$-0,00077fk$	$\cos(2\theta-2\pi-r)$	6,885196	$-0,000046$
$-0,00057fk$	$\cos(2\theta-2\pi+r)$	6,758625	$-0,000034$

et pro motu momentaneo

$$\frac{d\Phi}{dr} = \text{Praec.}$$

		Log. coeff.	Val. coeff.
$+0,00932fk$	$\cos r$	7,969416	$+0,000555$
$+0,01558fk$	$\cos(2\Phi-2\pi-r)$	8,192568	$+0,000928$
$+0,02142fkk$	$\cos(2\Phi-2\pi-2r)$	8,330819	$+0,000069$
$+0,00142fk$	$\cos(2\theta-2\pi-r)$	7,152288	$+0,000085$
$+0,00109fk$	$\cos 2\theta-2\pi+r)$	7,037426	$+0,000064$

§. 261. Pro longitudine autem lunae sequentes resolui debent aequationes:

$$+0,00932 = \mathfrak{J}' - 0,01766(\mathfrak{G}' + \mathfrak{H}') - 0,004241(\mathfrak{M}' + \mathfrak{G}' + \mathfrak{T}')$$

$$+0,01558 = 1,026834\,\mathfrak{M}' - \mathfrak{A}'s' + 0,01766\,\mathfrak{G}' + 0,00996\,\mathfrak{H}'$$
$$+0,004221\,\mathfrak{G}'$$

$$+0{,}02142 = 0{,}026834\,\mathfrak{O}' - \mathfrak{D}'s' + 2{,}6b' + 0{,}02127\,\mathfrak{G}' + 0{,}01064\,\mathfrak{H}' + 0{,}01766\,\mathfrak{M}' + 0{,}00996\,\mathfrak{S}'$$

$$+\delta{,}00142 = -0{,}840642\,\mathfrak{S}' - \mathfrak{A}'m' - \mathfrak{D}'b' - \mathfrak{E}'g' + 0{,}02114\,\mathfrak{G}' + 0{,}16853\,\mathfrak{H}' + 0{,}02396\,\mathfrak{M}'$$

$$+0{,}00109 = 1{,}159358\,\mathfrak{T}' - \mathfrak{D}'g' - \mathfrak{E}'b' - 0{,}35615\,\mathfrak{G}' + 0{,}16850\,\mathfrak{H}'$$

hincque prodit pro longitudine vera:

			Log. coeff.	Val. coeff. in min. sec.
$\Phi =$ Pr.	$+0{,}00932fk$	$\sin r$	7,969416	+115
	$+0{,}01521fk$	$\sin(2\Phi-2\pi-r)$	8,182130	+187
	$+0{,}79079fkk$	$\sin(2\Phi-2\pi-2r)$	9,898060	+529
	$-0{,}00121fk$	$\sin(2\theta-2\pi-r)$	7,083939	− 15
	$-0{,}00082fk$	$\sin(2\theta-2\pi+r)$	6,913527	− 10

§. 262. Ob inclinationem ergo orbitae lunaris ad eclipticam omnes correctiones huc redeunt, vt sit

I. Pro distantia lunae a terra:

$$u = \text{Praec.}$$

		Log. coeff.	Val. coeff.
$+0{,}000084f$	$\cos 2\eta$	5,926350	+0,000092
$+0{,}001832f$	$\cos(2\Phi-2\pi)$	7,262918	+0,002004
$+0{,}000011f$	$\cos(2\theta-2\pi)$	5,034111	+0,000012
$+0{,}000001f$	$\cos(4\theta-4\pi)$	4,011565	+0,000001
$-0{,}00462fk$	$\cos r$	7,664440	−0,000275
$-0{,}00773fk$	$\cos(2\Phi-2\pi-r)$	7,853230	−0,000424
$-0{,}01210fkk$	$\cos(2\Phi-2\pi-2r)$	8,082808	−0,000039
$-0{,}00077fk$	$\cos(2\theta-2\pi-r)$	6,885196	−0,000046
$-0{,}00057fk$	$\cos(2\theta-2\pi+r)$	1,758625	−0,000034

II. Pro

II. Pro motu momentaneo:

$\frac{d\Phi}{dr}$ = Praec.

		Log. coeff.	Val. coeff.
− 0,000169*f*	cos 2η	6,227887	− 0,000185
− 0,003697*f*	cos (2Φ−2π)	7,567849	− 0,004043
− 0,000227*f*	cos (2θ−2π)	6,356026	− 0,000248
− 0,000005*f*	cos (4θ−4π)	4,698970	− 0,000005
+ 0,00932*fk*	cos *r*	7,969416	+ 0,000555
+ 0,01558*fk*	cos (2Φ−2π−*r*)	8,192568	+ 0,000928
+ 0,02142*fkk*	cos (2Φ−2π−2*r*)	8,330819	+ 0,000069
+ 0,00142*fk*	cos (2θ−2π−*r*)	7,152288	+ 0,000085
+ 0,00109*fk*	cos (2θ−2π+*r*)	7,037426	+ 0,000064

III. Pro longitudine lunae vera

		Log. coeff.	Val. coeff. in min. sec.
Φ = Pr. − 0,000096*f*	sin 2η	5,984018	− 22″
− 0,001823*f*	sin (2Φ−2π)	7,260310	− 411
− 0,000910*f*	sin (2θ−2π)	6,959131	− 205
− 0,000028*f*	sin (4θ−4π−*r*)	5,450835	− 6
+ 0,00932*fk*	sin *r*	7,969416	+ 115
+ 0,01521*fk*	sin (2θ−2π−*r*)	8,182130	+ 187
+ 0,79079*fkk*	sin (2Φ−2π−2*r*)	9,898060	+ 529
− 0,00121*fk*	sin (2θ−2π−*r*)	7,083939	− 15
− 0,00082*fk*	sin (2θ−2π+*r*)	6,913527	− 10

CAPUT

CAPUT XVI.

EXPOSITIO INAEQUALITATUM LUNAE HACTENUS INUENTARUM.

§. 263.

Quas igitur inuenimus hactenus lunae inaequalitates eae primum, si originem earum spectemus, ad sex classes reducuntur. Quatenus enim luna in motu suo a regulis Keplerianis, in quibus quidem motum apogei compectimur, recedit, eius errores vel primo a solo lunae aspectu, seu eius distantia a sole pendent, seu quod eodem redit, per angulum η tantum definiuntur, quibus variatio lunae continetur. Ad secundam classem refero eas lunae inaequalitates, quae insuper ab excentricitate eius orbitae pendent. Tertia classis eas complectitur inaequalitates, quae ab excentricitate orbitae solis ortum trahunt. Quarta vero eas, quae per vtramque excentricitatem coniunctim determinantur. Quintae porro classi annumeramus eas inaequalitates, quae parallaxin solis inuoluunt, atque errores quatuor ante memoratorum generum implicant. Sexta denique classis suppeditat eas inaequalitates, quae praeterea ab inclinatione orbitae lunaris ad eclipticam pendent.

§. 264. Quodsi vero ad vsum harum inaequalitatum attendamus, prouti eae ad lunam accommodari debent, tum eae in quinque classes commodissime distribuuntur. Primo enim perpendendae sunt eae inaequaliqualitates, quarum ope vera diſtantia lunae a terra determinatur, vt inde porro tam lunae diameter apparens, quam eius parallaxis horizontalis aſſignari poſſit. Secundo loco formulae erunt collocandae illae, quae motui momentaneo definiendo inſeruiunt, ex quibus deinceps motus lunae horarius accurate exhiberi poterit. Tertium locum occupabunt eae inaequalitates, quae veram longitudinem lunae ad eclipticam relatam praebent. Quarto vero poſitio lineae nodorum lunae, ſeu longitudo nodi aſcendentis; ac quinto vera inclinatio orbitae lunaris ad eclipticam inueniri debebit; vt deinde vera lunae latitudo concludi poſſit. Manifeſtum autem eſt, has inaequalitates plurimum inter ſe permiſceri, ita vt vix vllum habeatur genus, cuius inaequalitates non a reliquis generibus pendeant; cui tamen incommodo facile medela adhibetur.

§. 265. Quanquam numerus inaequalitatum, quas ſumus conſecuti, tantopere increuit, vt calculus ſine maxima moleſtia expediri nequeat, tamen iam monui, non omnes inaequalitates, quibus motus Lunae perturbatur, eſſe definitas, ſed potius earum numerum omnino eſſe infinitum. Facile quidem intelligitur, plerasque has praetermiſſas inaequalitates nullius fere eſſe momenti, atque ſine notabili errore iis ſuperſederi poſſe: verum tamen ſunt inter eas nonnullae, quae ad plura minuta ſecunda aſſurgere videntur, quarum argumenta ſupra iam innui; ex quo omnino operae eſſet pretium in eas omni cura inquirere. Sed earum inueſtigatio tam eſt lubrica et incerta, vt leuiſſima omiſſio in calculo facta eas maxime afficiat. Cum igitur in calculo plurimos terminos reiicere cogamur, istam inuestigationem frustra plane susciperemus, quamdiu scilicet rem sine appropinquatione exequi non licet. Cuius defectus eximium habemus exemplum in inaequalitatibus postremo loco inuentis, quae statim atque in negligendo minus largi fueramus, mirum quantum prodierunt immutatae; ac nullum plane est dubium, si calculum adhuc accuratius prosequi liceret, quin valores inuenti notabilem insuper mutationem sint subiturae. Imprimis autem aequatio ab angulo $2\Phi - 2\pi - 2r$ seu a dupla distantia apogei a nodo pendens, est suspecta, ac minime pro certa haberi potest, cum leuissima circumstantia eam magnopere perturbare valeat.

§. 266. Si enim in causam inquiramus, cur analysis posterior tam diuersos valores pro his inaequalitatibus suppeditauerit, primo quidem statim patet, neglectum litterarum germanicarum $\mathfrak{J}$, $\mathfrak{M}$, $\mathfrak{O}$, etc. in calculo priori potissimum hoc discrimen produxisse: ingens enim valor litterae $\mathfrak{O}$ imprimis aequationem ab angulo $2\Phi - 2\pi - 2r$ pendentem tantopere auxit. Praeterea vero etiam non parum augmenti haec aequatio inde est nacta, quod in calculo posteriori rationem quoque habuimus termini $\cos(2\eta - 2r)$, qui tam in valore $\frac{d\Phi}{dr}$ quam $\frac{d\eta}{dr}$ inesse est deprehensus; vnde tuto colligere licet, si alios quoque terminos similes veluti $2\theta - 2\pi - 2r$, etsi per se sunt minimi, in calculum introduxissemus, coefficienefficientes terminorum $2\Phi-2\pi-2r$ non mediocrem inde mutationem subituros fuisse. Quamobrem plus hinc colligere non possumus, nisi inaequalitatem Lunae ab angulo hoc $2\Phi-2\pi-2r$ pendentem minime esse contemnendam, etiamsi fortasse tanta non sit, quam inuenimus. Vera autem eius quantitas certius ex obseruationibus quam ex Theoria colligi posse videtur.

§. 267. Quoniam vero hae inaequalitates omnes ad anomaliam Lunae veram referuntur, antequam eas ad vsum adhibere liceat, modum tradi conveniet ad quodvis tempus propositum anomaliam Lunae veram determinandi. Cognita autem excentricitate orbitae lunaris k et motu anomaliae mediae, inde ad quodvis tempus facile anomalia media p colligitur. Verum ex anomalia media p et excentricitate k anomalia vera r definiri debet ope huius aequationis $dp = \frac{(1-kk)^{\frac{3}{2}}\, dr}{(1-k\cos r)^2}$; vnde quidem non difficulter, si nota esset anomalia vera r, viciffim inveniri posset anomalia media p. Calculo enim peracto, si breuitatis gratia ponatur

$$\delta = \frac{1-\sqrt{(1-kk)}}{k} = \tfrac{1}{2}k + \frac{1.1}{2.4}k^3 + \frac{1.1.3}{2.4.6}k^5 + \text{etc.}$$

reperitur:

$$p = r + 2k\sin r + 2\delta(k-\tfrac{1}{2}\delta)\sin 2r + 2\delta\delta(k-\tfrac{2}{3}\delta)\sin 3r$$
$$+ 2\delta^3(k-\tfrac{3}{4}\delta)\sin 4r \text{ etc.}$$

cuius seriei progressio primo intuitu patet. Cum autem k ac proinde δ sit valde parvum, erit satis exacte:

 $p =$

$$p = r + 2k\,\text{ſin}\,r + (\tfrac{3}{4}kk + \tfrac{1}{8}k^4 + \tfrac{3}{64}k^6)\,\text{ſin}\,2r + (\tfrac{1}{3}k^3 + \tfrac{1}{8}k^5)\,\text{ſin}\,3r$$
$$+ (\tfrac{5}{32}k^4 + \tfrac{3}{32}k^6)\,\text{ſin}\,4r + \tfrac{3}{40}k^5\,\text{ſin}\,5r + \tfrac{7}{192}k^6\,\text{ſin}\,6r$$

§. 268. Data ergo anomalia media Lunae p, eius anomalia vera r elici debebit ex hac aequatione

$$r = p - 2k\,\text{ſin}\,r - (\tfrac{3}{4}kk + \tfrac{1}{3}k^4 + \tfrac{3}{64}k^6)\,\text{ſin}\,2r - (\tfrac{1}{3}k^3 + \tfrac{1}{8}k^5)\,\text{ſin}\,3r$$
$$- (\tfrac{5}{32}k^4 + \tfrac{3}{32}k^6)\,\text{ſin}\,4r - \tfrac{3}{40}k^5\,\text{ſin}\,5r - \tfrac{7}{192}k^6\,\text{ſin}\,6r$$

cuius quidem ope, ſi cognita fuerit excentricitas k, calculus non difficulter expedietur. Quoniam enim termini ſinus inuoluentes ſunt admodum parui, in iis ſtatim poni poterit $r = p$, vnde valor verior pro r eruetur, qui deinde iterum in his terminis adhibitus, iuſtiorem valorem pro r ſuppeditabit. Atque hoc modo poſt aliquot operationes verus tandem valor pro anomalia vera r obtinebitur. Interim tamen, quo iſte calculus facilius perfici queat, aequatio haec ita poteſt transformari, vt loco ſinuum anomaliae verae r, ſinus anomaliae mediae p introducantur; id quod ſequenti modo praeſtabitur.

§. 269 Ponatur breuitatis gratia:

$$\tfrac{3}{4} + \tfrac{1}{8}kk + \tfrac{3}{64}k^4 = \alpha;\quad \tfrac{1}{3} + \tfrac{1}{8}kk = \beta;\quad \tfrac{5}{32} + \tfrac{3}{32}kk = \gamma$$
$$\tfrac{3}{40} = \delta \quad \text{et} \quad \tfrac{7}{192} = \varepsilon$$

vt ſit:

$$r = p - 2k\,\text{ſin}\,r - \alpha kk\,\text{ſin}\,2r - \beta k^3\,\text{ſin}\,3r - \gamma k^4\,\text{ſin}\,4r - \delta k^5\,\text{ſin}\,5r - \varepsilon k^6\,\text{ſin}\,6r$$

ac ponatur denuo

$$2k\,\text{ſin}\,r + \alpha kk\,\text{ſin}\,2r + \beta k^3\,\text{ſin}\,3r + \gamma k^4\,\text{ſin}\,4r + \delta k^5\,\text{ſin}\,5r + \varepsilon k^6\,\text{ſin}\,6r = R$$

et

et cum sit $r = p - R$ habebitur:

$$\sin r = (1 - \tfrac{1}{2}RR + \tfrac{1}{24}R^4)\sin p - (R - \tfrac{1}{6}R^3 + \tfrac{1}{120}R^5)\cos p$$

$$\sin 2r = (1 - 2RR + \tfrac{2}{3}R^4)\sin 2p - (2R - \tfrac{4}{3}R^3)\cos 2p$$

$$\sin 3r = (1 - \tfrac{9}{2}RR)\sin 3p - (3R - \tfrac{9}{2}R^3)\cos 3p$$

$$\sin 4r = (1 - 8RR)\sin 4p - 4R\cos 4p$$

$$\sin 5r = \sin 5p - 5R\cos 5p$$

$$\sin 6r = \sin 6p$$

negligendo scilicet terminos, qui ipsius k potestates sexta altiores continent.

§. 270. Euolutio autem huius calculi fit maxime prolixa, si quidem ad sextam potestatem ipsius k ascendere velimus. Facile autem expedietur, si ad quartam subsistamus, tum autem reperietur:

$$r = p - (2k - \tfrac{1}{4}k^3)\sin p + (\tfrac{5}{4}kk - \tfrac{11}{24}k^4)\sin 2p - \tfrac{13}{12}k^3\sin 3p - \tfrac{103}{96}k^4\sin 4p$$

quae expressio satis accurate pro quauis anomalia media p conuenientem anomaliam veram indicabit. Calculus autem, si excentricitas k constet, facili negotio absoluetur. Formula haec quoque ita repraesentari poterit, vt sit

$$r = p - 2k(1 - \tfrac{1}{8}kk)\sin p + \tfrac{5}{4}kk(1 - \tfrac{11}{30}kk)\sin 2p - \tfrac{13}{12}k^3\sin 3p - \tfrac{103}{96}k^4\sin 4p$$

Hinc igitur tabulam construi conueniet, vnde pro quauis anomalia media proposita ipsi respondens anomalia vera excerpi queat.

§. 271. Cum autem inuenta fuerit anomalia vera r, longitudo Lunae regula Kepleriana inuenienda, quam

 supra

ſupra (206) per ζ indicauimus, ita exprimetur, vt ſit

$$\zeta = C + 1{,}0085272\,p$$

$$-1{,}0085272\left(\begin{array}{l}2k\sin r + \frac{3}{4}kk(1+\frac{5}{6}kk+\frac{1}{16}k^4)\sin 2r + \frac{1}{3}k^3(1+\frac{3}{8}kk)\sin 3r \\ \quad + \frac{5}{32}k^4(1+\frac{3}{5}kk)\sin 4r + \frac{3}{40}k^5\sin 5r + \frac{7}{192}k\sin 6r\end{array}\right)$$

vbi $C + 1{,}0085272\,p$ exhibet longitudinem Lunae mediam; quae ſi vocetur $= \xi$, atque in coefficientium partibus minimis pro k ſcribatur valor proximus 0,0545, erit

	log. coeff.	val. in min. ſec.
$\zeta = \xi - 2{,}0170544\,k\ \sin r$	0,304718	$22675'' = 6^\circ, 17', 55''$
$- 0{,}756770\,kk\ \sin 2r$	9,878964	$464 = 7, 44$
$- 0{,}336551\,k^3\ \sin 3r$	9,527051	$11\frac{1}{2}$
$- 0{,}15786\,k^4\ \sin 4r$	9,198282	$\frac{1}{5}$

vnde patet ſuperfluum futurum fuiſſe, ſi ſuperiores expreſſiones vltra quartam poteſtatem ipſius k extendere voluiſſemus.

CAPUT

CAPUT XVII.

INUESTIGATIO ELEMENTORUM MOTUS LUNAE.

§. 272.

Inuentis iam per Theoriam hisce inaequalitatibus, quibus motus Lunae perturbatur, antequam eas ad computum astronomicum accommodare liceat, elementa, quae in eas ingrediuntur, per obseruationes determinari oportet. Primo scilicet ad datam epocham cum longitudo Lunae media, tum eius anomalia media, ac locus nodi medius constitui debebit, vt eaedem res inde ad quoduis aliud tempus assignari queant. Deinde quoque ex obseruationibus verus valor excentricitatis lunaris colligi debet, a quo potissimum quantitas praecipuarum inaequalitatum pendet. Excentricitas autem orbitae solaris pro satis certa haberi poterit, cum sit $e = 0,0168$. Lunae vero excentricitas tam prope iam constat, vt inde sine errore ad quamlibet anomaliam mediam vera satis exacte assignari possit. Etsi enim in anomalia vera error aliquot minutorum primorum committitur, inaequalitates Lunae inde non vltra aliquot minuta secunda afficiuntur.

§. 273. Quodsi autem statim quasuis Lunae obseruationes ad hunc finem adhibere velimus, ob tam ingentem inaequalitatum numerum, inuestigatio elementorum maxime molesta redderetur. Quocirca ex obser-

obſeruationibus eas eligi conueniet, pro quibus numerus inaequalitatum multo fiat minor; dum ſcilicet diſtantia Lunae a ſole ſeu angulus η datum obtinet valorem. Commodiſſimae ergo erunt eae obſeruationes, quae in ipſis momentis coniunctionis vel oppoſitionis ſunt inſtitutae. Accuratas itaque obſeruationes eclipſium lunarium ad hoc negotium adhibebo, quoniam praeter haec tempora, vera vel coniunctionis vel oppoſitionis momenta non ſatis certo ex obſeruationibus colligi licet.

§. 274. Momento autem oppoſitionis verae Lunae et Solis, longitudo Lunae ſex ſignis diſtat a longitudine ſolis, ita vt ſit $\theta = \Phi \pm 180^\circ$, ideoque angulus $\eta = 180^\circ$. Poſito autem pro η hoc valore longitudo Lunae vera Φ ex media ξ per ſequentes formulas definietur, in quas formulae hactenus inuentae abeunt:

$$\begin{array}{llll}
\Phi = \xi & -2{,}0170544\,k\sin r & -0{,}756770\,kk\sin 2r & -0{,}33655\,k^3\sin 3r \\
& +0{,}0101460 & +0{,}004200 & \\
& +0{,}4202260 & -0{,}573280 & \\
& +0{,}0049920 & +0{,}003180 & \\
& -0{,}0052860 & +0{,}150830 & \\
& -0{,}000860 & -0{,}000002 & \\
& +1{,}1959\,\nu & & \\
& -0{,}0757\,\nu & & \\
& +0{,}00932\,f & &
\end{array}$$

$$\begin{array}{l}
+(0{,}201385+0{,}021889-0{,}016368-0{,}3959\nu+4{,}1738\nu)\,e\sin s \\
+(0{,}06645-0{,}02332+0{,}00840)\,ee\sin 2s \\
+(0{,}74760-0{,}81430-0{,}01420)\,ek\sin(r-s) \\
+(-0{,}61850-0{,}23960-0{,}00610)\,ek\sin(r+s)
\end{array}$$

$$-(0{,}002823 + 0{,}000910) f \sin(2\Phi - 2\pi)$$
$$-0{,}000028 f \sin(4\Phi - 4\pi)$$
$$+(0{,}01521 - 0{,}00121) fk \sin(2\Phi - 2\pi - r)$$
$$+0{,}79079 fkk \sin(2\Phi - 2\pi - 2r)$$

§. 275. Cum igitur sit $f = 1{,}09375$, et $\nu = \frac{1}{11}$, erit has formulas colligendo:

$$\Phi = \xi - 1{,}572993 k \sin r - 1{,}17186 kk \sin 2r - 0{,}3365 k^3 \sin 3r$$
$$+0{,}21998 e \sin s + 0{,}05123 ee \sin 2s - 0{,}0809 ek \sin(r-s)$$
$$-0{,}8642 ek \sin(r+s)$$
$$-0{,}002989 \sin(2\Phi - 2\pi) + 0{,}01531 k \sin(2\Phi - 2\pi - r)$$
$$-0{,}000031 \sin(4\Phi - 4\pi)$$
$$+0{,}86493 kk \sin(2\Phi - 2\pi - 2r)$$

et cum sit $e = 0{,}0168$, erit hoc valore substituto:

$$\Phi = \xi - 1{,}572993 k \sin r - 1{,}17186 kk \sin 2r - 0{,}3365 k^3 \sin 3r$$
$$+0{,}003697 \sin s + 0{,}000014 \sin 2s - 0{,}001359 k \sin(r-s)$$
$$-0{,}014523 k \sin(r+s)$$
$$-0{,}002989 \sin(2\Phi - 2\pi) + 0{,}01531 k \sin(2\Phi - 2\pi - r)$$
$$-0{,}000031 \sin(2\Phi - 4\pi)$$
$$+0{,}86493 kk \sin(2\Phi - 2\pi - 2r)$$

§. 276. Assumta iam hypothesi quapiam non nimis a vero aberrante, vnde ad datum quoduis tempus definiri possit tam longitudo lunae, quam eius anomalia media, ex qua praeterea ope excentricitatis proxime cognitae anomalia vera assignari queat: haec elementa correctione indigebunt, quam ex obseruationibus elici oporteat. Ponamus ergo longitudinem mediam ex tabulis desumtam augeri debere m minutis secundis. Tum vero excentricitas supposita, quae sit $= 0,0545$, augeri debeat $\frac{n}{10000}$, vt sit $k = 0,0545 + \frac{n}{10000}$: ipsa vero anomalia vera tabularis, quae sit $= v$, augmentum requirat μ minutorum secundorum, vt sit $r = v + \mu''$: eritque $\sin r = \sin v + \mu'' \cos v$; $\sin 2r = \sin 2v + 2\mu'' \cos 2v$; et $\sin 3r = \sin 3v$ in terminis enim minimis haec correctio praetermitti poterit.

§. 277. Quod si haec omnia in minuta secunda conuertantur, prodibit longitudo lunae vera

$$\Phi = \text{Long. med.} + m''$$

$$- 17682'' \sin v - 32,445\, n'' \sin v - 0,085728\, \mu'' \cos v$$

$$- 718'' \sin 2v - 2,635\, n'' \sin 2v - 0,006962\, \mu'' \cos 2v$$

$$- 11'' \sin 3v + 762'' \sin s + 3'' \sin 2s$$

$$- 15'' \sin (r-s) - 163'' \sin (r+s)$$

$$- 616'' \sin (2\Phi - 2\pi) + 172'' \sin (2\Phi - 2\pi - r)$$

$$- 6'' \sin (4\Phi - 4\pi) + 530 \sin (2\Phi - 2\pi - 2r)$$

Cum

Cum autem postremus terminus sit suspectus, loco eius coefficientis 530 malumus ponere coefficientem indefinitum $100y$, atque ex obseruationibus valorem ipsius y indagare. Deinde sit error anomaliae verae i minutorum primorum, vt calculus commodior reddatur, atque ob $\mu = 60i$, neglectis terminis minimis erit:

$$\Phi = \text{Long. med.} + m''$$
$$-17682'' \sin v - 32,445\, n'' \sin v - 5,143\, i'' \cos v$$
$$- 718'' \sin 2v - 2,635\, n \sin 2v - 0,417\, i \cos 2v$$
$$+ 762 \sin s - 15 \sin (r-s) - 163 \sin (r+s)$$
$$- 616 \sin (2\Phi - 2\pi) + 172'' \sin (2\Phi - 2\pi - r)$$
$$+ 100y \sin (2\Phi - 2\pi - 2r)$$

§. 278. Oblata autem obseruatione eclipsis lunae, quaeratur primum momentum medium huius eclipsis, pro quo colligatur longitudo solis, itemque longitudo nodi ascendentis. Punctum autem soli oppositum nondum erit longitudo lunae vera in ecliptica; verumtamen longitudo lunae pro hoc momento eclipsis medio inueniri poterit ope sequentis tabellae.

Subtrahatur longitudo nodi a longitudine solis, et aequatio tabulae secundum titulos adscriptos applicetur puncto soli opposito in ecliptica.

	{ O Sign. VI. Sign. } subtrahe	
gr.		
0	0′, 0″	30
1	0, 32	29
2	1, 6	28
3	1, 39	27
4	2, 12	26
5	2, 45	25
6	3, 17	24
7	3, 49	23
8	4, 21	22
9	4, 53	21
10	5, 24	20
11	5, 56	19
12	6, 26	18
	adde { V Sign. XI. Sign. }	gr.

§. 279. Quanquam autem hoc momento, ad quod lunae longitudinem hinc colligimus, non vera lunae oppositio existit, sed luna secundum longitudinem a puncto soli opposito distat particula, quam haec tabula monstrat; tamen tuto pro hoc momento ex formula nostra longitudinem lunae inuestigare poterimus, visuri, quam

quam exacte ea conueniat cum longitudine eius ad hoc tempus ex obseruatione conclusa. Cum enim luna hoc tempore nunquam vltra 5′ a vero oppositionis loco distet, si formula nostra generali vti vellemus, foret angulus η minor 5 minutis primis; vnde facile perspicitur, discrimen in loco lunae inde oriundum vix vnquam 12″ esse superaturum. Quoniam itaque medium cuiusque eclipsis momentum ipsum tam accurate definiri nequit, vt non error dimidii minuti primi sit pertimescendus, superfluum sane foret in calculo ad istiusmodi minutias attendere.

§. 280. Hanc ob causam quoque ex calculo, quem inibo, non summam praecisionem expectari conueniet; quia ipsae obseruationes, quibus vtar, non plenae accurationis sunt capaces. Plus igitur me non effecturum confido, quam vt satis prope tam excentricitatem orbitae lunaris, quam longitudinem et anomaliam lunae mediam ad datam epocham definiam. Quod cum fuerit factum maiori confidentia theoriam ad quasuis alias obseruationes transferre licebit; quae si nullis erroribus fuerint inquinatae, non admodum erit difficile reliquas elementorum correctiones, quibus formulae nostrae sunt innixae, inde concludere. Imprimis autem hic calculus veram excentricitatem orbitae lunaris satis exacte manifestabit, vt deinceps accuratius pro quauis anomalia media conuenientem anomaliam veram definire valeamus. Hunc igitur in finem nonnullas eclipses lunares Parisiis institutas calculo subiiciam.

§. 281. Primae igitur eclipsis medium contigisse reperio Parisiis A. 1712. Jan. $23^d, 7^h, 55', 16''$ temp. medio. Pro quo momento colligitur:

Longitudo solis θ	$10^s, 3^\circ, 0', 54''$
Anomalia vera solis s . . .	6, 24, 25, 13
Deinde ex tabulis meis -	
Longitudo lunae media . .	4, 7, 18, 55
Anomalia lunae media .	2, 0, 18, 20
Anomalia lunae vera v =	1, 25, 6, 27
Longitudo nodi vera π =	9, 24, 34, 32
Dist. nodi a sole $\theta - \pi$ =	0, 8, 26, 22
Hinc aequatio loci lunae .	—. — 4, 33
Ergo longitudo lunae vera Φ =	4, 2, 56, 21

§. 282. Hinc calculus sequenti modo instituetur:

v =	1, 25, 6, 27	;	$\sin v = + \sin 55^\circ, 6', 27''$ $\cos v = +$
$2v$ =	3, 20, 12, 54	;	$\sin 2v = + \sin 69, 47, 6$ $\cos 2v = -$
s =	6, 24, 25, 13	;	$\sin s = - \sin 24, 25, 13$
$v - s$ =	7, 0, 41, 14	;	$\sin = - \sin 30, 41, 14$
$v + s$ =	8, 19, 31, 40	;	$\sin = - \sin 79, 31, 40$
$\Phi - \pi$ =	6, 8, 21, 49		
$2\Phi - 2\pi$ =	0, 16, 43, 38	;	$\sin = + \sin 16, 43, 38$
r =	1, 25, 6, 27		
$2\Phi - 2\pi - r$ =	10, 21, 37, 11	;	$\sin = - \sin 38, 22, 49$
$2\Phi - 2\pi - 2r$ =	8, 26, 30, 44	;	$\sin = - \sin 86, 30, 44$

+

+ 9,91393	+ 9,9139	+ 9,7575
— 4,24753	− 1,5111	− 0,7104
— 4,16146	− 1,4250n	− 0,4679i
+ 9,9724	+ 9,9724	− 9,5385
— 2,8561	− 0,4208	− 9,6201
— 2,8285	− 0,3932n	+ 9,1586i
— 9,6163	− 9,7078	− 9,9927
+ 2,8819	− 1,1761	− 2,2122
— 2,4982	+ 0,8839	+ 2,2049
+ 9,4588	− 9,7930	
— 2,7896	+ 2,2355	− 99,8y
— 2,2484	− 2,0285	

aeq. +	aeq. —	− 26,6n
+ 8	− 14493	− 2,5n
+ 160	− 674	− 2,9i
+ 168	− 315	+ 0,1i
— 15766	− 177	− 99,8y
— 15598	− 107	
— 259′, 58″	− 15766	
— 4°, 19, 58 . . aequatio		

Long. med. 4,7,18,55 + m
aeq. − 4,19,58

4,2,58,57 − 29,1n − 2,8i − 99,8y = 4,2,56,21
4,2,56,21

Ergo o = 2,36 − 29,1n − 2,8i − 99,8y + m

§. 283.

§. 283. Secundae eclipſi medium contigit: Pariſiis A. 1713. Dec. $1^d, 15^h, 26', 34''$ temp. med. Pro quo momento colligitur

Longitudo Solis $\theta =$.	8, 9, 53, 40
Anomalia vera Solis $s =$.	5, 1, 46, 43
Longitudo Lunae media .	2, 5, 2, 26
Anomalia Lunae media .	9, 12, 27, 42
Anomalia Lunae vera $v =$.	9, 18, 24, 49
Longitudo nodi $\pi =$.	8, 17, 46, 10
Diſtantia Solis a nodo .	11, 22, 7, 30
Aequatio loci Lunae .	+ 4, 17
Longitudo Lunae vera $\Phi =$	2, 9, 57, 57

§. 284. Hinc calculus ſequens inſtituatur:

v	= 9, 18, 24, 49	; ſin v = − ſin 71, 35, 11
		coſ v = +
$2v$	= 7, 6, 49	; ſin $2v$ = − ſin 36, 49
		coſ $2v$ = −
s	= 5, 1, 46	; ſin s = + ſin 28, 14
$v - s$	= 4, 16, 39	; ſin = + ſin 43, 21
$v + s$	= 2, 20, 11	; ſin = + ſin 80, 11
$\Phi - \pi$	= 11, 22, 12	
$2\Phi - 2\pi$	= 11, 14, 24	; ſin = − ſin 15, 36
r	= 9, 18, 25	
$2\Phi - 2\pi - r$	= 1, 25, 59	; ſin = + ſin 55, 59
$2\Phi - 2\pi - 2r$	= 4, 7, 34	; ſin = + ſin 52, 26

− 9,97717	− 9,9772	+ 9,4996
− 4,24753	− 1,5111	− 0,7104
+ 4,22470	+ 1,4883 *n*	− 0,2100 *i*

− 9,7776	− 9,7776	− 9,9034
− 2,8561	− 0,4208	− 9,6201
+ 2,6337	+ 0,1984 *n*	+ 9,5235 *i*

+ 9,6749	+ 9,8366	+ 9,9936
+ 2,8819	− 1,1761	− 2,2122
+ 2,5568	− 1,0127	− 2,2058

− 9,4296	+ 9,9185	+ 79, 2 *y*
− 2,7866	+ 2,2355	
+ 2,2192	+ 2,1530	

aeq. aff.	aequat.	
+ 16776	− 10	+ 30, 8 *n*
+ 430	− 161	+ 1, 6 *n*
+ 360	− 171	− 1, 6 *i*
+ 166		+ 0, 3 *i*
+ 143		
+ 17875		
− 171		
+ 17704		
+ 2961′,4″	aequatio	

Long. media =	2, 5, 2, 26	+ *m*
aeq.	± 4, 55, 4	
Long. vera	2, 9, 57, 30,	+ *m*
obs.	2, 9, 57, 57	

Ergo $e = -0',27'' + m + 32,4n - 1,3i + 79,2y$

§. 285. Tertiae eclipsis medium contigit Parisiis A. 1717 Mart. $26^d, 15^h, 21', 20''$ temp. med. Pro quo tempore colligitur

Longitudo solis vera $\theta =$	$0^s, 6°, 19', 56''$,
Anomalia solis vera $s =$	8, 28, 0, 17
Longitudo media lunae . .	6, 1, 37, 2
Anomalia media lunae . .	8, 24, 7, 21
Anomalia lunae vera $v =$	9, 0, 19, 10
Longitudo nodi vera $\pi =$	6, 13, 30, 22
Distantia nodi a sole . .	5, 22, 49, 29
aeq. pro loco lunae	+ 3, 57
Ergo longitudo lunae vera $\phi =$	6, 6, 23, 53

§. 286. Calculus igitur ita se habebit

$v =$	9, 0, 19, 10	; sin $v = -$ sin $89°, 40', 50''$
		. cos. = +
$2v =$	6, 0, 38,	; sin $2v = -$ sin $0°, 38'$
		cos = −
$s =$	8, 28, 0	; sin $s = -$ sin $88°, 0'$
$v - s =$	0, 2, 19	; sin = + sin $2°, 19$
$v + s =$	5, 28, 19	; sin = + sin 1, 41
$\phi - \pi =$	5, 22, 53	
$2\phi - 2\pi =$	11, 15, 46	; sin = − sin 14, 14
$r =$	9, 0, 19	
$2\phi - 2\pi - r =$	2, 15, 27	; sin = + sin 75, 27
$2\phi - 2\pi - 2r =$	5, 15, 8	; sin = + sin 14, 52

— 9,99999	— 10,0000	+ 7,7425
— 4,24753	— 1,5111	— 0,7104
+ 4,24752	+ 1,5111*n*	— 8,4526*i*
— 8,0435	— 8,0435	— 9,9999
— 2,8561	— 0,4208	— 9,6201
+ 0,8996	+ 8,4643*n*	+ 9,6200*i*
— 9,9997	+ 8,6066	+ 8,4680
+ 2,8819	— 1,1761	— 2,2122
— 2,8816	— 9,7827	— 0,6802
— 9,3907	+ 9,9858	+ 25,65*y*
— 2,7893	+ 2,2355	
+ 2,1893	+ 2,2213	

aeq. aff.	aeq. neq.	
+ 17682	— 762	+ 32, 4*n*
+ 8	— 1	+ 0, 0*n*
+ 152	— 4	— 0, 0*i*
+ 166	— 767	+ 0, 2*i*
+ 18008	+ 18008	
	+ 17241	
	+ 287,21	
	+ 4,47,21 aequatio	

Long. media ☾ = 6, 1, 37, 2
aeq. + 4, 47, 21

Long. ☽ vera = 6, 6, 24, 23
obs. 6, 6, 23, 53

Ergo $o = + 30 + m + 32,4n + 0,2i + 25,65y$

§. 287. Quartae eclipsis medium erat Parisiis A. 1718 Sept. $9^d, 8^h, 1', 1''$ temp. medio Pro quo tempore colligitur

Longitudo solis vera $\theta =$	5, 16, 40, 58
Anomalia solis vera $s =$	2, 8, 19, 59
Longitudo lunae media	11, 17, 25, 16
Anomalia lunae media	0, 10, 41, 28
Anomalia lunae vera $v =$	0, 9, 36, 52
Longitudo nodi vera $\pi =$	5, 15, 59, 35
Distantia nodi a sole	0, 0, 41, 23
aeq. pro loco lunae	— 22
Longitudo lunae obs. $\phi =$	11, 16, 40, 36

§. 288. Calculus ergo sequens habebitur.

$v =$	0, 9, 36, 52	; sin $v = +$ sin 9, 36, 52
		cos $= +$
$2v =$	0, 19, 14	; sin $2v = +$ sin 19, 14
		cos $= +$
$s =$	2, 8, 20	; sin $s = +$ sin 68, 20
$v - s =$	10, 1, 17	; sin $= -$ sin 58, 43
$v + s =$	2, 17, 57	; sin $= +$ sin 77, 57
$\phi - \pi =$	0, 0, 41	
$2\phi - 2\pi =$	0, 1, 22	; sin $= +$ sin 1, 22
$r =$	0, 9, 37	
$2\phi - 2\pi - r =$	11, 21, 45	; sin $= -$ sin 8, 15
$2\phi - 2\pi - 2r =$	11, 12, 8	; sin $= -$ sin 17, 52

+

+ 9,22274	+ 9,2227	+ 9,9938
— 4,24753	− 1,5111	− 0,7104
— 3,47027	− 0,7338 *n*	− 0,7042 *i*
+ 9,5177	+ 9,5177	+ 9,9750
— 2,8561	− 0,4208	− 9,6201
— 2,3738	− 9,9385 *n*	− 9,5951 *i*
+ 9,9682	− 9,9318	+ 9,9903
+ 2,8819	− 1,1761	− 2,2122
+ 2,8501	+ 1,1079	− 2,2025
+ 8,3775	− 9,1568	− 30,68 *y*
— 2,7896	+ 2,2355	
— 1,1671	− 1,3923	

	aeq. aff.	aeq. neg.	
	+ 708	− 2953	− 5, 4 *n*
	+ 13	− 237	− 0, 8 *n*
	+ 721	− 159	− 5, 1 *i*
	— 3389	− 15	− 0, 4 *i*
	— 2668	− 25	
aeq. =	— 44′, 28″	− 3389	

Long. ☾ med.		11, 17, 25, 16
	aeq.	— 44, 28
Long.	vera	11, 16, 40, 48
	obs.	11, 16, 40, 36

Ergo $0 = +12 + m - 6{,}2n - 5{,}5i - 30{,}68y$

§. 289. Quintae eclipsis medium erat: Parisiis A. 1719 Aug. $29^d, 8^h, 33', 19''$ temp. med. Pro quo tempore colligitur:

Longitudo solis vera θ =	5,	5,	47,	14
Anomalia solis vera s =	1,	27,	25,	24
Longitudo lunae media .	11,	2,	9,	40
Anomalia lunae media . .	10,	15,	59,	25
Anomalia lunae vera v =	10,	20,	5,	19
Longitudo nodi vera π =	4,	27,	44,	39
Distantia nodi a sole =	0,	8,	2,	35
Aequ. pro loco lunae . .	—	—	4,	22
Long. lunae obseruata . .	11,	5,	42,	52

§. 290. Calculus ergo ita se habebit:

$v = 10, 20, 5, 19$	;	$\sin = -\sin 39, 54, 41$ $\cos = +$
$2v = 9, 10, 11$	;	$\sin 2v = -\sin 79, 49$ $\cos = +$
$v = 10, 20, 5$		
$s = 1, 27, 25$	;	$\sin s = +\sin 57, 25$
$v-s = 8, 22, 40$	;	$\sin = -\sin 82, 40$
$v+s = 0, 17, 30$	;	$\sin = +\sin 17, 30$
$\Phi-\pi = 0, 7, 58$		
$2\Phi-2\pi = 0, 15, 16$	;	$\sin = +\sin 15, 56$
$r = 10, 20, 5$		
$2\Phi-2\pi-r = 1, 25, 15$	;	$\sin = +\sin 55, 51$
$2\Phi-2\pi-2r = 3, 5, 46$	;	$\sin = +\sin 84, 14$

— 9,80726	— 9,8073	+ 9,8849
— 4,24753	— 1,5111	— 0,7104
+ 4,05479	+ 1,3184n	— 0,5953i

— 9,9931	— 9,9931	+ 9,2475
— 2,8561	— 0,4208	— 9,6201
+ 2,8492	+ 0,4139n	— 8,8676i

+ 9,9256	— 9,9964	+ 9,4781
+ 2,8819	— 1,1761	— 2,2122
+ 2,8075	+ 1,1725	— 1,6903

+ 9,4386	+ 9,9178	+ 99,5y
— 2,7896	+ 2,2355	
— 2,2282	+ 2,1533	

aeq. aff.	aeq. neg.	
+ 11345	— 49	+ 20,8n
+ 707	— 170	+ 2,6n
+ 642	— 219	— 3,9i
+ 15	+ 12851	— 0,1i
+ 142	+ 12632	
+ 12851	210,32	
	+ 3,30,32	aequatio

Long. lunae med.	11, 2, 9, 40
aeq.	+ 3, 30, 32
Long. lunae vera	11, 5, 40, 12
obs.	11, 5, 42, 52

$0 = -2,40 + m + 23,4n - 4,0i + 99,5y$

§. 291.

§. 291. Sextae eclipfis medium erat Parifiis A. 1722. Jun. 28ᵈ, 13ʰ, 58′, 41″ temp. med. Pro quo tempore habetur:

Longitudo folis vera θ =	3, 6, 51, 7
Anomalia folis vera s =	11, 28, 26, 56
Longitudo lunae media .	9, 9, 31, 50
Anomalia lunae media .	4, 28, 8, 18
Anomalia lunae vera v =	4, 24, 39, 53
Longitudo nodi vera π =	3, 2, 36, 2
Diftantia nodi a fole . .	0, 4, 15, 5
Aequatio loci lunae . .	— 2, 20
Longitudo lunae obferuata	9, 6, 48, 47

§. 292. Calculus ergo ita fe habebit

$v =$	4, 24, 39, 53	;	$\sin v = + \sin 35°, 20', 7''$
			$\cos v = -$
$2v =$	9, 19, 20	;	$\sin 2v = - \sin 70, 40$
			$\cos 2v = +$
$r =$	4, 24, 40		
$s =$	11, 28, 27	;	$\sin s = - \sin 1, 33$
$r-s =$	4, 26, 13	;	$\sin = + \sin 33, 47$
$r+s =$	4, 23, 7	;	$\sin = + \sin 36, 53$
$\Phi-\pi =$	0, 4, 13		
$2\Phi-2\pi =$	0, 8, 26	;	$\sin = + \sin 8, 26$
$r =$	4, 24, 40		
$2\Phi-2\pi-r =$	7, 13, 46	;	$\sin = - \sin 43, 46$
$2\Phi-2\pi-2r =$	2, 19, 6	;	$\sin = + \sin 79, 6$

+

+	9,76220	+	9,7622	−	9,9116
−	4,24753	−	1,5111	−	9,7104
−	4,00973	−	1,2733*n*	+	0,6220*i*
−	9,9748	−	9,9748	+	9,5199
−	2,8561	−	0,4208	−	9,6201
+	2,8309	+	0,3956*n*	−	9,1400*i*
−	8,4321	+	9,7451	+	9,7783
+	2,8819	−	1,1761	−	2,2122
−	1,3140	−	0,9212	−	1,9905
+	9,1663	−	9,8399	+	98,2*y*
−	2,7896	+	2,2355		
−	1,9559	−	2,0754		

aeq. aff.	aeq.neg.	
+ 678	− 10227	− 18, 8*n*
− 10563	− 21	+ 2, 5*n*
− 9885	− 8	
164,45	− 98	+ 4, 2*i*
aeq. = − 2,44,45	− 90	− 0, 1*i*
	− 119	
	−101563	

Long. lun. med.	9, 9, 31, 50
	− 2, 44, 45
Long. lun.	9, 6, 47, 5
obs.	9, 6, 48, 47

Ergo $\bullet = - 1,42 + m - 16,3n + 4,1i + 98,9y$

§. 293. Septimae eclipsis medium obseruatum est Parisiis A. 1724 Oct. 31^d, 15^h, $34'$, $17''$ temp. med. Pro quo tempore colligitur

Longitudo solis vera θ =	7, 8, 56, 1
Anomalia solis vera s =	4, 0, 29, 44
Longitudo lunae media	1, 9, 23, 59
Anomalia lunae media	5, 22, 38, 2
Anomalia lunae vera v =	5, 21, 46, 51
Longitudo nodi vera	1, 16, 36, 22
Distantia nodi a sole	5, 22, 19, 39
aequatio loci lunae	+ 4, 10
Long. lunae obseruata	1, 9, 0, 11

§. 294. Calculus ergo ita ineatur:

v =	5, 21, 46, 51	; sin v = + sin 8°, 13′, 9″
		cos = −
$2v$ =	11, 13, 34,	; sin $2v$ = − sin 16°, 26′
r =	5, 21, 47	cos = +
s =	4, 0, 30	; sin s = + sin 59°, 30′
$r - s$ =	1, 21, 17	; sin = + sin 51, 17
$r + s$ =	9, 22, 17	; sin = − sin 67, 43
$\Phi - \pi$ =	5, 22, 24	
$2\Phi - 2\pi$ =	11, 14, 48	; sin = − sin 15, 12
r =	5, 21, 47	
$2\Phi - 2\pi - r$ =	5, 23, 1	; sin = + sin 6, 59
$2\Phi - 2\pi - 2r$ =	0, 1, 14	; sin = + sin 1, 14

+

+	9, 15520	+	9, 1552	−	9, 9955
−	4, 24753	−	1, 5111	−	0, 7104
−	3, 40273	−	0, 6663n	+	0, 7059i
−	9, 4516	−	9, 4516	+	9, 9819
−	2, 8561	−	0, 4208	−	9, 6201
+	2, 3077	+	9, 8724n	−	9, 6020i
+	9, 9353	+	9, 8922	−	9, 9663
+	2, 8819	−	1, 1761	−	2, 2122
−	2, 8172	−	1, 0683	+	2, 1785
−	9, 4186	+	9, 0849	+	2, 15y
−	2, 7896	+	2, 2355		
+	2, 2082	+	1, 3024		

aeq. aff.	aeq. neg.	
+ 203	− 2528	− 4, 6m
+ 657	− 11	− 7, 5n
+ 151	− 2539	
+ 161	+ 1193	+ 5, 1i
+ 21	− 1346	− 0, 4i
+ 1193	− 22, 26	aequatio

Long. ☽	med.	1, 9, 23, 59
		− 22, 26
Long.	calc.	1, 9, 1, 33
Long.	obf.	1, 9, 0, 11

$$0 = +1,22 + m - 3,9n + 4,7i + 2,15y$$

§. 295. Octauae eclipsis medium obseruatum est Parisiis A. 1729. Febr. $13^d, 9^h, 6', 56''$ temp. med. Pro quo tempore colligitur:

Longitudo solis vera θ =	$10^s, 25^\circ, 13', 23''$
Anomalia solis vera s =	7, 16, 43, 34
Longitudo lunae media .	5, 0, 5, 27
Anomalia lunae media .	3, 18, 53, 24
Anomalia lunae vera v =	3, 12, 54, 9
Longitudo nodi vera π =	10, 24, 4, 30
Distantia nodi a sole =	0, 1, 8, 53
aequatio pro long. lunae .	— 0, 37
Longitudo lunae obseruata	4, 25, 12, 46

§. 296. Calculus ergo ita se habebit:

v =	3, 12, 54, 9	; sin v = + sin $77^\circ, 5', 51''$
		cos v = −
$2v$ =	6, 25, 48	; sin $2v$ = − sin 25, 48
r =	3, 12, 54	cos $2v$ = −
s =	7, 16, 44	; sin s = − sin 46, 44
$r-s$ =	7, 26, 10	; sin = − sin 56, 10
$r+s$ =	10, 29, 38	; sin = − sin 30, 22
$\Phi-\pi$ =	0, 1, 8,	
$2\Phi-2\pi$ =	0, 2, 16,	; sin = + sin 2, 16
r =	3, 12, 54	
$2\Phi-2\pi-r$ =	8, 19, 22	; sin = − sin 79, 22
$2\Phi-2\pi-2r$ =	5, 6, 28	; sin = + sin 23, 32

+ 9,98889	+ 9,9889	− 9,3488
− 4,24757	− 1,5111	− 0,7104
− 4,23642	− 1,5000*n*	+ 0,0502 *i*

− 9,6387	− 9,6387	− 9,9544
− 2,8561	− 0,4208	− 9,6201
+ 2,4948	+ 0,0595 *n*	+ 9,5745 *i*

− 9,8622	− 9,9194	− 9,7037
+ 2,8819	− 1,1761	− 2,2122
− 2,7441	+ 1,0955	+ 1,9159

+ 8,5971	− 9,9925	+ 39, 9*y*
− 2,7896	+ 2,2355	
− 1,3867	− 2,2280	

aeq. afl.	aeq. neg.	
+ 312	− 17235	− 31, 7*n*
+ 12	− 555	+ 1, 1*n*
+ 82	− 24	+ 1, 1 *i*
+ 406.	− 169	+ 0, 3 *i*
	− 17983	
	+ 406	
	− 17577	
	− 292, 57	
	− 4,52,57 aequatio	

Long. lunae media	= 5, 0, 5, 27
aeq.	− 4, 52, 57
Long. lunae calc.	4, 25, 12, 30
Long. lunae obf.	4, 25, 12, 46

$e = -16 + m - 30,6n + 1,4i + 39,9y$

§. 297. Nonae eclipsis medium obseruatum est Parisiis A. 1729. Aug. 8^d, 13^h, $14'$, $14''$ temp. med. Pro quo tempore reperitur.

Longitudo solis vera ϕ	=	4, 16, 17, 29
Anomalia solis vera s	=	1, 7, 47, 12
Longitudo lunae media		10, 11, 23, 57
Anomalia lunae media		8, 10, 36, 19
Anomalia lunae vera v	=	8, 16, 34, 40
Longitudo nodi vera π	=	10, 14, 58, 21
Distantia nodi a sole		6, 1, 19, 8
Aequatio pro loco lunae		— 43
Long. lunae obseruata		10, 16, 16, 46

§. 298. Calculus ergo ita instituetur:

v	=	8, 16, 34, 40	; $\sin v = -\sin 76, 34, 40$	
			$\cos v = -$	
$2v$	=	5, 3, 9	; $\sin 2v = +\sin 26, 51$	
			$\cos 2v = -$	
r	=	8, 16, 35		
s	=	1, 7, 47	; $\sin s = +\sin 37, 47$	
$r-s$	=	7, 8, 48	; $\sin = -\sin 38, 48$	
$r+s$	=	9, 24, 22	; $\sin = -\sin 65, 38$	
$\Phi-\pi$	=	6, 1, 19		
$2\Phi-2\pi$	=	0, 2, 38	; $\sin = +\sin 2, 38$	
r	=	8, 16, 35		
$2\Phi-2\pi-r$	=	3, 16, 3	; $\sin = +\sin 73, 57$	
$2\Phi-2\pi-2r$	=	6, 29, 28	; $\sin = -\sin 29, 28$	

— 9,93797	— 9,9880	— 9,3655
— 4,24755	— 1,5111	— 0,7104
+ 4,23550	+ 1,4991*n*	+ 0,0759*i*

+ 9,6548	+ 9,6548	— 9,9505
— 2,8561	— 0,4208	— 9,6201
— 2,5109	— 0,0756*n*	+ 9,5706*i*

+ 9,7872	— 9,7970	— 9,9595
+ 2,8819	— 1,1761	— 2,2122
+ 2,6691	+ 0,9731	+ 2,1717

+ 8,6622	+ 9,9827	— 49, 2*y*
— 2,7896	+ 2,2355	
— 1,4518	+ 2,2182	

aeq. aff.	aeq. neg.	
+ 17199	— 324	+ 31, 6*n*
+ 467	— 28	— 1, 2*n*
+ 9	— 352	+ 1, 2*i*
+ 148	+ 17988	+ 0, 4*i*
+ 165	+ 17636	
+ 17988	+ 293, 56	
	+ 4, 53, 56	aequatio

Long. ☽ med.	=	10, 11, 23, 57
aeq.		+ 4, 53, 56
Long. ☽ calc.	=	10, 16, 17, 53
Long. ☽ obs.		10, 16, 16, 46

$$0 = + 1, 7 + m + 30, 4n + 1, 6i - 49, 2y$$

§. 299.

§. 299. Decimae eclipsis medium obseruatum est Parisiis A. 1731. Jun. 19^d, 13^h, 55$'$, 13$''$. temp. med. Pro quo tempore colligitur

Longitudo solis vera θ =	2^s,	28°,	5$'$,	41$''$
Anomalia solis vera s =	11,	19,	48,	47
Longitudo lunae media .	9,	1,	45,	1
Anomalia lunae media .	4,	15,	9,	43
Anomalia lunae vera v =	4,	10,	34,	21
Longitudo nodi vera π =	9,	8,	6,	38
Distantia nodi a sole . . .	5,	19,	59,	3
Aequatio pro loco lunae .	+		5,	24
Longitudo lunae obseruata	8,	28,	11,	5

§. 300. Calculus ergo ita instituetur

$v =$	4, 10, 34, 21	;	$\sin v = + \sin$ 49, 25, 39
			$\cos v = -$
$2v =$	8, 21, 9	;	$\sin 2v = - \sin$ 81, 9
			$\cos 2v = -$
$r =$	4, 10, 34		
$s =$	11, 19, 49	;	$\sin s = - \sin$ 10, 11
$r - s =$	4, 20, 45	;	$\sin\ = + \sin$ 39, 15
$r + s =$	4, 0, 23	;	$\sin\ = + \sin$ 59, 37
$\Phi - \pi =$	5, 20, 4		
$2\Phi - 2\pi =$	11, 10, 8	;	$\sin\ = - \sin$ 29, 52
$r =$	4, 10, 34		
$2\Phi - 2\pi - r =$	6, 29, 34	;	$\sin\ = - \sin$ 29, 34
$2\Phi - 2\pi - 2r =$	2, 19, 0	;	$\sin\ = + \sin$ 79, 0

+

+	9,88057	+	9,8806	−	9,8131
−	4,24753	−	1,5111	−	0,7104
−	4,12810	−	1,3917 *n*	+	0,2100 *i*
−	9,9948	−	9,9948	−	9,1871
−	2,8561	−	0,4208	−	9,6201
+	2,8509	+	0,4156 *n*	+	8,8072 *i*
−	9,2475	+	9,8012	+	9,9358
+	2,8819	−	1,1761	−	2,2122
−	2,1294	−	0,9773	−	2,1480
−	9,5313	−	9,6932	+	98, 1 *y*
−	2,7896	+	2,2355		
+	2,3209	−	1,9287		

	aeq. aff.	aeq. neg.		
	+ 709	− 13431	−	24, 8 *n*
	+ 209	− 135	+	2, 6 *n*
	+ 918	− 9	+	3, 3 *i*
	− 13801	− 141	+	0, 1 *i*
	− 12883	− 85		
	− 214, 43	− 13801		
aeq.	− 3, 34, 43			

Long. lunae media	9, 1, 45, 1
aeq.	− 3, 34, 43
Long. lunae calc.	8, 28, 10, 18,
Long. lunae obs.	8, 28, 11, 5

Ergo $e = -47'' + m - 22,2n + 3,4i + 98,1y$

 §. 301.

§. 301. Eclipsis undecimae medium obseruatum est Parisiis A. 1732 Dec. $1^d, 9^h, 48', 23''$ temp. med. Pro quo tempore colligitur:

Longitudo solis vera θ =	8,	10,	3,	6
Anomalia solis vera s =	5,	1,	29,	50
Longitudo lunae media .	2,	6,	8,	19
Anomalia lunae media . . .	7,	19,	24,	12
Anomalia lunae vera v =	7,	24,	19,	39
Longitudo nodi vera π =	8,	10,	41,	14
Distantia nodi a sole =	11,	29,	21,	52
Aequ. pro loco lunae . .	+			21
Long. lunae obseruata . .	2,	10,	3,	27

§. 302. Calculus ergo ita se habebit:

$v =$	7, 24, 19, 39	; $\sin v = - \sin 54, 19, 39$
		$\cos v = -$
$2v =$	3, 18, 39	; $\sin 2v = + \sin 71, 21$
		$\cos = -$
$r =$	7, 24, 20	
$s =$	5, 1, 30	; $\sin s = + \sin 28, 30$
$r - s =$	2, 22, 50	; $\sin = + \sin 82, 50$
$r + s =$	0, 25, 50	; $\sin = + \sin 25, 50$
$\Phi - \pi =$	11, 29, 22	
$2\Phi - 2\pi =$	11, 28, 44	; $\sin = + \sin 1, 16$
$r =$	7, 24, 40	
$2\Phi - 2\pi - r =$	4, 4, 4	; $\sin = + \sin 55, 56$
$2\Phi - 2\pi - 2r =$	8, 9, 24	; $\sin = + \sin 69, 24$

— 9,90975	— 9,9097	— 9,7657
— 4,24753	— 1,5111	— 0,7104
+ 4,15728	+ 1,4208 *n*	+ 0,4761 *i*
+ 9,9766	+ 9,9766	— 9,5048
— 2,8561	— 0,4208	— 9,6201
— 2,8327	— 0,3974 *n*	+ 9,1249 *i*
+ 9,6787	+ 9,9969	+ 9,6444
+ 2,8819	— 1,1761	— 2,2122
+ 2,5606	— 1,1730	— 1,8566
— 8,3445	+ 9,9182	— 93, 6 *y*
— 2,7896	+ 2,2355	
+ 1,1341	+ 2,1537	

aeq. aff.	aeq. neq.	
+ 14364	— 680	+ 26, 4 *n*
+ 364	— 15	— 2, 5 *n*
+ 14	— 72	+ 3, 0 *i*
+ 142	— 767	+ 0, 1 *i*
+ 14884	+ 14884	
	+ 14117	
	+ 235, 17	
	+ 3, 55, 17	aequatio

Long. lunae media		2, 6, 8, 19
aeq.	+	3, 55, 17
Long. lunae calc.		2, 10, 3, 36
obs.		2, 10, 3, 27

$$0 = +9 + m + 23{,}9n + 3{,}1i - 93{,}6y$$

§. 303

§. 303. Eclipsis duodecimae medium obseruatum est Parisiis A. 1736 Mart. $26^d, 12^h, 14', 36''$ temp. med. Pro quo tempore colligitur

Longitudo solis vera θ =	0^s,	6°,	$35'$,	$42''$
Anomalia solis vera s =	8,	27,	58,	24
Longitudo lunae media	6,	4,	5,	0
Anomalia lunae media	7,	3,	25,	43
Anomalia lunae vera v =	7,	7,	2,	56
Longitudo nodi vera π =	6,	6,	24,	31
Distantia nodi a sole	6,	0,	11,	11
aeq. pro long. lunae	—			6
Longitudo lunae obs.	6,	6,	35,	36

§. 304. Calculus ergo ita instituatur.

v =	7,	7,	2, 56	; $\sin v = -\sin 57, 2, 56$	
				$\cos = -$	
$2v$ =	2,	14,	6	; $\sin 2v = +\sin 74, 6$	
				$\cos = +$	
r =	7,	7,	3		
s =	8,	27,	58	; $\sin s = -\sin 87, 58$	
$r-s$ =	10,	9,	5	; $\sin = -\sin 50, 55$	
$r+s$ =	4,	5,	1	; $\sin = +\sin 54, 59$	
$\phi-\pi$ =	6,	0,	11		
$2\phi-2\pi$ =	0,	0,	22	; $\sin = +\sin 0, 22$	
r =	7,	7,	3		
$2\phi-2\pi-r$ =	4,	23,	19	; $\sin = +\sin 36, 41$	
$2\phi-2\pi-2r$ =	9,	16,	16	; $\sin = -\sin 73, 44$	

— 9,77995	— 9,7799	— 9,9021
— 4,24753	— 1,5111	— 0,7104
+ 4,02748	+ 1,2910n	+ 0,6125i

+ 9,9831	+ 9,9831	+ 9,4377
— 2,8561	— 0,4208	— 9,6201
— 2,8392	— 0,4039n	— 9,0578i

— 9,9997	— 9,8900	+ 9,9133
+ 2,8819	— 1,1761	— 2,2122
— 2,8816	+ 1,0661	— 2,1255

+ 7,8061	+ 9,7763	— 96, 0y
— 2,7896	+ 2,2355	
— 0,5957	+ 2,0118	

	aeq. aff.	aeq. neg.	
	+ 10653	— 691	+ 19, 6n
	+ 12	— 761	— 2, 5n
	+ 103	— 133	+ 4, 1i
	+ 10768	— 4	— 0, 1i
	— 1589	— 1589	
	+ 9179		
	+ 152, 59		
aeq. =	+ 2,32,59		

Long. ☾ med.		6, 4, 5, 0
	aeq.	+ 2, 32, 59
Long. ☽ calc.		6, 6, 37, 59
	obſ.	6, 6, 35, 36

$o = + 2', 23'' + m + 17,1n + 4,0i - 96,0y$

§. 305. Elipsis decima tertiae medium obseruatum est Parisiis A. 1736 Sept. 19^d, 14^h, $59'$, $36''$ temp. med. Pro quo tempore colligitur

Longitudo solis vera θ =	5^s, 27°, $21'$, $39''$,
Anomalia solis vera s =	2^s, 18, 43, 51
Longitudo lunae media . .	11, 27, 48, 53
Anomalia lunae media . .	0, 7, 25, 42
Anomalia lunae vera v =	0, 6, 40, 44
Longitudo nodi vera	5, 27, 15, 4
Distantia nodi a sole . .	0, 0, 6, 35
aeq. pro long. lunae	— 4
Longitudo lunae obseruata	11, 27, 21, 35

§. 306. Calculus ergo ita instituetur

$v =$	0, 6, 40, 44	; $\sin v = + \sin 6, 40, 50$
		$\cos v = +$
$2v =$	0, 13, 21	; $\sin 2v = + \sin 13, 21$
		$\cos 2v = +$
$r =$	0, 6, 41	
$s =$	2, 18, 44	; $\sin s = + \sin 88, 44$
$r - s =$	9, 17, 57	; $\sin = - \sin 72, 3$
$r + s =$	2, 25, 25	; $\sin = + \sin 85, 25$
$\Phi - \pi =$	0, 0, 6	
$2\Phi - 2\pi =$	0, 0, 12	; $\sin = + \sin 0, 12$
$r =$	0, 6, 41	
$2\Phi - 2\pi - r =$	11, 23, 31	; $\sin = - \sin 6, 29$
$2\Phi - 2\pi - 2r =$	11, 16, 50	; $\sin = - \sin 13, 10$

+

+	9,06561	+	9,0656	+	9,9970
—	4,24753	−	1,5111	−	0,7104
—	3,31314	−	0,5767n	−	0,7074i
+	9,3634	+	9,3634	+	9,9881
—	2,8561	−	0,4208	−	9,6201
—	2,2195	−	9,7842n	−	9,6082i
+	9,9915	−	9,9783	+	9,9986
+	2,8819	−	1,1751	−	2,2122
+	2,8734	+	1,1534	−	2,2108
+	7,5429	−	9,0527	−	22,6y
—	2,7896	+	2,2355		
—	0,3325	−	1,2882		

aeq. aff.	aeq. neg.	
+ 747	− 2057	− 3,8n
+ 14	− 166	− 0,6n
+ 761	− 2	− 5,1i
— 2406	− 19	− 0,4i
— 1645	− 2406	
— 27′,25″ aequatio		

Long. ☽ med.	11, 27, 48, 53
aeq.	− 27, 25
Long. ☽ calc.	11, 27, 21, 28
Long. ☽ obs.	11, 27, 21, 35

$0 = -0,7'' + m - 4,4n - 5,5i - 22,6y$

§. 307.

§. 307. Ex his ergo tredecim eclipsibus nacti sumus aequationes, ex quibus cum tabularum, quibus sum vsus, correctiones, tum verus valor aequationis ab angulo $2\Phi - 2\pi - 2r$ pendentis definiri debebit:

Aequationes autem inde ortae sunt sequentes.

I. $$0 = +156'' + m - 29{,}1n - 2{,}8i - 99{,}8y$$

II. $$0 = -27 + m + 32{,}4n - 1{,}3i + 79{,}2y$$

III. $$0 = +30 + m + 32{,}4n + 0{,}2i + 25{,}6y$$

IV. $$0 = +12 + m - 6{,}2n - 5{,}5i - 30{,}7y$$

V. $$0 = -160 + m + 23{,}4n - 4{,}0i + 99{,}5y$$

VI. $$0 = -102 + m - 16{,}3n + 4{,}1i + 98{,}2y$$

VII. $$0 = +82 + m - 3{,}9n + 4{,}7i + 2{,}1y$$

VIII. $$0 = -16 + m - 30{,}6n + 1{,}4i + 39{,}9y$$

IX. $$0 = +67 + m + 30{,}4n + 1{,}6i - 49{,}2y$$

X. $$0 = -47 + m - 22{,}2n + 3{,}4i - 98{,}1y$$

XI. $$0 = +9 + m + 23{,}9n + 3{,}1i - 93{,}6y$$

XII. $$0 = +143 + m + 17{,}1n + 4{,}0i - 96{,}0y$$

XIII. $$0 = -7 + m - 4{,}4n - 5{,}5i - 22{,}6y$$

§. 308. Hic statim commode euenit, vt errores calculi ab obseruationibus infra tria minuta prima subsistant, qui autem infra sesquiminutum primum deprimuntur, simul ac litterae y valor tribuitur vnitati fere aequalis. Hincque ergo cognoscimus valorem ipsius y, quem quinario maiorem inueneramus, merito nobis fuisse suspectum, cum iam perspiciamus, eum vnitatem superare non posse. Quamobrem ponamus $y = 1$, seu in formula nostra pro longitudine lunae scribamus terminum $100''$ sin $(2\Phi - 2\pi - 2r)$. Quod autem ad litteras m, n et i attinet, tentanti mox patebit, quoscunque ipsis valores tribuamus, errores inde non admodum posse diminui; interim tamen decem circiter minutis secundis diminuentur, si ponatur $y = \frac{1}{2}$; $n = \frac{1}{2}$; $i = -3$ et $m = -4$; quo facto errores vix vnum minutum primum superabunt.

CAPUT XVIII.

CONSTITUTIO ELEMENTORUM PRO TABULIS LUNARIBUS.

§. 309.

Tabulae autem, quibus in praecedenti calculo sum vsus, praebent pro meridiano Parisino ad epocham 1701 seu ad meridiem diei vltimi anni 1700 tempore medio

Longitudinem Lunae mediam	5ˢ, 20°, 19′, 47″
et Anomaliam Lunae mediam	6, 13, 26, 51

Hinc accuratius habebimus haec elementa pro eodem tempore eodemque loco scilicet

Longitudinem Lunae mediam	5ˢ, 20°, 19′, 43″
Anomaliam Lunae mediam	6, 13, 24, 0
unde Longitudo Apogei	11, 6, 55, 43

§. 310. Si haec elementa comparemus cum Tabulis astronomicis Cel. Cassini et Monnierii, reperiemus pro eodem tempore et loco

	Cassini	Monnier
Long. mediam Lunae	5, 20, 18, 19	5, 20, 19, 28
Anom. mediam Lunae	6, 13, 10, 48	6, 13, 13, 2
Long. Apogei	11, 7, 7, 27	11, 7, 6, 26

Hic quidem longitudo media satis conuenit cum ea, quam ex obseruationibus conclusimus; verum anomalia media inuenta superat Cassinianam 13′, 12″, Monnierianam autem 11′, quod discrimen satis est notabile.

Verum

Verum si perpendamus motum lunae a tam multis variisque inaequalitatibus perturbari, mirum sane non est, anomaliam mediam per solas obseruationes accuratius definiri non potuisse; praesertim cum error 15′ in anomalia media commissus in loco lunae ad summum errorem 1′, 45″ gignere valeat.

§. 311. Excentricitatem autem orbitae lunaris, quam statueram $= 0,0545$ iam $\frac{1}{10000}$ vel $0,0001$ augeri oportet, ita vt nunc sit excentricitatis valor $k = 0,05455$; qui a supra assumto tam parum discrepat, vt anomalia vera inde ex media collecta pro satis exacta haberi possit: aequationes autem ab excentricitate pendentes aliquod augmentum capient, quod nunc quidem diligentius definiri oportet. Primum ergo formulam pro longitudine lunae inuentam hinc corrigamus; deinde vero etiam formulas pro distantia lunae a terra, pro eius motu momentaneo, et pro loco nodi veraque inclinatione orbitae lunaris ad eclipticam hinc euoluamus.

§. 312. Ante omnia autem oportebit formulam exhibere, cuius ope ex data quauis anomalia lunae media p elicere liceat, conuenientem anomaliam veram r. Ac substituto quidem pro k vero eius valore nunc invento, coefficientibusque in minuta secunda conuersis, formula supra (§. 306) exhibita sequentem induet formam:

$$r = p - 22495'' \sin p + 766'' \sin 2p - 36'' \sin 3p$$

$$\qquad 4,352086 \qquad\qquad 2,884229 \qquad\qquad 1,55630$$

 Huius

Huius ergo formulae ope haud difficulter tabula computabitur, quae ad singulos anomaliae mediae gradus exhibeat valores anomaliae verae.

§. 313. Inuenta autem anomalia vera r, si habeatur quoque anomalia vera solis s, vna cum angulo η et longitudinibus Φ, θ, π saltem proxime, formula longitudinem veram Φ datae mediae ξ respondentem exhibens, sequenti modo habebitur expressa:

	log. coeff.	
$\Phi = \xi - 22466'' \sin r$	4,351535	I
$- 462 \sin 2r$	2,66456	
$- 11 \sin 3r$	1,0518	
$+ 701 \sin s$	2,84572	II
$+ 4 \sin 2s$	0,602	
$+ 141 \sin (r-s)$	2,1492	III
$- 118 \sin (r+s)$	2,0719	IV
$- 175 \sin \eta$	2,2430	V
$+ 2115 \sin 2\eta$	3,32531	
$+ 4 \sin 3\eta$	0,602	
$- 8 \sin 4\eta$	0,903	
$+ 59 \sin (\eta-r)$	1,7708	VI
$+ 352 \sin (2\eta-2r)$	2,5465	
$- 2729 \sin (2\eta-r)$	3,67477	VII
$- 93 \sin (4\eta-2r)$	1,9685	
$+ 56 \sin (2\eta+r)$	1,7482	VIII
$+ 59 \sin (4\eta-r)$	1,7708	IX
$- 49 \sin (\eta+s)$	1,6902	X
$- 76 \sin (2\eta-s)$	1,8808	XI
$- 57 \sin (2\eta+s)$	1,7559	XII
$+ 154 \sin (2\eta-r+s)$	2,1875	XIII

+

+	45 sin $(2\eta-\nu-s)$	1, 6532	XIV
—	411 sin $(2\Phi-2\pi)$	2, 6138	XV
—	205 sin $(2\theta-2\pi)$	2, 3117	XVI
—	6 sin $(4\theta-4\pi)$	0, 778	
+	187 sin $(2\Phi-2\pi-r)$	2, 2718	XVII
+	80 sin $(2\Phi-2\pi-2r)$	1, 9031	XVIII
—	15 sin $(2\theta-2\pi-r)$	1, 176	XIX
—	10 sin $(2\theta-2\pi+r)$	1, 000	XX

§. 314. Inaequalitates has ita dispoſui, vt eas, quae vna tabula comprehendi poſſunt, coniunctim expoſuerim, quo facilius calculus expediri queat. Hinc igitur patet omiſſis iis inaequalitatibus, quae 10″ non ſuperant, locum lunae per viginti inaequalitates corrigi debere, antequam vera eius longitudo obtineatur.

§. 315. Haec autem expreſſio adhuc iſto defectu laborat, quod pleraeque inaequalitates ipſam lunae longitudinem veram Φ, quae tamen demum quaeritur, inuoluant, ideoque calculus, cum longitudo lunae etiamnunc eſt incognita, commode expediri non poſſit. Quoniam tamen ſufficit longitudinem lunae proxime tantum noſſe, cum longitudo media per quatuor priores inaequalitates fuerit correcta, ea pro ſequentibus inaequalitatibus loco longitudinis verae vſurpari poterit, ſicque tandem longitudo lunae multo exactior reperietur. Quo facto ſi accuratior deſideretur, omnes inaequalitates poſt 4 priores denuo ad calculum reuocari conueniet, iisque euolutis longitudo lunae vera prodibit, quae nulla amplius correctione indigebit. Interim tamen ne calculum per se satis taediosum bis repetere opus sit, non difficulter hanc expressionem ita transformare licet, vt locus lunae per quatuor tantum priores inaequalitates correctus sine errore in sequentibus loco φ adhiberi possit.

§. 316. Cum autem longitudo lunae iam per obseruationes fuerit cognita, haec expressio sine vlla immutatione ad calculum accommodabitur, vt hoc modo consensus theoriae cum veritate exploretur. In inaequalitatibus enim determinandis pro littera φ vbique longitudo lunae obseruata introducetur, calculoque peracto patebit, quantum locus lunae per calculum definitus etiamnunc discrepet ab eius loco vero obseruato. Atque si hoc modo plurimae obseruationes calculo subiiciantur, ex aberrationibus a veritate non solum elementa, quibus haec formula innititur, accuratius definire licebit, sed etiam inaequalitates, quae nondum satis certae videntur, inde emendari poterunt. Quin etiam nouae inaequalitates, quas per Theoriam determinare non licuerat, hoc modo forte certius colligi poterunt.

§. 317. Antequam autem huiusmodi calculi specimen exhiberi queat, necesse est vt aequationem pro loco nodi vero inueniendo ad calculum accommodemus. Formulae autem supra (219) exhibitae, si pro r substituamus valorem inuentum $r = p - 2k \sin p - \frac{3}{4} kk \sin 2r$, pars: Const. $- 0,004053 p$ indicabit longitudinem nodi mediam. Hincque longitudo nodi vera erit $r =$

	Log. coeff.
$\varpi = \text{Long. med.} - 107'' \sin r$	2, 0294
$- 6 \sin 2r$	0, 778
$+ 551 \sin s$	2, 7411
$- 453 \sin 2\eta$	2, 6561
$- 129 \sin (2\eta - r)$	2, 1106
$- 33 \sin (2\eta + r)$	1, 518
$+ 55 \sin (2\eta - 2r)$	1, 740
$+ 420 \sin (2\Phi - 2\pi)$	2, 6232
$+ 98 \sin (2\Phi - 2\pi - r)$	1, 991
$+ 30 \sin (2\Phi - 2\pi + r)$	1, 477
$+ 235 \sin (2\Phi - 2\pi - 2r)$	2, 3711
$+ 5426 \sin (2\theta - 2\pi)$	3, 73448
$+ 75 \sin (4\theta - 4\pi)$	1, 875
$- 53 \sin (2\theta - 2\pi - r)$	1, 724
$+ 53 \sin (2\theta - 2\pi + r)$	1, 724
$- 90 \sin (2\theta - 2\pi - s)$	1, 954
$- 32 \sin (2\theta - 2\pi + s)$	1, 505

§. 318. In hoc calculo plerasque inaequalitates omittere licet, siquidem tantum longitudinem lunae investigare sit propositum: manifestum enim est, etiamsi in loco nodi error plurium minutorum primorum committatur, inde vix errorem aliquot minutorum secundorum in longitudinem lunae redundare. Quodsi vero eclipsis cuiuspiam omnia phaenomena diligenter definire velimus, tum locum nodi exactissime cognitum esse oportet. Praeterea vero pro latitudine assignanda vera inclinatio orbitae lunaris ad eclipticam ex media ε accuratissime erit definienda ope huius formulae:

$\varrho = \varepsilon$

		Log. coeff.
$\varrho = \epsilon$	$- 2'' \cos r$	0, 30
	$- 48 \cos 2\eta$	1, 681
	$+ 11 \cos (2\eta - r)$	1, 041
	$+ 3 \cos (2\eta + r)$	0, 48
	$+ 36 \cos (2\Phi - 2\pi)$	1, 556
	$+ 9 \cos (2\Phi - 2\pi - r)$	0, 95
	$+ 3 \cos (2\Phi - 2\pi + r)$	0, 48
	$+ 23 \cos (2\Phi - 2\pi - 2r)$	1, 362
	$+ 484 \cos (2\theta - 2\pi)$	2, 6848
	$+ 9 \cos (4\theta - 4\pi)$	0, 95
	$- 5 \cos (2\theta - 2\pi - r)$	0, 70
	$+ 5 \cos (2\theta - 2\pi - + r)$	0, 70
	$- 7 \cos (2\theta - 2\pi - s)$	0, 84
	$- 3 \cos (2\theta - 2\pi + s)$	0, 48

Tabula autem pro diſtantia lunae a terra, vnde eius parallaxis et diameter apparens definiatur, ex formulis ſupra exhibitis facile conſtruetur.

ADDI-

ADDITAMENTUM
CONTINENS ALIAS METHODOS INUESTIGANDI MOTUS LUNAE INAEQUALITATES.

Qui methodum ante descriptam accuratius euoluerit, eam quidem in se spectatam satis bonam atque plerisque lunae inaequalitatibus definiendis aptam deprehendet; interim tamen fateri cogor, eam non solum maxime esse operosam, sed etiam ita comparatam, vt plures inaequalitates, quae tamen motum lunae imprimis afficere videntur, non satis exacte exhibeat, et quasi in dubio relinquat. Causa huius incertitudinis manifesto in hoc est sita, quod omnes inaequalitates ita inter se sunt connexae, vt nullius valor verus accurate definiri possit, quin simul reliquae inaequalitates omnes fuerint cognitae. Cum igitur eiusmodi methodo approximandi sim vsus, vt primo quasdam inaequalitates tanquam cognitas assumserim, ex quibus deinceps reliquas definiuerim, probe notandum est ab his inuentis iterum priores, quae erant assumtae, leuem quandam mutationem pati; quae si statim ab initio nota fuisset, etiam reliquarum valores aliquantillum mutati prodiissent: at quaedam inaequalitates adeo sunt lubricae, vt facta vel minima mutatione in iis, a quibus pendent, inde non exiguam alterationem trahant. Huc imprimis pertinet motus apogei, cuius inuestigatio omnes omnino inaequalitates implicat, ita vt ſine harum cognitione neutiquam accurate definiri queat.

Cum igitur haec methodus iſtis tantis incommodis ſit obnoxia, aliam maxime diuerſam tentaui viam, quae ab iis eſſet libera, etiamſi negare nequeam, etiam hanc ſuis non carere incommodis, quae tamen prorſus alius ſunt generis. Ex quo confido his duabus diuerſis methodis combinandis haud exiguum fructum in veram motuum lunarium cognitionem eſſe redundaturum. Praecipuum autem diſcrimen verſatur in electione anomaliae, quae in ſuperiore methodo non ita eſt aſſumta, vt diſtantia lunae a terra fieret vel maxima vel minima, ſi anomalia vel $= 0$ vel $= 180°$ ſtatuatur: neque enim differentiale diſtantiae dx euaneſcit, quando ſinus anomaliae in nihilum abit, ſed praeterea etiamnunc ab elongatione ſolis a luna ſeu angulo η pendet. Ita ſecundum hanc methodum neque apogaeum lunae neque perigaeum ibi ſtatuitur, ubi angulus, quem motus lunae directio cum radio vectore facit, eſt rectus; ſed plerumque in alia puncta incidunt, quae ab iis locis, vbi luna terrae vel eſt proxima, vel ab ea maxime remota, notabiliter ſint diuerſa. Etſi autem in hoc calculo non verae lineae abſidum poſitio conſideratur, hinc tamen methodus minime vitioſa eſt reputanda; propterea quod non eſt quaeſtio, quo nomine quaepiam orbitae lunaris puncta appellentur, dummodo cunctae inaequalitates recte exprimantur. Sed quoniam circa has ipſas inaequalitates nonnulla grauiora dubia ſunt orta, haud abs re fore arbitror, et alteram methodum hic proponere.

I. Si

I.

Sit igitur vt ante: Massa solis $= \odot$; terrae $= ♁$ et lunae $= ☽$; atque vis attractiua terrae in distantia d vt $\frac{1}{dd} - \frac{1}{bb}$; manente vi solis quadrato distantiae exacte proportionali. Tum vero sit

Longitudo lunae $= \phi$; latitudo $= \psi$; et distantia curtata $= x$

Longitudo solis $= \theta$; eiusque a terra distantia $= y$

Longitudo nodi ascendentis lunae $= \pi$ et inclinatio ad eclipticam $= \rho$

ac ponatur breuitatis ergo elongatio lunae a sole $\phi - \theta = \eta$ et distantia $\sqrt{(xx \sec. \psi^2 - 2xy \cos \eta + yy)} = z$.

Quibus positis supra §. 20. vidimus motum lunae his quatuor aequationibus contineri:

$$\text{I.}\quad 2dxd\phi + xdd\phi = -\tfrac{1}{2} dt^2 . \odot \left(\frac{y}{z^3} - \frac{1}{yy}\right) \sin \eta$$

$$\text{II.}\quad ddx - xd\phi^2 = -\tfrac{1}{2} dt^2 (♁ + ☽) \cos \psi^3 \left(\frac{1}{xx} - \frac{1}{bb}\right)$$

$$- \tfrac{1}{2} dt^2 . \odot \left(\frac{x - y \cos \eta}{z^3} + \frac{\cos \eta}{yy}\right)$$

$$\text{III.}\quad d\pi = -\tfrac{1}{2} dt^2 . \odot \left(\frac{y}{z^3} - \frac{1}{yy}\right) \frac{\sin(\phi - \pi) \sin(\theta - \pi)}{xd\phi}$$

$$\text{IV.}\quad dl \tang \rho = \frac{d\pi}{\tang(\phi - \pi)}, \text{ et } \tang \psi = \operatorname{tg} \rho \cos(\phi - \pi)$$

vbi elementum temporis dt sumtum est pro constante.

II.

Quatenus hic motus solis ingreditur, is pro regulari atque regulis Kepleri conformi haberi poterit: habebimus ergo

$$2dyd\theta + ydd\theta = 0 \text{ et } ddy - yd\theta^2 = -\tfrac{1}{2}dt^2 . \frac{☉+♁}{yy}$$

vnde si ponamus orbitae solaris:

semiparametrum $= c$; excentricitatem $= e$ et anomaliam veram $= u$

erit $y = \frac{c}{1-e\cos u}$; $du = d\theta = \frac{dt}{yy} V \tfrac{1}{2} c (☉+♁)$

Sit a semiaxis transuersus orbitae solis, ac tempore $= t$ sol motu medio absoluat angulum $= \omega$, quo pro mensura temporis t vtamur: erit ergo $d\omega = \frac{dt}{aa} V \tfrac{1}{2} a (☉+♁)$

ideoque $\tfrac{1}{2}dt^2 = \frac{a^3 d\omega^2}{☉+♁}$. At est $a = \frac{c}{1-ee}$. Hinc ergo fit

$$du = d\theta = \frac{ad\omega}{yy} V ac = \frac{aad\omega}{yy} V(1-ee) = \frac{d\omega(1-e\cos u)^2}{(1-ee)V(1-ee)},$$

sicque tam du quam $d\theta$ per elementum $d\omega$ loco temporis introductum expressimus. Quia autem massa solis ☉ massam terrae ♁ tam enormiter excedit, sine errore pro $\tfrac{1}{2}dt^2$ scribi poterit $\frac{a^3 d\omega^2}{☉}$, eruntque nostrae aequationes pro luna:

I. $2dxd\Phi + xdd\Phi = -a^3 d\omega^2 \left(\frac{y}{z^3} - \frac{1}{yy}\right) \sin \eta$

II. $ddx - xd\Phi^2 = -\frac{a^3(♁+☽)d\omega}{☉} \cos\psi^2 \left(\frac{1}{xx} - \frac{1}{bb}\right)$

$$-a^3 d\omega^2 \left(\frac{x-y\cos\eta}{z^3} + \frac{\cos\eta}{yy}\right)$$

III. dx

III. $d\pi = -\frac{a^3 d\omega^2}{x d\Phi}\left(\frac{y}{z^3} - \frac{1}{yy}\right)$ fin $(\Phi - \pi)$ fin $(\theta - \pi)$

IV. $d.l$ tang $\varrho = \frac{d\pi}{tg(\Phi-\pi)}$; atque ob tg $\psi =$ tg ϱ cf$(\Phi - \pi)$, habebitur proxime cof $\psi^3 = 1 - \frac{3}{4}$ tg $\varrho^2 - \frac{3}{4}$ tg ϱ^2 cf $2(\Phi-\pi)$.

III.

Incipiamus a duabus aequationibus prioribus, ac ponamus breuitatis gratia

$$a^3\left(\frac{y}{z^3} - \frac{1}{yy}\right) \text{fin } \eta = \text{M} \quad \text{et}$$

$$\frac{a^3(♁+☽)}{☉} \text{cof}\psi^3\left(\frac{1}{xx} - \frac{1}{bb}\right) + a^3\left(\frac{x - y\text{cof}\eta}{z^3} + \frac{\text{cof}\eta}{yy}\right) = \frac{\text{A}}{xx} + \text{N}$$

quandoquidem haec posterior expressio terminum inuoluit formae $\frac{\text{A}}{xx}$ prae ceteris incomparabiliter maiorem; atque habebimus has duas aequationes:

$$2dxd\Phi + xdd\Phi = -\text{M}d\omega^2 \quad \text{et} \quad ddx - xd\Phi^2 = -\frac{\text{A}d\omega^2}{xx} - \text{N}d\omega^2$$

quarum prior per $2x^3 d\Phi$ multiplicata ob $d\omega$ constans habebit integrale:

$$x^4 d\Phi^2 = -2d\omega^2 \int \text{M}x^3 d\Phi$$

Tum prior multiplicata per $2xd\Phi$ addatur ad posteriorem per $2dx$ multiplicatam, eritque, aggregatum:

$$2xdxd\Phi^2 + 2xxd\Phi dd\Phi + 2dxddx = -2\text{M}xd\omega^2 d\Phi - \frac{2\text{A}d\omega^2 dx}{xx} - 2\text{N}d\omega^2 dx$$

Cuius integrale erit:

$$dx^2 + xxd\Phi^2 = +\frac{2\text{A}d\omega^2}{x} - 2d\omega^2 \int(\text{M}xd\Phi + \text{N}dx)$$

IV.

Ponantur formulae integrales, quae in his expressionibus insunt:

$$-\int M x^3 d\Phi = P \quad \text{et} \quad -\int (M x d\Phi + N dx) = Q$$

vt habeamus has duas aequationes:

$$x^4 d\Phi^2 = 2 P d\omega^2 \quad \text{et} \quad dx^2 + xx d\Phi^2 = \frac{2 A d\omega^2}{x} + 2 Q d\omega^2$$

vnde cum sit $xx d\Phi^2 = \frac{2 P d\omega^2}{xx}$ erit

$$dx^2 = 2 d\omega^2 \left(Q + \frac{A}{x} - \frac{P}{xx}\right) \text{ et } dx = \pm d\omega \sqrt{2\left(Q + \frac{A}{x} - \frac{P}{xx}\right)}$$

sicque differentiale dx per $d\omega$ exprimitur. Deinde vero habetur

$$d\Phi = \frac{d\omega}{xx} \sqrt{2P}$$

estque per hypothesin:

$$dP = -M x d\omega \sqrt{2P} \text{ et } dQ = -\frac{M d\omega}{x} \sqrt{2P} \mp N d\omega \sqrt{2\left(Q + \frac{A}{x} - \frac{P}{xx}\right)}$$

vbi quidem signorum ambiguorum inferius locum habere statuamus, quia motum ab apogeo numerare in animo est, ita vt hinc exeundo distantia x minuatur.

V.

Cum igitur differentiale dx in apogeo et perigeo euanescat, necesse est vt his locis formula irrationalis $\sqrt{\left(Q + \frac{A}{x} - \frac{P}{xx}\right)}$ in nihilum abeat, in reliquis autem locis valorem sortiatur realem. Commodissime ergo haec formula per sinum cuiuspiam anguli v exhibebitur, qui cum in apogeo euanescat, in perigeo autem duobus re-

ctis

ctis aequalis fiat, anomaliam lunae referet: idque ſenſu vero, ita vt diſtantia x in apogeo prodeat maxima, in perigeo vero minima. Sit igitur vt formam motus regularis ſequamur:

ſemilatus rectum orbitae lunaris $= p$
excentricitas orbitae $= q$
et anomalia vera lunae $= v$

eritque hinc per eandem legem diſtantia $x = \frac{p}{1 - q \operatorname{cof} v}$. Verum hic quantitates p et q, quae in motu regulari eſſent conſtantes, nunc pro variabilibus ſunt habendae, earumque variabilitas per variabilitatem quantitatum P et Q, quae in motu regulari itidem ſunt conſtantes, determinari debebit.

VI.

Subſtituamus ergo valorem aſſumtum $x = \frac{p}{1 - q \operatorname{cof} v}$ in formula irrationali $\sqrt{\left(Q + \frac{A}{x} - \frac{P}{xx}\right)}$, quae abibit in

$$\frac{1}{p} \sqrt{(Qpp + Ap(1 - q \operatorname{cof} v) - P(1 - q \operatorname{cof} v)^2)}$$

et euoluta dabit

$$\frac{1}{p} \sqrt{(Qpp + Ap - P - Apq \operatorname{cof} v + 2Pq \operatorname{cof} v - Pqq \operatorname{cof} v^2)}$$

quae vt reducatur ad formam $V \operatorname{fin} v$, ſtatuatur

primo $2P - Ap = 0$
tum vero $Qpp + Ap - P = Pqq$

ac noſtra formula fiet $= \frac{1}{p} \sqrt{Pqq \operatorname{fin} v^2} = \frac{q \operatorname{fin} v}{p} \sqrt{P}$, habebimusque

$$dx = -\frac{q\,d\omega \sin v}{p} V 2P \quad \text{et}$$

$$dQ = -\frac{M\,d\omega}{x} V 2P + \frac{Nq\,d\omega \sin v}{p} V 2P$$

VII.

Cum iam fit $2P - Ap = o$; erit $P = \frac{1}{2}Ap$: quo valore in altera formula fubftituto orietur:

$$Qpp + \frac{1}{2}Ap = \frac{1}{2}Apqq \quad \text{feu} \quad Q = -\frac{A}{2p}(1-qq)$$

Sumantur nunc differentialia; eritque

$dP = -Mx\,d\omega V 2P = \frac{1}{2}A\,dp$, quae ob $2P = Ap$ abit in hanc

$$-Mx\,d\omega V Ap = \frac{1}{2}A\,dp, \text{ fiue } dp = -\frac{2Mx\,d\omega}{A} V Ap$$

vel etiam $V Ap = -\int M x\,d\omega$

Simili modo erit

$$dQ = +\frac{A\,dp(1-qq)}{2pp} + \frac{Aq\,dq}{p} = -\frac{Mx\,d\omega(1-qq)}{pp} V Ap + \frac{Aq\,dq}{p}$$

ideoque

$$\frac{Aq\,dq}{p} = M\,d\omega\left(\frac{x(1-qq)}{pp} - \frac{1}{x}\right) V Ap + \frac{Nq\,d\omega \sin v}{p} V Ap$$

At eft $\frac{x(1-qq)}{pp} - \frac{1}{x} = \frac{x}{pp}\left(1 - qq - \frac{pp}{xx}\right) = \frac{x}{pp}(1 - qq - 1 + 2q\cos v - qq\cos v^2)$

fiue $\frac{x(1-qq)}{pp} - \frac{1}{x} = \frac{qx}{pp}(2\cos v - q - q\cos v^2)$

Hinc ergo colligitur:

$$dq = \frac{Mx\,d\omega}{Ap}(2\cos v - q - q\cos v^2) V Ap + \frac{N\,d\omega \sin v}{A} V Ap \text{ fiue}$$

$$dq = d\omega\left(\frac{M}{A}\left(2\cos v - \frac{q \sin v^2}{1 - q\cos v}\right) + \frac{N}{A}\sin v\right) V Ap$$

VIII.

VIII.

Inuenta iam relatione differentialium dx, dp et dq ad differentiale temporis $d\omega$ scilicet:

$$dx = -\frac{q\,d\omega \sin v}{q} \sqrt{Ap}; \quad dp = -\frac{2Mx\,d\omega}{A}\sqrt{Ap}$$

$$\text{et } dq = d\omega\left(\frac{M}{A}\left(2\cos v - \frac{q \sin v^2}{1-q\cos v}\right) + \frac{N}{A}\sin v\right)\sqrt{Ap}$$

superest, vt quoque relationem elementi anomaliae dv definiamus. Cum igitur sit

$x = \frac{p}{1-q\cos v}$, erit $1 - q\cos v = \frac{p}{x}$; hincque differentiando

$$q\,dv \sin v = dq \cos v + \frac{dp}{x} - \frac{p\,dx}{xx};$$

substituantur valores pro dq, dp et dx inuenti; ac diuisione facta per $q \sin v$ prodibit

$$dv = \frac{d\omega}{xx}\sqrt{Ap} - \frac{d\omega}{q}\left(\frac{M}{A}\left(2\sin v + \frac{q\sin v\cos v}{1-q\cos v}\right) - \frac{N}{A}\cos v\right)\sqrt{Ap}$$

Pro elemento autem longitudinis $d\Phi$ ob $2P = Ap$, ex antecedentibus habemus:

$$d\Phi = \frac{d\omega}{xx}\sqrt{Ap} = \frac{d\omega(1-q\cos v)^2}{pp}\sqrt{Ap}$$

IX.

Ex his formulis statim se offert motus apogei; cum enim longitudo apogei sit $= \Phi - v$, erit eius differentiale pro tempusculo $d\omega$:

$$d\Phi - dv = \frac{d\omega}{q}\left(\frac{M}{A}\left(2\sin v + \frac{q\sin v\cos v}{1-q\cos v}\right) - \frac{N}{A}\cos v\right)\sqrt{Ap}$$

cuius ergo integrale praebebit verum motum apogei cum omnibus inaequalitatibus, quibus perturbatur. Vnde

 qui-

quidem perſpicitur, quod per ſe eſt manifeſtum, ſi quantitates M et N euaneſcerent, motum apogei fore nullum, ſeu apogeum perpetuo in loco fixo eſſe permanſurum. Deinde etiam iuuabit notaſſe has formulas:

$$d.\ q\cos v = -q\,d\Phi \sin v + \frac{2M}{A}\, d\omega \sqrt{Ap}$$

$$d.\ q\sin v = +q\,d\Phi \cos v + d\omega\left(\frac{N}{A} - \frac{M}{A}\cdot\frac{q\sin v}{1-q\cos v}\right)\sqrt{Ap}$$

Tandem quoque habemus ex motu ſolis $du = d\theta = \frac{d\omega(1-e\cos u)^2}{(1-ee)\sqrt{(1-ee)}}$ ideoque

$$d\eta = d\Phi - d\theta = d\omega\left(\frac{(1-q\cos v}{pp}\sqrt{Ap} - \frac{(1-e\cos u)^2}{(1-ee)\sqrt{(1-ee)}}\right)$$

X.

Inuentis nunc omnium differentialium relationibus ad elementum temporis $d\omega$, euoluamus valores litterarum M et N, ac primo quidem cum ſit $z = \sqrt{(yy - 2xy\cos\eta + xx\ \text{ſec.}\psi^2)}$; quoniam quantitas z nonniſi in terminis minimis occurrit, pro ſec. ψ tuto unitas ſcribi poterit, et quia y tantopere excedit x, erit proxime

$$\frac{1}{z^3} = \frac{1}{y^3} + \frac{3x}{y^4}\cos\eta + \frac{3xx}{2y^5}(5\cos\eta^2 - 1) \quad \text{ſiue}$$

$$\frac{1}{z^3} = \frac{1}{y^3} + \frac{3x}{y^4}\cos\eta + \frac{3xx}{4y^5}(3 + 5\cos 2\eta)$$

Ideoque hinc habebitur:

$$\frac{y}{z^3} - \frac{1}{yy} = \frac{3x}{y^3}\cos\eta + \frac{3xx}{4y^4}(3 + 5\cos 2\eta)$$

Vnde

Vnde obtinemus:

$$M = a^3\left(\frac{3x}{2y^3}\sin 2\eta + \frac{3xx}{8y^4}(\sin\eta + 5\sin 3\eta)\right)$$

$$N = \frac{a^3(♁ + ☽)}{☉}\cos\psi^3\left(\frac{1}{xx} - \frac{1}{bb}\right) - \frac{A}{xx}$$

$$- a^3\left(\frac{x}{2y^3}(1+3\cos 2\eta) + \frac{3xx}{8y^4}(3\cos\eta + 5\cos 3\eta)\right)$$

XI.

Cum sit proxime $\cos\psi^3 = 1 - \frac{3}{4}\tang\rho^2 - \frac{3}{4}\tang\rho^2\cos 2(\phi-\pi)$, eius valor vnitate erit minor, atque ex parte constante, et parte variabili constabit, quae illa multo erit minor. Ponatur ergo

$\cos\psi^3 = \lambda + \Pi$; vt sit $\Pi = 1 - \lambda - \frac{3}{4}\tang\rho^2 - \frac{3}{4}\tang\rho^2\cos 2(\phi-\pi)$

vbi λ denotat partem constantem vnitate proxime aequalem, Π vero partem variabilem.

Erit ergo:

$$N = \frac{\lambda a^3(♁ + ☽)}{☉}\left(\frac{1}{xx} - \frac{1}{bb}\right) - \frac{A}{xx} + \frac{a^3(♁ + ☽)}{☉}\Pi\left(\frac{1}{xx} - \frac{1}{bb}\right)$$

$$- a^3\left(\frac{x}{2y^3}(1+3\cos 2\eta) + \frac{3xx}{8y^4}(3\cos\eta + 5\cos 3\eta)\right)$$

Statuatur nunc $A = \frac{\lambda a^3(♁ + ☽)}{☉}$; vt fiat

$$N = -\frac{A}{bb} + A\Pi\left(\frac{1}{xx} - \frac{1}{bb}\right)$$

$$- a^3\left(\frac{x}{2y^3}(1+3\cos 2\eta) + \frac{3xx}{8y^4}(3\cos\eta + 5\cos 3\eta)\right)$$

ac ponatur breuitatis gratia: $p = b(1+\xi)$

erit $\frac{\sqrt{Ap}}{pp} = \sqrt{\frac{A}{b^3(1+\xi)^3}} = (1-\frac{3}{2}\xi+\frac{15}{8}\xi^2)\sqrt{\frac{A}{b^3}}$

ob ξ prae 1 vehementer paruum, fitque porro:

$$\sqrt{\frac{A}{b^3}} = \sqrt{\frac{\lambda a^3(♁+☽)}{☉}} = m,$$

atque habebitur $d\Phi = m\,d\omega\,(1-\frac{3}{2}\xi+\frac{15}{8}\xi^2)(1-q\cos v)^2$

XII.

Substituantur nunc pro x et y valores $\frac{p}{1-q\cos v}$ et $\frac{c}{1-e\cos u}$, eritque

$$M = a^3\left(\frac{3p(1-e\cos u)^3}{2c^3(1-q\cos v)}\sin 2\eta + \frac{3pp(1-e\cos u)^4}{8c^4(1-q\cos v)^2}(\sin\eta+5\sin 3\eta)\right)$$

$$N = -\frac{A}{bb} + A\Pi\left(\frac{(1-q\cos v)^2}{pp} - \frac{1}{bb}\right)$$

$$-a^3\left(\frac{p(1-e\cos u)^3}{2c^3(1-q\cos v)}(1+3\cos 2\eta) + \frac{3pp(1-e\cos u)^4}{8c^4(1-q\cos v)^2}(3\cos\eta+5\cos 3\eta)\right)$$

vbi quidem quoque terminus $\frac{A\Pi}{bb}$ prae termino $\frac{A}{bb}$ omitti potest. Nunc vt hinc valores $\frac{M}{A}\sqrt{Ap}$ et $\frac{N}{A}\sqrt{Ap}$ commode exprimantur, erit

$$\frac{a^3 b}{Ac^3}\sqrt{Ab} = \frac{a^3}{mc^3} = \frac{1}{m(1-ee)^3} = \frac{1+3ee}{m}$$

quoniam in his terminis minimis pro $1-ee$ scribere licet 1. Tum vero sit $\frac{b}{c} = n$, eritque n fractio valde parua.

XIII.

XIII.

Factis ergo his substitutionibus, ob $p = b(1+\xi)$ habebimus:

$$\frac{M}{A}\sqrt{Ap} = \frac{3(1+3ee)}{2m}\,\frac{(1-e\cos u)^3}{1-q\cos v}\,(1+\tfrac{3}{2}\xi)\sin 2\eta$$
$$+\frac{3n}{8m}\,\frac{(1-e\cos u)^4}{(1-q\cos v)^2}\,(1+\tfrac{5}{2}\xi)(\sin\eta + 5\sin 3\eta)$$

Pro altera valore $\frac{N}{A}\sqrt{Ap}$ statuatur terminus minimus:

$$\frac{\sqrt{Ab}}{bb} = \sqrt{\frac{\lambda a^3 b(\delta+\mathfrak{D})}{\odot b^4}} = i \;;\; \text{eritque}$$

$$\frac{N}{A}\sqrt{Ap} = -\frac{(1+3ee)}{2m}\,\frac{(1-e\cos u)^3}{1-q\cos v}\,(1+\tfrac{3}{2}\xi)(1+3\cos 2\eta)$$
$$-\frac{3n}{8m}\,\frac{(1-e\cos u)^4}{(1-q\cos v)^2}\,(1+\tfrac{5}{2}\xi)(3\cos\eta + 5\cos 3\eta)$$
$$+ m(1-q\cos v)^2(1-\tfrac{3}{2}\xi)\,\Pi - i$$

vbi notari oportet, terminos per n multiplicatos ratione praecedentium esse minimos; tum vero quantitates ξ et Π atque multo magis i esse fractiones prae vnitate fere euanescentes.

XIV.

Quoniam hi ipsi termini quantitates M et N inuoluentes sunt valde parui, in iis sine errore altiores potestates vtriusque excentricitatis q et e negligi possunt. In terminis ergo primis simpliciter per m diuisis excentricitates tantum ad duas dimensiones introducantur, in terminis autem per $\frac{n}{m}$ multiplicatis penitus omittantur, quia fractio n iam fere quadrato excentricitatis q aequiualet. In termino autem littera minima Π affecto, quia is per numerum m satis magnum, vtpote 13 fere, est multiplicatus, excentricitas q vnius dimensionis retineatur.

His obseruatis habebimus:

$$\frac{M}{A}\sqrt{Ap}=\left\{\begin{array}{l}
+\frac{3}{2m}(1+\frac{9}{2}ee+\frac{1}{2}qq)\sin 2\eta+\frac{3q}{4m}\sin(2\eta-v)\\
+\frac{3q}{4m}\sin(2\eta+v)-\frac{9e}{4m}\sin(2\eta-u)-\frac{9e}{4m}\sin(2\eta+u)\\
+\frac{3qq}{8m}\sin(2\eta-2v)+\frac{3qq}{8m}\sin(2\eta+2v)\\
+\frac{9ee}{8m}\sin(2\eta-2u)+\frac{9ee}{8m}\sin(2\eta+2u)\\
-\frac{9eq}{8m}\sin(2\eta-v+u)-\frac{9eq}{8m}\sin(2\eta+v-u)\\
-\frac{9eq}{8m}\sin(2\eta-v-u)-\frac{9eq}{8m}\sin(2\eta+v+u)\\
+\frac{9}{4m}\xi\sin 2\eta+\frac{9}{8m}q\xi\sin(2\eta-v)+\frac{9}{8m}q\xi\sin(2\eta+v)\\
-\frac{27}{8m}e\xi\sin(2\eta-u)-\frac{27}{8m}e\xi\sin(2\eta+u)\\
+\frac{3n}{8m}\sin\eta+\frac{15n}{8m}\sin 3\eta
\end{array}\right.$$

$\frac{N}{A}\sqrt{Ap}=$

$$
\frac{N}{A} v A p = \left\{
\begin{array}{l}
-\frac{1}{2m}\left(1+\frac{3}{2}ee+\frac{1}{2}qq\right)-\frac{3}{2m}\left(1+\frac{3}{2}ee+\frac{1}{2}qq\right)\cos 2\eta \\
-\frac{q}{2m}\cos v+\frac{3e}{2m}\cos u \\
-\frac{3q}{4m}\cos(2\eta-v)-\frac{3q}{4m}\cos(2\eta+v) \\
+\frac{9e}{4m}\cos(2\eta-u)+\frac{9e}{4m}\cos(2\eta+u) \\
-\frac{qq}{4m}\cos 2v+\frac{3eq}{4m}\cos(v-u)+\frac{3eq}{4m}\cos(v+u) \\
-\frac{3ee}{4m}\cos 2u \\
-\frac{3qq}{8m}\cos(2\eta-2v)-\frac{3qq}{8m}\cos(2\eta+2v) \\
-\frac{9ee}{8m}\cos(2\eta-2u)-\frac{9ee}{8m}\cos(2\eta+2u) \\
+\frac{9eq}{8m}\cos(2\eta-v+u)+\frac{9eq}{8m}\cos(2\eta+v-u) \\
+\frac{9eq}{8m}\cos 2\eta-v-u)+\frac{9eq}{8m}\cos(2\eta+v+u) \\
-\frac{3}{4m}\xi-\frac{9}{4m}\xi\cos 2\eta-\frac{3q}{8m}\xi\cos v \\
-\frac{9q}{8m}\xi\cos(2\eta-v)-\frac{9q}{8m}\xi\cos(2\eta+v) \\
+\frac{9e}{4m}\xi\cos u+\frac{27e}{8m}\xi\cos(2\eta-u) \\
+\frac{27e}{8m}\xi\cos(2\eta+u)-\frac{9n}{8m}\cos\eta-\frac{15n}{8m}\cos 3\eta \\
+m\Pi-2mq\Pi\cos v-\frac{3}{2}m\xi\Pi-i
\end{array}
\right.
$$

XV.

XV.

Quaeramus igitur valores euolutos nostrorum differentialium ad elementum temporis applicatorum: ac primo quidem habebimus:

$$\frac{d\Phi}{d\omega} = m(1+\tfrac{1}{2}qq) - 2mq\cos v + \tfrac{3}{2}mqq\cos 2v - \tfrac{3}{2}m(1+\tfrac{1}{2}qq)\xi + 3mq\xi\cos v - \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{15}{8}m\xi\xi$$

$\frac{du}{d\omega} = \frac{d\theta}{d\omega} = 1 + 2ee - 2e\cos u + \tfrac{5}{2}ee\cos 2u$; vnde concludimus

$$\frac{d\eta}{d\omega} = m(1+\tfrac{1}{2}qq) - 1 - 2ee - 2mq\cos v + 2e\cos u + \tfrac{1}{2}mqq\cos 2v - \tfrac{1}{2}ee\cos 2u - \tfrac{3}{2}m(1+\tfrac{1}{2}qq)\xi + 3mq\xi\cos v - \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{15}{8}m\xi\xi$$

Deinde cum sit $\frac{dp}{d\omega} = -2x.\frac{M}{A}\sqrt{Ap} = -\frac{2b(1+\xi)}{1-q\cos v}.\frac{M}{A}\sqrt{Ap}$, ob $p = b(1+\xi)$ erit $\frac{d\xi}{d\omega} =$

$$\left(-2(1+\tfrac{1}{2}qq) - 2q\cos v - qq\cos 2v - 2\xi - 2q\xi\cos v\right)\frac{M}{A}\sqrt{Ap}$$

ac valorem pro $\frac{M}{A}\sqrt{Ap}$ inuentum substituendo obtinebimus sequentes formulas:

$\frac{d\xi}{d\omega} =$

$$\frac{d\xi}{d\omega} = \left\{ \begin{array}{l} -\frac{3}{m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\sin 2\eta - \frac{3q}{m}\sin(2\eta-v) - \frac{3q}{m}\sin(2\eta+v) \\ +\frac{9e}{2m}\sin(2\eta-u) + \frac{9e}{2m}\sin(2\eta+u) \\ -\frac{9qq}{4m}\sin(2\eta-2v) - \frac{9qq}{4m}\sin(2\eta+2v) \\ -\frac{9ee}{4m}\sin(2\eta-2u) - \frac{9ee}{4m}\sin(2\eta+2u) \\ +\frac{9eq}{2m}\sin(2\eta-v+u) + \frac{9eq}{2m}\sin(2\eta+v-u) \\ +\frac{9eq}{2m}\sin(2\eta-v-u) + \frac{9eq}{2m}\sin(2\eta+v+u) \\ -\frac{15}{2m}\xi\sin 2\eta - \frac{15q}{2m}\xi\sin(2\eta-v) - \frac{15q}{2m}\xi\sin(2\eta+v) \\ +\frac{45e}{4m}\xi\sin(2\eta-u) + \frac{45e}{4m}\xi\sin(2\eta+u) \\ -\frac{3n}{4m}\sin\eta - \frac{15n}{4m}\sin 3\eta \end{array} \right.$$

XVI.

Porro cum ſit $\frac{q\sin v^2}{1-q\cos v} = \frac{q-q\cos 2v}{2(1-q\cos v)} =$ $\frac{1}{2}q - \frac{1}{2}q\cos 2v + \frac{1}{4}qq\cos v - \frac{1}{4}qq\cos 3v$; erit

$$\frac{dq}{d\omega} = (2\cos v - \frac{1}{2}q + \frac{1}{2}q\cos 2v - \frac{1}{4}qq\cos v + \frac{1}{4}qq\cos 3v)\frac{M}{A}\sqrt{Ap} + \sin v.\frac{N}{A}\sqrt{Ap}$$

Facta ergo ſubſtitutione valorum pro $\frac{M}{A}\sqrt{Ap}$ et $\frac{N}{A}\sqrt{Ap}$ inuentorum, habebitur:

 $\frac{dq}{d\omega} =$

$$\frac{dq}{d\omega} = \left\{ \begin{array}{l}
+ \frac{9}{4m}\left(1+\frac{9}{2}ee+\frac{5}{12}qq\right)\sin(2\eta-v) + \frac{3}{4m}\left(1+\frac{9}{2}ee+\frac{3}{4}qq\right) \\
\quad \sin(2\eta+v) - \frac{1}{2m}\left(1+\frac{9}{2}ee+\frac{1}{4}qq\right)\sin v \\
+ \frac{3q}{4m}\sin 2\eta + \frac{3q}{2m}\sin(2\eta-2v) + \frac{3q}{4m}\sin(2\eta+2v) - \frac{q}{4m}\sin 2v \\
- \frac{27e}{8m}\sin(2\eta-v-u) - \frac{27e}{8m}\sin(2\eta-v+u) - \frac{9e}{8m}\sin(2\eta+v-u) \\
- \frac{9e}{8m}\sin(2\eta+v+u) + \frac{3e}{4m}\sin(v-u) + \frac{3e}{4m}\sin(v+u) \\
+ \frac{15qq}{16m}\sin(2\eta-3v) + \frac{9qq}{16m}\sin(2\eta+3v) - \frac{qq}{8m}\sin 3v \\
- \frac{9eq}{8m}\sin(2\eta-u) - \frac{9eq}{8m}\sin(2\eta+u) - \frac{9eq}{4m}\sin(2\eta-2v+u) \\
- \frac{9eq}{4m}\sin(2\eta-2v-u) - \frac{9eq}{8m}\sin(2\eta+2v-u) - \frac{9eq}{8m}\sin(2\eta+2v+u) \\
+ \frac{27ee}{16m}\sin(2\eta-v-2u) + \frac{27ee}{16m}\sin(2\eta-v+2u) \\
+ \frac{9ee}{16m}\sin(2\eta+v-2u) + \frac{9ee}{16m}\sin(2\eta+v+2u) \\
+ \frac{3eq}{8m}\sin(2v-u) + \frac{3eq}{8m}\sin(2v+u) \\
- \frac{3ee}{8m}\sin(v-2u) - \frac{3ee}{8m}\sin(v+2u) \\
+ \frac{27}{8m}\xi\sin(2\eta-v) + \frac{9}{8m}\xi\sin(2\eta+v) - \frac{3}{4m}\xi\sin v
\end{array} \right.$$

$$+ \frac{9q}{8m}$$

$$+\frac{9q}{8m}\xi\sin 2\eta+\frac{9q}{4m}\xi\sin(2\eta-2v)+\frac{9q}{8m}\xi\sin(2\eta+2v)$$
$$-\frac{3q}{8m}\xi\sin 2v+\frac{9e}{8m}\xi\sin(v-u)+\frac{9e}{8m}\xi\sin(v+u)$$
$$-\frac{81e}{16m}\xi\sin(2\eta-v-u)-\frac{81e}{16m}\xi\sin(2\eta-v+u)$$
$$-\frac{27e}{16}\xi\sin(2\eta+v-u)-\frac{27e}{16m}\xi\sin(2\eta+v+u)$$
$$+\frac{15n}{16m}\sin(\eta-v)-\frac{3n}{16m}\sin(\eta+v)$$
$$+\frac{45n}{16m}\sin(3\eta-v)+\frac{15n}{16m}\sin(3\eta+v)$$
$$+m\Pi\sin v-mq\Pi\sin 2v-\tfrac{3}{2}m\xi\Pi\sin v-i\sin v$$

XVII.

Deinde cum sit $\frac{q\sin v\cos v}{1-q\cos v}=\frac{q\sin 2v}{2(1-q\cos v)}=$ $=\frac{1}{2}q\sin 2v+\frac{1}{4}qq\sin v+\frac{1}{4}qq\sin 3v$; erit pro motu elementari apogei:

$$\frac{q(d\Phi-dv)}{d\omega}=(2\sin v+\tfrac{1}{2}q\sin 2v+\tfrac{1}{4}qq\sin v$$
$$+\tfrac{1}{4}qq\sin 3v)\frac{M}{A}\sqrt{Ap}-\cos v.\frac{N}{A}\sqrt{Ap}$$

ac facta substitutione obtinebitur:

$\frac{q(d\Phi-dv)}{d\omega}=$

$$\frac{q(d\Phi - dv)}{dw} = \left\{ \begin{array}{l}
+ \frac{9}{4m}(1 + \frac{9}{2}ee + \frac{7}{12}qq)\cos(2\eta - v) - \frac{3}{4m}(1 + \frac{9}{2}ee + \frac{1}{4}qq) \\
\quad \cos(2\eta + v) + \frac{1}{2m}(1 + \frac{9}{2}ee + \frac{3}{4}qq)\cos v \\
+ \frac{3q}{4m}\cos 2\eta + \frac{3q}{2m}\cos(2\eta - 2v) - \frac{3q}{4m}\cos(2\eta + 2v) \\
+ \frac{q}{4m}\cos 2v + \frac{q}{4m} - \frac{3e}{4m}\cos(v - u) - \frac{3e}{4m}\cos(v + u) \\
- \frac{27e}{8m}\cos(2\eta - v - u) - \frac{27e}{8m}\cos(2\eta - v + u) \\
+ \frac{9e}{8m}\cos(2\eta + v - u) + \frac{9e}{8m}\cos(2\eta + v + u) \\
+ \frac{15qq}{16m}\cos(2\eta - 3v) - \frac{9qq}{16m}\cos(2\eta + 3v) + \frac{qq}{8m}\cos 3v \\
- \frac{9eq}{8m}\cos(2\eta - u) - \frac{9eq}{8m}\cos(2\eta + u) - \frac{3eq}{4m}\cos u \\
- \frac{3eq}{8m}\cos(2v - u) - \frac{3eq}{8m}\cos(2v + u) \\
- \frac{9eq}{4m}\cos(2\eta - 2v + u) - \frac{9eq}{4m}\cos(2\eta - 2v - u) \\
+ \frac{9eq}{8m}\cos(2\eta + 2v - u) + \frac{9eq}{8m}\cos(2\eta + 2v + u) \\
+ \frac{27ee}{16m}\cos(2\eta - v - 2u) + \frac{27ee}{16m}\cos(2\eta - v + 2u) \\
- \frac{9ee}{16m}\cos(2\eta + v - 2u) - \frac{9ee}{16m}\cos(2\eta + v + 2u) \\
+ \frac{3ee}{8m}\cos(v - 2u) + \frac{3ee}{8m}\cos(v + 2u)
\end{array} \right.$$

$$+ \frac{27}{8m}$$

$$
\left|\begin{array}{l}
+\frac{27}{8m}\xi\cos(2\eta-v)-\frac{9}{8m}\xi\cos(2\eta+v)+\frac{3}{4m}\xi\cos v \\
+\frac{9q}{8m}\xi\cos 2\eta+\frac{9q}{4m}\xi\,\mathrm{cs}(2\eta-2v)-\frac{9q}{8m}\xi\,\mathrm{cs}(2\eta+2v) \\
+\frac{3q}{8m}\xi+\frac{3q}{8m}\xi\cos 2v \\
-\frac{81e}{16m}\xi\cos(2\eta-v-u)+\frac{27e}{16m}\xi\cos(2\eta+v-u) \\
-\frac{81e}{16m}\xi\cos(2\eta-v+u)+\frac{27e}{16m}\xi\cos(2\eta+v+u) \\
-\frac{9e}{8m}\xi\cos(v-u)-\frac{9e}{8m}\xi\cos(v+u) \\
+\frac{15n}{16m}\cos(\eta-v)+\frac{3n}{16m}\cos(\eta+v) \\
+\frac{45n}{16m}\cos(3\eta-v)-\frac{15n}{16m}\cos(3\eta+v) \\
-m\Pi\cos v+mq\Pi+mq\Pi\cos 2v+\frac{3}{2}m\xi\Pi\cos v+i\,\mathrm{cs}\,v
\end{array}\right.
$$

XVIII.

Euoluamus ſimili modo valorem differentialium $d\pi$ et $d\varphi$, et cum ſit $\frac{y}{z^3}-\frac{1}{yy}=\frac{3x}{y^3}\,\mathrm{cs}\,\eta+\frac{3xx}{4y^4}(3+5\,\mathrm{cs}\,2\eta)$ et $d\Phi=\frac{d\omega}{xx}\sqrt{Ap}$; erit

$$d\pi=-\frac{a^3 x\,d\omega}{\sqrt{Ap}}\left(\frac{3x}{y^3}\cos\eta+\frac{3xx}{4y^4}(3+5\cos 2\eta)\right)\sin(\theta-\pi)\sin(\Phi-\pi)$$

Subſtitutis autem valoribus $x=\frac{p}{1-q\,\mathrm{cs}\,v}$, $y=\frac{c}{1-e\,\mathrm{cs}\,u}$; $p=b(1+\xi)$; $\sqrt{Ap}=m\sqrt{b^3p}=mbb(1+\frac{1}{2}\xi)$, $\frac{a^3}{c^3}=\frac{1}{(1-ee)^3}=1+3ee$ et $\frac{b}{c}=n$, erit

$$d\pi=-\frac{d\omega\sin(\theta-\pi)\sin(\Phi-\pi)}{m}\left[\frac{3(1+\frac{3}{2}\xi)(1+3ee)(1-e\,\mathrm{cs}\,u)^3}{(1-q\cos v)^2}\,\mathrm{cs}\,\eta+\frac{1}{4}n(3+5\,\mathrm{cs}\,2\eta)\right]$$

 Negle-

Neglectis igitur terminis, qui nullum valorem sensibilem continent, habebimus

$$\frac{d\pi}{d\omega} = \left\{ \begin{array}{l}
-\frac{3}{4m}(1+\frac{3}{2}ee+\frac{3}{2}qq) - \frac{3}{4m}(1+\frac{3}{2}ee+\frac{3}{2}qq)\cos 2\eta \\
+\frac{3}{4m}(1+\frac{3}{2}ee+\frac{3}{2}qq)\cos 2(\phi-\pi) + \frac{3}{4m}(1+\frac{3}{2}ee+\frac{3}{2}qq)\cos 2(\theta-\pi) \\
-\frac{3q}{2m}\cos v - \frac{3q}{4m}\cos(2\eta-v) - \frac{3q}{4m}\cos(2\eta+v) \\
+\frac{9e}{4m}\cos u + \frac{9e}{8m}\cos(2\eta-u) + \frac{9e}{8m}\cos(2\eta+u) \\
+\frac{3q}{4m}\cos(2\phi-2\pi-v) + \frac{3q}{4m}\cos(2\phi-2\pi+v) \\
+\frac{3q}{4m}\cos(2\theta-2\pi-v) + \frac{3q}{4m}\cos(2\theta-2\pi+v) \\
-\frac{9e}{8m}\cos(2\phi-2\pi-u) - \frac{9e}{8m}\cos(2\phi-2\pi+u) \\
-\frac{9e}{8m}\cos(2\theta-2\pi-u) - \frac{9e}{8m}\cos(2\theta-2\pi+u) \\
-\frac{9}{8m}\xi - \frac{9}{8m}\xi\cos 2\eta \\
+\frac{9}{8m}\xi\cos(2\phi-2\pi) + \frac{9}{8m}\xi\cos(2\theta-2\pi) \\
-\frac{11n}{16m}\cos\eta + \frac{3n}{8m}\cos(\phi+\theta-2\pi) - \frac{5n}{16m}\cos 3\eta \\
+\frac{5n}{16m}\cos(3\phi-\theta-2\pi) + \frac{5n}{16m}\cos(3\theta-\phi-2\pi)
\end{array} \right.$$

XIX.

XIX.

Simili autem modo praecedentem valorem per tang $(\phi-\pi)$ diuidendo prodibit differentiale logarithmi tangentis inclinationis ϱ, erit enim

$$\frac{d.l\,\mathrm{tang}\,\varrho}{du} = \left\{ \begin{aligned}
&+ \frac{3}{4m}(1+\tfrac{9}{2}ee+\tfrac{3}{2}qq)\sin 2\eta - \frac{3}{4m}(1+\tfrac{9}{2}ee+\tfrac{3}{2}qq) \\
&\qquad \sin 2(\phi-\pi) - \frac{3}{4m}(1+\tfrac{9}{2}ee+\tfrac{3}{2}qq)\sin 2(\theta-\pi) \\
&+ \frac{3q}{4m}\sin(2\eta-v) + \frac{3q}{4m}\sin(2\eta+v) \\
&- \frac{9e}{8m}\sin(2\eta-u) - \frac{9e}{8m}\sin(2\eta+u) \\
&- \frac{3q}{4m}\sin(2\phi-2\pi-v) - \frac{3q}{4m}\sin(2\phi-2\pi+v) \\
&- \frac{3q}{4m}\sin(2\theta-2\pi-u) - \frac{3q}{4m}\sin(2\theta-2\pi+u) \\
&+ \frac{9e}{8m}\sin(2\phi-2\pi-u) + \frac{9e}{8m}\sin(2\phi-2\pi+u) \\
&+ \frac{9e}{8m}\sin(2\theta-2\pi-u) + \frac{9e}{8m}\sin(2\theta-2\pi+u) \\
&+ \frac{9}{8m}\xi\sin 2\eta - \frac{9}{8m}\xi\sin(2\phi-2\pi) - \frac{9}{8m}\xi\sin(2\theta-2\pi) \\
&+ \frac{n}{16m}\sin\eta - \frac{3n}{8m}\sin(\phi+\theta-\pi) + \frac{5n}{16m}\sin 3\eta \\
&- \frac{5n}{16m}\sin(3\phi-\theta-2\pi) - \frac{5n}{16m}\sin(3\theta-\phi-2\pi)
\end{aligned} \right.$$

XX.

Quo iam facilius has formulas admodum complicatas euoluere queamus, quadruplicis generis terminos distingui conuenit. Primum scilicet genus eos complectitur terminos, qui tantum ab excentricitate orbitae lunaris pendent, neque excentricitatem solis, neque parallaxin solis seu literam n, neque inclinationem orbitae lunaris seu litteram Π inuoluunt. Ad secundum genus refero terminos, qui ad primum genus insuper excentricitatem solis adiungunt. Ad tertium autem eos, qui praeterea parallaxin solis seu litteram n inducunt. In quarto autem eas inaequalitates, quae insuper ab obliquitate orbitae lunaris proueniunt, complexurus sum. Ab inaequalitatibus ergo primi generis exordiar, ideoque cum excentricitatem solis e, tum eius parallaxin, tum quoque obliquitatem orbitae lunaris reiiciam

INUESTIGATIO INAEQUALITATUM LUNAE PRIMI GENERIS.

XXI.

Neglectis ergo excentricitate solis cum eius parallaxi et obliquitate orbitae lunaris, has habebimus aequationes:

$\frac{d\xi}{d\omega} =$

$$\frac{d\xi}{d\omega} = \begin{cases} -\frac{3}{m}(1+\frac{1}{2}qq)\sin 2\eta - \frac{3q}{m}\sin(2\eta-v) - \frac{3q}{m}\sin(2\eta+v) \\ -\frac{9qq}{4m}\sin(2\eta-2v) - \frac{9qq}{4m}\sin(2\eta+2v) \\ -\frac{15}{2m}\xi\sin 2\eta - \frac{15q}{2m}\xi\sin(2\eta-v) - \frac{15q}{2m}\xi\sin(2\eta+v) \end{cases}$$

$$\frac{dq}{d\omega} = \begin{cases} +\frac{9}{4m}(1+\frac{5}{12}qq)\sin(2\eta-v) + \frac{3}{4m}(1+\frac{3}{5}qq)\sin(2\eta+v) \\ -\frac{1}{2m}(1+\frac{1}{4}qq)\sin v - i\sin v \\ +\frac{3q}{4m}\sin 2\eta + \frac{3q}{2m}\sin(2\eta-2v) + \frac{3q}{4m}\sin(2\eta+2v) \\ -\frac{q}{4m}\sin 2v \\ +\frac{15qq}{16m}\sin(2\eta-3v) + \frac{9qq}{16m}\sin(2\eta+3v) - \frac{qq}{8m}\sin 3v \\ +\frac{27}{8m}\xi\sin(2\eta-v) + \frac{9}{8m}\xi\sin(2\eta+v) - \frac{3}{4m}\xi\sin v \\ +\frac{9q}{8m}\xi\sin 2\eta + \frac{9q}{4m}\xi\sin(2\eta-2v) + \frac{9q}{8m}\xi\sin(2\eta+2v) \\ -\frac{3q}{8m}\xi\sin 2v \end{cases}$$

$$\frac{q(d\Phi - dv)}{d\omega} = \left\{ \begin{array}{l}
+\frac{9}{4m}(1+\frac{7}{12}qq)\cos(2\eta - v) - \frac{3}{4m}(1+\frac{1}{4}qq)\cos(2\eta+v) \\
+\frac{1}{2m}(1+\frac{3}{4}qq)\cos v + i\cos v \\
+\frac{3q}{4m}\cos 2\eta + \frac{3q}{2m}\cos(2\eta - 2v) - \frac{3q}{4m}\cos(2\eta+2v) \\
+\frac{q}{4m}\cos 2v + \frac{q}{4m} \\
+\frac{15qq}{16m}\cos(2\eta - 3v) - \frac{9qq}{16m}\cos(2\eta+3v) + \frac{qq}{8m}\cos 3v \\
+\frac{27}{8m}\xi\cos(2\eta - v) - \frac{9}{8m}\xi\cos(2\eta+v) + \frac{3}{4m}\xi\cos v \\
+\frac{9q}{8m}\xi\cos 2\eta + \frac{9q}{4m}\xi\cos(2\eta - 2v) - \frac{9q}{8m}\xi\cos(2\eta+2v) \\
+\frac{3q}{8m}\xi + \frac{3q}{8m}\xi\cos 2v
\end{array} \right.$$

$$\frac{d\Phi}{d\omega} = m(1+\tfrac{1}{2}qq) - 2mq\cos v + \tfrac{1}{2}mqq\cos 2v - \tfrac{3}{2}m(1+\tfrac{1}{2}qq)\xi + 3mq\xi\cos v - \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{15}{8}m\xi\xi$$

$$\frac{d\eta}{d\omega} = m(1+\tfrac{1}{2}qq) - 1 - 2mq\cos v + \tfrac{1}{2}mqq\cos 2v - \tfrac{3}{2}m(1+\tfrac{1}{2}qq)\xi + 3mq\xi\cos v - \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{15}{8}m\xi\xi$$

XXII.

Hic autem primo patet valores litterarum ξ et q fine cognitis rationibus $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ definiri non posse, has autem vicissim ipsas quantitates ξ et q inuoluere. Cum autem ad valores ξ et q inueniendos non opus sit rationes

tiones $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ eo praecisionis gradu nosse, quo ipsi illi valores desiderantur; patet si valores ξ et q prope tantum veri constent, iis in rationibus $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ adhibitis, eosdem multo exactiores repertum iri. Cum igitur, si motus esset regularis, foret $\xi = 0$ et $q =$ constanti, hinc primam hypothesin constituamus. Sit ergo

$$\xi = 0 \quad \text{et} \quad q = g$$

et neglectis terminis, qui ob harum litterarum errores affici possent, vtpote valde paruis prae reliquis, habebimus proxime

$$\frac{d\Phi}{d\omega} = m(1 + \tfrac{1}{2}gg) - 2mg \cos v; \quad \frac{d\eta}{d\omega} = (1 + \tfrac{1}{2}gg) - 1 - 2mg \cos v$$

et
$$\frac{d\Phi - dv}{d\omega} = \frac{9}{4mg}(1 + \tfrac{7}{12}gg)\cos(2\eta - v) - \frac{3}{4mg}(1 + \tfrac{1}{4}gg)\cos(2\eta + v)$$
$$+ \frac{1}{2mg}(1 + \tfrac{3}{4}gg)\cos v + \frac{i}{g}\cos v + \frac{1}{4m}$$

ideoque

$$\frac{dv}{d\omega} = m(1 + \tfrac{1}{2}gg) - \frac{1}{4m} - \left(2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g}\right)\cos v$$
$$- \frac{9}{4mg}(1 + \tfrac{7}{12}gg)\cos(2\eta - v) + \frac{3}{4mg}(1 + \tfrac{1}{4}gg)\cos(2\eta + v)$$

XXIII.

Ponamus ad has formulas abbreuiandas:

$$m(1 + \tfrac{1}{2}gg) - 1 = \alpha; \qquad 2mg = \gamma$$

$$m(1 + \tfrac{1}{2}gg) - \frac{1}{4m} = \beta; \quad 2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g} = \delta$$

 et

et neglectis quadratis gg in reliquis terminis, habebimus has formulas simpliciores:

$$\frac{d\eta}{d\omega} = \alpha - \gamma \cos v$$

$$\frac{dv}{d\omega} = \beta - \delta \cos v - \frac{9}{4mg}\cos(2\eta - v) + \frac{3}{4mg}\cos(2\eta + v)$$

Tum vero pro valoribus ξ et q propius inueniendis has aequationes:

$$\frac{d\xi}{d\omega} = -\frac{3}{m}(1+\tfrac{3}{4}gg)\sin 2\eta - \frac{3g}{m}\sin(2\eta - v) - \frac{3g}{m}\sin(2\eta + v)$$

$$\frac{dq}{d\omega} = +\frac{9}{4m}(1+\tfrac{5}{12}gg)\sin(2\eta - v) + \frac{3}{4m}(1+\tfrac{3}{4}gg)\sin(2\eta + v)$$

$$-\frac{1}{2m}(1+\tfrac{1}{4}gg)\sin v - i\sin v$$

$$+\frac{3g}{4m}\sin 2\eta + \frac{3g}{2m}\sin(2\eta - 2v) + \frac{3g}{4m}\sin(2\eta + 2v) - \frac{g}{4m}\sin 2v$$

XXIV.

Fingamus ergo primo:

$$\xi = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos(2\eta - v) + \mathfrak{C}\cos(2\eta + v)$$

vbi notandum est terminos binos posteriores, vti in differentiali, multo esse minores primo. Quare cum etiam in differentialibus $d\eta$ et dv duplicis generis termini occurrant, quorum posteriores prae primis sint valde parui, in differentiatione solius primi termini totum differentialis $d\eta$ valorem pono, in duobus vero reliquis tantum valorem principalem; sic prodibit

$$\frac{d\xi}{d\omega} = -2\alpha\mathfrak{A}\sin 2\eta + \gamma\mathfrak{A}\sin(2\eta - v) + \gamma\mathfrak{A}\sin(2\eta + v)$$

$$-(2\alpha - \beta)\mathfrak{B} \quad -(2\alpha + \beta)\mathfrak{C}$$

Collato ergo hoc differentiali cum forma proposita obtinetur: $\mathfrak{A} =$

$$\mathfrak{A} = \frac{3}{2ma}(1+\tfrac{1}{2}gg)$$

$$(2\alpha-\beta)\mathfrak{B} = \gamma\mathfrak{A} + \frac{3g}{m} \quad \text{ergo} \quad \mathfrak{B} = \frac{3(\gamma+2\alpha g)}{2ma(2\alpha-\beta)}$$

$$(2\alpha+\beta)\mathfrak{C} = \gamma\mathfrak{A} + \frac{3g}{m} \quad \text{ergo} \quad \mathfrak{C} = \frac{3(\gamma+2\alpha g)}{2ma(2\alpha+\beta)}$$

XXV.

Simili modo fingatur:

$$q = g + \text{A}\cos(2\eta-v) + \text{B}\cos(2\eta+v) + \text{C}\cos v$$
$$+ \text{D}\cos 2\eta + \text{E}\cos(2\eta-2v) + \text{F}\cos(2\eta+2v) + \text{G}\cos 2v$$
$$+ \text{H}\cos 4\eta + \text{J}\cos(4\eta-2v) + \text{K}\cos(4\eta+2v)$$

vbi linea prior continet terminos multo maiores, quam binae inferiores. Hinc ergo fit differentiando secundum regulam supra datam:

$$\frac{dq}{d\omega} = -(2\alpha-\beta)\text{A}\sin(2\eta-v) - (2\alpha+\beta)\text{B}\sin(2\eta+v) - \beta\text{C}\sin v$$
$$+\left(\tfrac{1}{2}(2\gamma-\delta)\text{A} + \tfrac{1}{2}(2\gamma+\delta)\text{B} + \frac{3}{2mg}\text{C} - 2\alpha\text{D}\right)\sin 2\eta$$
$$+\left(\tfrac{1}{2}(2\gamma-\delta)\text{A} - \frac{9}{8mg}\text{C} - 2(\alpha-\beta)\text{E}\right)\sin(2\eta-2v)$$
$$+\left(\tfrac{1}{2}(2\gamma+\delta)\text{B} - \frac{3}{8mg}\text{C} - 2(\alpha+\beta)\text{F}\right)\sin(2\eta+2v)$$
$$+\left(\tfrac{1}{2}\delta\text{C} - \frac{3\text{A}+9\text{B}}{8mg} - 2\beta\text{G}\right)\sin 2v$$
$$+\left(\frac{3\text{A}+9\text{B}}{8mg} - 4\alpha\text{H}\right)\sin 4\eta$$
$$+\left(-\frac{9\text{A}}{8mg} - 2(2\alpha-\beta)\text{J}\right)\sin(4\eta-2v)$$
$$+\left(-\frac{3\text{B}}{8mg} - 2(2\alpha+\beta)\text{K}\right)\sin(4\eta+2v)$$

Hincque elicientur sequentes coefficientium valores:

$$(2\alpha-\beta)\,A = -\frac{9}{4m}\left(1+\tfrac{5}{12}gg\right)$$

$$(2\alpha+\beta)\,B = -\frac{3}{4m}\left(1+\tfrac{3}{4}gg\right)$$

$$\beta\,C = \frac{1}{2m}\left(1+\tfrac{1}{4}gg\right)+i$$

$$2\alpha\,D = \tfrac{1}{2}(2\gamma-\delta)A+\tfrac{1}{2}(2\gamma+\delta)B+\frac{3}{2mg}C-\frac{3g}{4m}$$

$$2(\alpha-\beta)\,E = \tfrac{1}{2}(2\gamma-\delta)A-\frac{9}{8mg}C-\frac{3g}{2m}$$

$$2(\alpha+\beta)\,F = \tfrac{1}{2}(2\gamma+\delta)A-\frac{3}{8mg}C-\frac{3g}{4m}$$

$$2\beta G = -\frac{3A+9B}{8mg}+\tfrac{1}{2}\delta C+\frac{g}{4m}$$

$$4\alpha H = \frac{3A+9\beta}{8mg};\quad 2(2\alpha-\beta)J = -\frac{9}{8mg}A$$

$$2(2\alpha+\beta)K = -\frac{3}{8mg}B$$

XXVI.

Cum igitur his inuentis valoribus sit multo verius:

$$z = \mathfrak{A}\cos 2\eta \quad \text{et} \quad q = g + A\cos(2\eta-v) + B\cos(2\eta+v) + C\cos v$$

vbi terminos minores data opera adhuc omitto, quia fortasse correctione egent, praecedentes operationes multo accuratius instituere atque ad ordinem terminorum vlteriorem progredi poterimus. Obtinebimus ergo:

$$\frac{d\Phi}{d\omega} = m\left(1+\tfrac{1}{2}gg-C\right) - 2mg\cos v$$
$$-m\left(\tfrac{3}{2}\mathfrak{A}+A+B\right)\cos 2\eta - mA\cos(2\eta-2v) - mB\cos(2\eta+2v) + m\left(\tfrac{1}{2}gg-C\right)\cos 2v$$

hincque $\frac{d\eta}{d\omega} = \frac{d\Phi}{d\omega} - 1.$

Porro

Porro ob $\frac{1}{q} = \frac{1}{g} - \frac{A}{gg}\cos(2\eta - v) - \frac{B}{gg}\cos(2\eta + v) - \frac{C}{gg}\cos v$ erit

$$\frac{d\Phi - dv}{d\omega} = \frac{9}{4mg}(1 + \tfrac{7}{12}gg)\cos(2\eta - v) - \frac{3}{4mg}(1 + \tfrac{1}{4}gg)\cos(2\eta + v)$$
$$+ \frac{1}{2mg}(1 + \tfrac{3}{2}gg)\cos v + \frac{i}{g}\cos v + \frac{1}{4m}\left(1 - \frac{9A + 3B - 2C}{2gg}\right)$$
$$+ \frac{1}{4m}\left(3 - \frac{A - B - 3C}{gg}\right)\cos 2\eta + \frac{1}{4m}\left(6 - \frac{2A - 9C}{2gg}\right)\cos(2\eta - 2v)$$
$$- \frac{1}{4m}\left(3 + \frac{2B - 3C}{2gg}\right)\cos(2\eta + 2v) + \frac{1}{4m}\left(1 + \frac{3A - 9B - 2C}{2gg}\right)\cos 2v$$
$$+ \frac{3A - 9B}{8mgg}\cos 4\eta - \frac{9A}{8mgg}\cos(4\eta - 2v) + \frac{3B}{8mgg}\cos(4\eta + 2v)$$

XXVII.

Ponatur ad abbreuiandum:

$$m(1 + \tfrac{1}{2}gg - C) - 1 = \alpha \quad ; \quad 2mg = \gamma$$
$$m(\tfrac{1}{2}\mathfrak{A} + A + B) = \varepsilon \quad ; \quad \text{vt fit}$$

$$\frac{d\eta}{d\omega} = \alpha - \gamma\cos v - \varepsilon\cos 2\eta$$
$$- mA\cos(2\eta - 2v) - mB\cos(2\eta + 2v) + m(\tfrac{1}{4}gg - C)\cos 2v$$

Porro fit

$$m(1 + \tfrac{1}{2}gg - C) - \frac{1}{4m}\left(1 - \frac{9A + 3B - 2C}{2gg}\right) = \mathfrak{C}$$

$$2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g} = \delta$$

$$+ m(\tfrac{1}{2}\mathfrak{A} + A + B) + \frac{1}{4m}\left(3 - \frac{A - B - 3C}{gg}\right) = \zeta$$

mA

$$mA + \frac{1}{4m}\left(6 - \frac{2A-9C}{2gg}\right) = \eta$$

$$mB - \frac{1}{4m}\left(3 + \frac{2B-3C}{2gg}\right) = \theta$$

$$m(C - \tfrac{1}{2}gg) + \frac{1}{4m}\left(1 + \frac{3A-9B-2C}{2gg}\right) = \varkappa$$

vt habeatur

$$\frac{dv}{d\omega} = \beta - \delta \operatorname{cos} v - \frac{9}{4mg}\operatorname{cos}(2\eta - v) + \frac{3}{4mg}\operatorname{cos}(2\eta + v)$$
$$- \zeta \operatorname{cos} 2\eta - \eta \operatorname{cos}(2\eta - 2v) - \theta \operatorname{cos}(2\eta + 2v) - \varkappa \operatorname{cos} 2v$$
$$- \frac{3A+9B}{8mgg}\operatorname{cos} 4\eta + \frac{9A}{8mgg}\operatorname{cos}(4\eta - 2v) - \frac{3B}{8mgg}\operatorname{cos}(4\eta + 2v)$$

vbi caueatur, ne coefficientes η, θ, cum angulis cognominibus confundantur.

XXVIII.

Opus plane non eſt, vt valores litterarum ζ et η accuratius determinemus, atque ad plures terminos, quam ante inuenimus, expediamus; verum hos ipſos terminos, quos ante inuenimus, accuratius obtinebimus, ſi litteris α et β eos valores tribuemus, quos nunc eis conuenire collegimus. Pluribus autem terminis non indigebimus tam ad longitudinem lunae Φ, quam ad eius anomaliam veram v ſatis exacte definiendam. Verum ad hoc ipſum negotium valores differentiales $\frac{d\Phi}{d\omega}$ et $\frac{d\Phi - dv}{d\omega}$, ac praecipue hunc poſteriorem, quo motus apogei continetur, accuratius euolui oportet, quoniam imprimis in motu medio apogei minimae particulae ingentis momenti eſſe poſſunt.

XXIX.

XXIX.

Cum igitur accuratius quam adhuc assumsimus sit

$\xi = \mathfrak{A} \cos 2\eta + \mathfrak{B} \cos(2\eta - v) + \mathfrak{C} \cos(2\eta + v)$ et

$$q = g + A \cos(2\eta - v) + B \cos(2\eta + v) + C \cos v$$
$$+ D \cos 2\eta + E \cos(2\eta - 2v) + F \cos(2\eta + 2v) + G \cos 2v$$
$$+ H \cos 4\eta + J \cos(4\eta - 2v) + K \cos(4\eta + 2v)$$

erit terminus ad quartum vsque ordinem extensis

$$\overset{\text{I.}}{\frac{d\Phi}{d\omega} = m(1 + \tfrac{1}{2}gg - C)} \quad \overset{\text{II.}}{- (2mg - \tfrac{3}{2}mgC + mG)\cos v}$$

III.

$$-m(\tfrac{3}{2}\mathfrak{A} + A + B)\cos 2\eta - mA\cos(2\eta - 2v) - mB\cos(2\eta + 2v)$$
$$- m(C - \tfrac{1}{2}gg)\cos 2v$$

IV.

$$+ m(\tfrac{3}{2}g\mathfrak{A} - \tfrac{3}{2}\mathfrak{B} + gA + \tfrac{1}{2}gB - D - E)\cos(2\eta - v)$$
$$+ m(\tfrac{3}{2}g\mathfrak{A} - \tfrac{3}{2}\mathfrak{C} + gB + \tfrac{1}{2}gA - D - F)\cos(2\eta + v)$$
$$+ m(\tfrac{1}{2}gA - E)\cos(2\eta - 3v) + m(\tfrac{1}{2}gB - F)\cos(2\eta + 3v)$$
$$+ m(\tfrac{1}{2}gC - G)\cos 3v$$
$$- m(H + J)\cos(4\eta - v) - m(H + K)\cos(4\eta + v) - mJ\cos(4\eta - 3v)$$
$$- mK\cos(4\eta + 3v)$$

vnde cum esset ante $\gamma = 2mg$, nunc accuratius erit

$$\gamma = 2mg - \tfrac{3}{2}mgC + mG$$

XXX.

Deinde cum nunc quoque fit accuratius:

$$\overset{\text{II.}}{\frac{1}{q} = \left(\frac{1}{g} + \frac{AA+BB+CC}{2g^3}\right)}$$

$$\overset{\text{III.}}{-\frac{A}{gg}\cos(2\eta - v) - \frac{B}{gg}\cos(2\eta + v) - \frac{C}{gg}\cos v}$$

$$\overset{\text{IV.}}{+\left(\frac{(A+B)C}{g^3} - \frac{D}{gg}\right)\cos 2\eta + \left(\frac{AC}{g^3} - \frac{E}{gg}\right)\cos(2\eta - 2v)}$$

$$+\left(\frac{BC}{g^3} - \frac{F}{gg}\right)\cos(2\eta + 2v) + \left(\frac{2AB+CC}{2g^3} - \frac{G}{gg}\right)\cos 2v$$

$$+\left(\frac{AB}{g^3} - \frac{H}{gg}\right)\cos 4\eta + \left(\frac{AA}{2g^3} - \frac{J}{gg}\right)\cos(4\eta - 2v)$$

$$+\left(\frac{BB}{g2^3} - \frac{K}{gg}\right)\cos(4\eta + 2v)$$

Hinc quoque ad terminos quarti ordinis vsque valor formulae $\frac{d\Phi - dv}{d\omega}$ definiri poſſet, ſed expreſſio prodiret tantopere complicata, vt eius euolutio ſummam requireret patientiam; neque tamen hic labor vllius foret vſus, niſi forte in motu apogei exactius eruendo: ipſae enim inaequalitates nullius forent momenti; propterea quod error in anomalia commiſſus multo minorem errorem in longitudine producit.

XXXI.

XXXI.

Ponatur ergo longitudo apogei:

$$\phi - v = \text{Const.}$$
$$+ A' \sin(2\eta - v) + B' \sin(2\eta + v) + C' \sin 2v$$
$$+ \Delta\omega + D' \sin 2\eta + E' \sin(2\eta - 2v) + F' \sin(2\eta + 2v) + G' \sin 2v$$
$$+ H' \sin 4\eta + J \sin(4\eta - 2v) + K \sin(4\eta + 2v)$$

et erit differentiando:

$$\frac{d\phi - dv}{d\omega} =$$
$$(2\alpha - \beta) A' \cos(2\eta - v) + (2\alpha + \beta) B' \cos(2\eta + v) + \beta C' \cos v$$
$$+ \Delta - \tfrac{1}{2}\delta C' + \frac{9A'}{8mg} + \frac{3B'}{8mg} - \epsilon D' - (mA - \eta) E'$$
$$- (mB + \theta) F' - \kappa G' - \frac{9AJ'}{8mgg} - \frac{3BK'}{3mgg}$$
$$\cos 2\eta \left(-\tfrac{1}{2}(2\gamma - \delta) A' - \tfrac{1}{2}(2\gamma + \delta) B' - \frac{3C'}{4mg} + 2\alpha D' \right.$$
$$\cos(2\eta - 2v) \left(-\tfrac{1}{2}(2\gamma - \delta) A' - \frac{9C'}{8mg} + 2(\alpha - \beta) E' \right.$$
$$\cos(2\eta + 2v) \left(-\tfrac{1}{2}(2\gamma + \delta) B' + \frac{3C'}{8mg} + 2(\alpha + \beta) F' \right.$$
$$\cos 2v \left(-\tfrac{1}{2}\delta C' - \frac{3A'}{8mg} - \frac{9B'}{8mg} + 2\beta G' \right.$$
$$\cos 4\eta \left(-\frac{3A'}{8mg} - \frac{9B'}{8mg} + 4\alpha H' \right.$$
$$\cos(4\eta - 2v) \left(\frac{9A'}{8mg} + 2(2\alpha - \beta) J' \right.$$
$$\cos(4\eta + 2v) \left(\frac{3B'}{8mg} + 2(2\alpha + \beta) K' \right.$$

XXXIII.

Calculo autem praecipue in primis terminis accuratius expedito est:

$$\frac{d\Phi - dv}{d\omega} =$$

$$+ \frac{1}{4mg} \cos(2\eta - v) \left(9 + \tfrac{21}{4} gg + \frac{27\,AA + 18\,BB + 15\,CC}{4gg} - \frac{3\,AB + 2\,AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g}\right)$$

$$+ \frac{1}{4mg} \cos(2\eta + v) \left(-3 - \tfrac{3}{4} gg - \frac{6\,AA - 9\,BB + 15\,CC}{4gg} + \frac{9AB + AC + 2BC}{gg} - \frac{2D - 2F - 9G - 9H + 3K}{2g}\right)$$

$$+ \frac{1}{2mg} \cos v \left(1 + \tfrac{3}{4} gg + \frac{2\,AA + 2\,BB + 3\,CC}{4gg} + \frac{AB + 6AC + 6BC}{2gg} - \frac{3D - 3E - 3F - G}{2g} + 2mi\right)$$

$$+ \frac{1}{4m} \left(1 - \frac{9\,A + 3\,B - 2\,C}{2gg}\right)$$

$$+ \frac{1}{4m} \left(3 - \frac{A - B - 3C}{gg}\right) \cos 2\eta + \frac{1}{4m} \left(1 + \frac{3A - 9B - 2C}{2gg}\right) \cos 2v$$

$$+ \frac{1}{4m} \left(6 - \frac{2A - 9C}{2gg}\right) \cos(2\eta - 2v) - \frac{1}{4m} \left(3 + \frac{2B - 3C}{2gg}\right) \cos(2\eta + 2v)$$

$$+ \frac{3A - 9B}{8mgg} \cos 4\eta - \frac{9A}{8mgg} \cos(4\eta - 2v) + \frac{3B}{8mgg} \cos(4\eta + 2v)$$

Simili

Simili autem modo ex valore ipſius $\frac{dq}{d\omega}$ accuratius erit

$$(2\alpha-\beta)\,\mathrm{A} = -\frac{1}{4m}\left(9+\tfrac{15}{4}gg+\tfrac{9}{2}\mathrm{C}\right)$$

$$(2\alpha+\beta)\,\mathrm{B} = -\frac{1}{4m}\left(3+\tfrac{9}{4}gg+3\mathrm{C}\right)$$

$$\beta\,\mathrm{C} = \frac{1}{2m}\left(1+\tfrac{1}{4}gg+\tfrac{3}{4}\mathrm{A}+2mi\right)$$

XXXIII.

Comparatione autem inſtituta reperitur:

$$(2\alpha-\beta)\,\mathrm{A}' = \frac{1}{4mg}\left(9+\tfrac{21}{4}gg+\frac{27\mathrm{AA}+18\mathrm{BB}+15\mathrm{CC}}{4gg} - \frac{3\mathrm{AB}+2\mathrm{AC}+\mathrm{BC}}{gg} - \frac{2\mathrm{D}-2\mathrm{E}+3\mathrm{G}+3\mathrm{H}-9\mathrm{J}}{2g}\right)$$

ſeu

$$(2\alpha-\beta)\mathrm{A}' = -\frac{1}{g}(2\alpha-\beta)\mathrm{A}+\frac{1}{4mg}\left(\tfrac{3}{2}gg-\tfrac{9}{2}\mathrm{C}+\frac{27\mathrm{AA}+18\mathrm{BB}+15\mathrm{CC}}{4gg} - \frac{3\mathrm{AB}+2\mathrm{AC}+\mathrm{BC}}{gg} - \frac{2\mathrm{D}-2\mathrm{E}+3\mathrm{G}+3\mathrm{H}-9\mathrm{J}}{2g}\right)$$

$$(2\alpha+\beta)\mathrm{B}' = +\frac{1}{g}(2\alpha+\beta)\mathrm{B}+\frac{1}{4mg}\left(\tfrac{1}{2}gg+3\mathrm{C}-\frac{6\mathrm{AA}-9\mathrm{BB}+15\mathrm{CC}}{4gg} + \frac{9\mathrm{AB}+\mathrm{AC}+2\mathrm{BC}}{gg} - \frac{2\mathrm{D}-2\mathrm{F}-9\mathrm{G}-9\mathrm{H}+3\mathrm{K}}{2g}\right)$$

$$\beta\,\mathrm{C}' = +\frac{1}{g}\beta\mathrm{C}+\frac{1}{2mg}\left(\tfrac{1}{2}gg-\tfrac{3}{4}\mathrm{A}+\frac{2\mathrm{AA}+2\mathrm{BB}+3\mathrm{CC}}{4gg} + \frac{\mathrm{AB}+6\mathrm{AC}+6\mathrm{BC}}{2gg} - \frac{3\mathrm{D}-3\mathrm{E}-3\mathrm{F}-\mathrm{G}}{2g}\right)$$

Quibus valoribus ſubſtitutis obtinebitur pro apogei motu medio, qui in termino $\Delta\omega$ continetur:

$$\Delta = \frac{1}{4m} + \frac{2mg\delta - 1}{4mgg} C$$

$$+ \frac{\delta}{4\beta mg}\left(\tfrac{1}{2}gg - \tfrac{3}{4}A + \frac{2AA + 2BB + 9CC}{4gg} + \frac{AB + 6AC + 6BC}{2gg} - \frac{3D - 3E - 3F - G}{2g}\right)$$

$$- \frac{9}{32(2\alpha - \beta)mmgg}\left(\tfrac{3}{2}gg - \tfrac{9}{2}C + \frac{27AA + 18BB + 15CC}{4gg} - \frac{3AB + 2AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g}\right)$$

$$- \frac{3}{32(2\alpha + \beta)mmgg}\left(\tfrac{3}{2}gg + 3C - \frac{6AA - 9BB + 15CC}{4gg} + \frac{9AB + AC + 2BC}{gg} - \frac{2D - 2F - 9G - 9H + 3K}{2g}\right)$$

$$+ \varepsilon D' + (mA - \eta)E' + (mB + \theta)F' + \varkappa G' + \frac{9AJ'}{8mgg} + \frac{3BK'}{8mgg}$$

Quae expreſſio, cum omnino ſit ſimilis illi, quae methodo praecedente eſt inuenta, nullum etiam dubium relinquit, quin et hinc motus apogei proditurus ſit obſeruationibus conformis; ideoque littera illa ι omitti poterit.

XXXIV.

Hinc igitur patet ad motum apogei definiendum valores litterarum A, B, C et A′, B′, C′ ſumma accuratione inueſtigari debere, qui cum conſtent partibus duplicis ordinis, etiam ſi partes poſterioris ordinis prae primo

primo admodum videantur paruae, eas tamen omni cura euolui oportet, propterea quod pro motu apogei partes primi ordinis se destruunt. Quod cum in determinatione reliquorum coefficientium vsu non eueniat, in his quoque non erit opus, vt partes istae minores in computum ducantur, sed sufficiet partibus principalibus vti. Scilicet etsi determinatio litterae Δ maxime est lubrica, atque a reliquorum coefficientium exactissimis valoribus pendet, reliqui tamen coefficientes tantam sollertiam minime requirunt, sed satis exacte sine tanta opera definiri possunt.

XXXV.

Valores ergo reliquorum coefficientium sequenti modo neglectis exiguis particulis ita se habebunt,

$$(2\alpha-\beta)\,A' = \frac{9}{4mg}\,; \quad (2\alpha+\beta)\,B' = -\frac{3}{4mg}\,; \quad \beta\,C' = \frac{1}{2mg}.$$

$$2\alpha D' = \tfrac{1}{2}(2\gamma-\delta)A' + \tfrac{1}{2}(2\gamma+\delta)B' + \frac{3C'}{4mg} + \frac{1}{4m}\left(3 - \frac{A-B-3C}{gg}\right)$$

$$2(\alpha-\beta)\,E' = \tfrac{1}{2}(2\gamma-\delta)\,A' + \frac{9C'}{8mg} + \frac{1}{4m}\left(6 - \frac{2A-9C}{2gg}\right)$$

$$2(\alpha+\beta)\,F' = \tfrac{1}{2}(2\gamma+\delta)\,B' - \frac{3C'}{8mg} - \frac{1}{4m}\left(3 + \frac{2B-3C}{2gg}\right)$$

$$2\beta\,G' = \tfrac{1}{2}\delta C' + \frac{3A'+9B'}{8mg} + \frac{1}{4m}\left(1 + \frac{3A-9B-2C}{2gg}\right)$$

$$4\alpha H' = \frac{3A'+9B'}{8mg} + \frac{8A-9B}{8mgg} = 0$$

$$2(2\alpha-\beta)J' = -\frac{9A'}{8mg} - \frac{9A}{8mgg} = 0 \;\Big|\; 2(2\alpha+\beta)K' = -\frac{3B'}{8mg} + \frac{3B}{8mgg} = 0$$

ob $A' = -\frac{A}{g}$; $B' = \frac{B}{g}$ et $C' = \frac{C}{g}$ proxime.

XXXVI.

XXXVI.

Cum autem ſit proxime: $\gamma = 2mg$; $\delta = 2mg + \frac{1}{2mg}$; his valoribus quoque ſubſtitutis fiet:

$(2\alpha - \beta) A' = \frac{9}{4mg}$; $(2\alpha + \beta) B' = -\frac{3}{4mg}$; $\beta C' = \frac{1}{2mg}$; ſiue:

$$A' = -\frac{A}{g}\ ;\quad B' = \frac{B}{g}\ ;\quad C' = \frac{C}{g};$$

$$2\alpha D' = \frac{3}{4m} - mA + 3mB;$$

$$2(\alpha - \beta) E' = \frac{3}{2m} - mA$$

$$2(\alpha + \beta) F' = -\frac{3}{4m} + 3mB$$

$$2\beta G' = \frac{1}{4m} + mC$$

et reliqui coefficientes H', J', K' pro euaneſcentibus ſunt habendi. Valores autem litterarum A, B, C, etc. §. 25. ſunt exhibiti.

XXXVII.

Quaeramus nunc quoque longitudinem lunae Φ, huncque in finem fingamus:

$$\Phi = \text{Conſt.}$$

$$+\mathfrak{A}'\omega + \mathfrak{B}'\sin v + \mathfrak{C}'\sin 2\eta + \mathfrak{D}'\sin(2\eta - 2v) + \mathfrak{E}'\sin(2\eta + 2v) + \mathfrak{F}'\sin 2v$$

$$+\mathfrak{G}'\sin(2\eta - v) + \mathfrak{H}'\sin(2\eta + v) + \mathfrak{J}'\sin(2\eta - 3v) + \mathfrak{K}'\sin(2\eta + 3v) + \mathfrak{L}'\sin 3v$$

$$+\mathfrak{M}'\sin(4\eta - v) + \mathfrak{N}'\sin(4\eta + v) + \mathfrak{P}'\sin(4\eta - 3v) + \mathfrak{Q}'\sin(4\eta + 3v)$$

ac

ac differentiatione instituta obtinebimus:

$$\frac{d\Phi}{d\omega} = + \mathfrak{A}' - \tfrac{1}{2}\delta\mathfrak{B}'$$

$$+ \cos v \left(\beta\mathfrak{B}' - \tfrac{1}{2}\kappa\mathfrak{B}' + \frac{9\mathfrak{D}'}{4mg} + \frac{3\mathfrak{E}'}{4mg} - \delta\mathfrak{F}' \right.$$

$$+ \cos 2\eta \left(- \frac{3\mathfrak{B}'}{4mg} + 2\alpha\mathfrak{C}' \right.$$

$$+ \cos(2\eta - 2v) \left(- \frac{9\mathfrak{B}'}{8mg} + 2(\alpha - \beta)\mathfrak{D}' \right.$$

$$+ \cos(2\eta + 2v) \left(+ \frac{3\mathfrak{B}'}{8mg} + 2(\alpha + \beta)\mathfrak{E}' \right.$$

$$+ \cos 2v \left(- \tfrac{1}{2}\delta\mathfrak{B}' + 2\beta\mathfrak{F}' \right.$$

$$+ \cos(2\eta - v) \left(- \tfrac{1}{2}\zeta\mathfrak{B}' - \tfrac{1}{2}\eta\mathfrak{B}' - \gamma\mathfrak{C}' - (\gamma - \delta)\mathfrak{D}' + \frac{3\mathfrak{F}'}{4mg} + (2\alpha - \beta)\mathfrak{G}' \right.$$

$$+ \cos(2\eta + v) \left(- \tfrac{1}{2}\zeta\mathfrak{B}' - \tfrac{1}{2}\theta\mathfrak{B}' - \gamma\mathfrak{C}' - (\gamma + \delta)\mathfrak{E}' - \frac{9\mathfrak{F}'}{4mg} + (2\alpha + \beta)\mathfrak{H}' \right.$$

$$+ \cos(2\eta - 3v) \left(- \tfrac{1}{2}\eta\mathfrak{B}' - (\gamma - \delta)\mathfrak{D}' - \frac{9\mathfrak{F}'}{4mg} + (2\alpha - 3\beta)\mathfrak{I}' \right.$$

$$+ \cos(2\eta + 3v) \left(- \tfrac{1}{2}\theta\mathfrak{B}' - (\gamma + \delta)\mathfrak{E}' + \frac{3\mathfrak{F}'}{4mg} + (2\alpha + 3\beta)\mathfrak{K}' \right.$$

$$+ \cos 8v \left(- \tfrac{1}{2}\kappa\mathfrak{B}' - \frac{3\mathfrak{D}'}{4mg} - \frac{9\mathfrak{E}'}{4mg} - \delta\mathfrak{F}' + 3\beta\mathfrak{L}' \right.$$

$$+ \cos(4\eta - v) \left(- \frac{3A + 9B}{16mgg}\mathfrak{B}' + \frac{9A}{16mgg}\mathfrak{B}' - \frac{3\mathfrak{D}'}{4mg} + (4\alpha - \beta)\mathfrak{M}' \right.$$

$$+ \cos(4\eta + v) \left(- \frac{3A + 9B}{16mgg}\mathfrak{B}' - \frac{3B}{16mgg}\mathfrak{B}' - \frac{9\mathfrak{E}'}{4mg} + (4\alpha + \beta)\mathfrak{N}' \right.$$

$$+ \cos(4\eta - 3v) \left(+ \frac{9A}{16mgg}\mathfrak{B}' + \frac{9\mathfrak{D}'}{4mg} + (4\alpha - 3\beta)\mathfrak{P}' \right.$$

$$+ \cos(4\eta + 3v) \left(- \frac{3B}{16mgg}\mathfrak{B}' + \frac{3\mathfrak{E}'}{4mg} + (4\alpha + 3\beta)\mathfrak{Q}' \right.$$

XXXVIII.

Comparata iam hac forma cum valore ipſius $\frac{d\Phi}{d\omega}$ in §. 29. exhibito, obtinebitur

$$\mathfrak{A}' = \tfrac{1}{2}\delta\mathfrak{B}' + m(1 + \tfrac{1}{2}gg - C)$$

$$\beta\mathfrak{B}' = \tfrac{1}{2}\kappa\mathfrak{B}' - \frac{9\mathfrak{D}' - 3\mathfrak{E}'}{4mg} + \delta\mathfrak{F}' - 2mg + \tfrac{3}{2}mg\,C - mG$$

$$2\alpha\mathfrak{C}' = \frac{3\mathfrak{B}'}{4mg} - m(\tfrac{3}{2}\mathfrak{A} + A + B)$$

$$2(\alpha - \beta)\mathfrak{D}' = \frac{9\mathfrak{B}'}{8mg} - mA$$

$$2(\alpha + \beta)\mathfrak{E}' = -\frac{3\mathfrak{B}'}{8mg} - mB$$

$$2\beta\mathfrak{F}' = \tfrac{1}{2}\delta\mathfrak{B}' - m(C - \tfrac{1}{2}gg)$$

$$(2\alpha - \beta)\mathfrak{G}' = \tfrac{1}{2}(\zeta + \eta)\mathfrak{B}' + \gamma\mathfrak{C}' + (\gamma - \delta)\mathfrak{D}' - \frac{3\mathfrak{F}'}{4mg}$$
$$+ m(\tfrac{3}{2}g\mathfrak{A} - \tfrac{3}{2}\mathfrak{B} + gA + \tfrac{1}{2}gB - D - E)$$

$$(2\alpha + \beta)\mathfrak{H}' = \tfrac{1}{2}(\zeta + \theta)\mathfrak{B}' + \gamma\mathfrak{C}' + (\gamma + \delta)\mathfrak{E}' + \frac{9\mathfrak{F}'}{4mg}$$
$$+ m(\tfrac{3}{2}g\mathfrak{A} - \tfrac{3}{2}\mathfrak{C} + gB + \tfrac{1}{2}gA - D - F)$$

$$(2\alpha - 3\beta)\mathfrak{I}' = \tfrac{1}{2}\eta\mathfrak{B}' + (\gamma - \delta)\mathfrak{D}' + \frac{9\mathfrak{F}'}{4mg} + m(\tfrac{1}{2}gA - E)$$

$$(2\alpha + 3\beta)\mathfrak{K}' = \tfrac{1}{2}\theta\mathfrak{B}' + (\gamma + \delta)\mathfrak{E}' - \frac{3\mathfrak{F}'}{4mg} + m(\tfrac{1}{2}gB - F)$$

$$3\beta\mathfrak{L}' = \tfrac{1}{2}\kappa\mathfrak{B}' + \frac{3\mathfrak{D}' + 9\mathfrak{E}'}{4mg} + \delta\mathfrak{F}' + m(\tfrac{1}{2}gC - G)$$

$$(4\alpha - \beta)\mathfrak{M}' = -\frac{6A - 9B}{16mgg}\mathfrak{B}' + \frac{3\mathfrak{D}'}{4mg} - m(H + J)$$

$$(4\alpha + \beta)\mathfrak{N}' = +\frac{3A - 6B}{16mgg}\mathfrak{B}' + \frac{9\mathfrak{E}'}{4mg} - m(H + K)$$

$$(4\alpha - 3\beta)\mathfrak{P}' = -\frac{9A}{16mgg}\mathfrak{B}' - \frac{9\mathfrak{D}'}{4mg} - mJ$$

$$(4\alpha + 3\beta)\mathfrak{Q}' = +\frac{3B}{16mgg}\mathfrak{B}' - \frac{3\mathfrak{E}'}{4mg} - mK$$

XXXIX.

XXXIX.

Inuentis iam valoribus litterarum $p = b(1+\xi)$ et q vna cum anomalia vera v, diſtantia curtata lunae a terra $x = \frac{p}{1-q\cos v}$ cognoſcetur: ac ſi deinceps latitudinis lunae ψ ratio habebitur, erit diſtantia vera $= \frac{p}{(1-q\,\mathrm{cs}\,v)\,\mathrm{cs}\,\psi}$. In Aſtronomia autem non tam diſtantia lunae, quam eius diameter apparens et parallaxis requiri ſolet; quarum vtraque cum ſit diſtantiae lunae a terra reciproce proportionalis, erit tam diameter apparens quam parallaxis vt $\frac{(1-q\cos v)\cos\psi}{p}$; vnde ſi vtriusque valor medius ex obſeruationibus fuerit definitus, ad quoduis tempus valor verus aſſignari poterit. Sit igitur ſiue diametri apparentis ſiue parallaxis horizontalis valor medius $= \sigma$, eritque is pro tempore dato $= \frac{b\sigma}{p}(1-q\cos v)\cos\psi$. Eſt autem proxime $\cos\psi = 1 - \frac{1}{4}\operatorname{tang}\rho^2 - \frac{1}{4}\operatorname{tg}\rho^2\,\mathrm{cs}\,2\,(\phi-\pi) = \frac{2}{3} + \frac{\lambda+\Pi}{3}$, et $\frac{b}{p} = 1 - \xi + \xi\xi$: vnde fit diameter ſeu parallaxis $= \frac{1}{3}\sigma\,(2+\lambda+\Pi)\,(1-\xi+\xi\xi)\,(1-q\cos v)$, quae euoluitur in hanc expreſſionem: ob $\frac{2+\lambda}{3} = 1$ proxime:

$$\tfrac{1}{3}(2+\lambda)\,\sigma\,[1 - q\cos v - \xi + q\xi\cos v + \xi\xi + \tfrac{1}{3}\Pi - \tfrac{1}{3}q\Pi\cos v]$$

XL.

Pro praeſenti ergo caſu, quo parallaxin ſolis, eiusque excentricitatem vna cum inclinatione orbitae luna-

 ris

ris negligimus, erit lunae diameter apparens vel parallaxis horizontalis

$$= \tfrac{1}{3}(2+\lambda)\sigma \text{ mult. per}$$

$$1-\tfrac{1}{2}C-(g+\tfrac{1}{2}G)\cos v$$
$$-\tfrac{1}{2}(2\mathfrak{A}+A+B)\cos 2\eta-\tfrac{1}{2}A\cos(2\eta-2v)-\tfrac{1}{2}B\cos(2\eta+2v)-\tfrac{1}{2}C\cos 2v$$
$$-\tfrac{1}{2}(2\mathfrak{B}+D+E)\cos(2\eta-v)-\tfrac{1}{2}(2\mathfrak{C}+D+F)\cos(2\eta+v)$$
$$-\tfrac{1}{2}E\cos(2\eta-3v)-\tfrac{1}{2}F\cos(2\eta+3v)-\tfrac{1}{2}G\cos 3v$$
$$-\tfrac{1}{2}(H+J)\cos(4\eta-v)-\tfrac{1}{2}(H+K)\cos(4\eta+v)-\tfrac{1}{2}J\cos(4\eta-3v)-\tfrac{1}{2}K\cos(4\eta+3v)$$

vbi quidem factor constans $\tfrac{1}{3}(2+\lambda)\sigma$ omitti potest, siquidem tantum proportio vel diametri apparentis vel parallaxis horizontalis desideretur.

XLI.

Si nunc hos valores in numeris euoluere velimus, ex obseruationibus primum colligimus has determinationes:

$$\mathfrak{A}' = 13,3682;\ \Delta = 0,1123 \text{ et proxime } g = 0,05445$$

ac postremo quidem valore ipsius g tantum in terminis minimis vtar, in maioribus ipsam litteram g relicturus, vt deinceps ex collatione calculi cum obseruationibus accuratius fortasse determinari possit. Habemus ergo

$$13,3682 = \tfrac{1}{2}\delta\mathfrak{B}' + m(1+\tfrac{1}{2}gg-C)$$

vnde ob $\mathfrak{B}'$, gg et C numeros admodum paruos, statim prope colligitur $m = 13,3682$. Tum vero est prope

$$\mathfrak{B}' = -\frac{2mg}{\mathfrak{G}};\ \mathfrak{G} = m \text{ et } \delta = 2mg + \frac{1}{2mg} \text{ seu } \mathfrak{G} = 13,3682;$$

$\delta = 2,$

$\delta = 2,1419$; hinc $\frac{1}{2}\delta\mathfrak{B}' = -0,1165$, ergo accuratius

$$m(1+\tfrac{1}{2}gg - C) = 13,4847 = \alpha+1 \text{ et } \alpha = 12,4847$$

Porro ob $C = \frac{1}{2mm}$ erit satis exacte . $m = 13,5039$

vnde ex valoribus A et B proxime collectis fit $\beta = 13,0644$

$$\gamma = 26,9524g.$$

At valor ipsius δ duabus constat partibus, altera per g multiplicata altera diuisa, quibus separatim expressis erit

$$\delta = 27,0355g + 0,0370.\frac{1}{g} = 2,1521 \text{ proxime.}$$

XLII.

Hinc iam computo instituto sequentes supra assumtorum coefficientium eruuntur valores numerici:

$\mathfrak{A} = 0,008931$; $\mathfrak{B} = 0,03895g = 0,00209$; $\mathfrak{C} = 0,01219g = 0,00064$ ideoque

$$\xi = 0,008931\,\text{cs}\,2\eta + 0,03895g\,\text{cs}(2\eta - v) + 0,01219g\,\text{cs}(2\eta + v)$$

Deinde reperitur:

$A = -0,013995$; $B = -0,001460$; $C = +0,002834$

$D = -0,001213.g + 0,00002198.\frac{1}{g} = -0,000280$ proxime

$E = +0,25834.g - 0,00001889.\frac{1}{g} = +0,014012$ proxime

$F = -0,00225.g - 0,00000207.\frac{1}{g} = -0,000161$ proxime

$G = +0,00218.g + 0,00001221.\frac{1}{g} = +0,000340$ proxime

$H = -0,00001022.\frac{1}{g} = -0,000184$

$J = +0,00004897.\frac{1}{g} = +0,000882$

$K = +0,00000053.\frac{1}{g} = +0,000009$

XLIII.

XLIII.

Hinc porro pro motu apogei eiusque inaequalitatibus colligitur: $\Delta = 0,1123$; qui quidem valor ex obseruationibus est desumtus

$$A' = -0,013995.\frac{1}{g} = -0,25703 \text{ proxime}$$

$$B' = -0,001460.\frac{1}{g} = -0,02682 \text{ proxime}$$

$$C' = +0,002834.\frac{1}{g} = +0,05205 \text{ proxime}$$

$D' = +0,007432$	$F' = -0,002249$
$E' = -0,259170$	$G' = -0,002176$

in minutis secundis

A'	$= -53018''$	$= -14°, 43', 38''$
B'	$= -5532$	$= -1°, 32, 12$
C'	$= +10736$	$= +2, 58, 56$
D'	$= +1533$	$= +0, 25, 33$
E'	$= -53459$	$= -14, 50, 59$
F'	$= -464$	$= -0, 7, 44$
G'	$= +449$	$= +0, 7, 29$

Ergo longitudo apogei in minutis secundis

$$\phi - v = \text{Const.}$$

$$\begin{aligned} &+0,1123\,\omega - 53018''\sin(2\eta - v) + 1533''\sin 2\eta \\ &\qquad - 5532\sin(2+v) - 53459\sin(2\eta - 2v) \\ &\qquad + 10736\sin v - 464\sin(2\eta + 2v) \\ &\qquad\qquad + 449\sin 2v \end{aligned}$$

XLIV.

XLIV.

Iam pro longitudine ipſa inuenienda habentur primo ex §. 27. valores: $\gamma = 1,46756$

$\delta = +2,15210$; $\epsilon = -0,027791$; $\zeta = +0,07138$

$\eta = -0,06974$; $\theta = -0,039809$; $\kappa = -0,070920$

Deinde cum ſit proxime

$$6\mathfrak{B}' = -2mg \quad \text{ſeu} \quad \mathfrak{B}' = -0,11256$$

erit quoque proxime

$$\mathfrak{C}' = -0,003485 \quad ; \quad \mathfrak{D}' = -0,014456$$

$$\mathfrak{E}' = +0,001509 \quad ; \quad \mathfrak{F}' = -0,005334$$

Hinc ergo accuratius elicietur $\mathfrak{B}' = -0,11019$, ideoque hic et reliqui conficientes tam abſolute quam in numeris ſecundis erunt:

abſolute		in minutis ſecundis	
$\mathfrak{B}' = -0,11019$	. .	$\mathfrak{B}' = -22728''$	$= -6°, 18', 4''$
$\mathfrak{C}' = -0,00339$	. .	$\mathfrak{C}' = -700$	$= -0, 11, 40$
$\mathfrak{D}' = -0,01742$	. .	$\mathfrak{D}' = -3594$	$= -0, 59, 54$
$\mathfrak{E}' = +0,00149$	. .	$\mathfrak{E}' = +306$	$= +0, 5, 6$
$\mathfrak{F}' = -0,00524$	. .	$\mathfrak{F}' = -1081$	$= -0, 18, 1$
$\mathfrak{G}' = -0,01824$	. .	$\mathfrak{G}' = -3762$	$= -1, 2, 42$
$\mathfrak{H}' = -0,00056$	. .	$\mathfrak{H}' = -115$	$= -0, 1, 55$
$\mathfrak{I}' = +0,01368$	. .	$\mathfrak{I}' = +2823$	$= +0, 47, 3$
$\mathfrak{K}' = +0,00023$	. .	$\mathfrak{K}' = +47$	$= +0, 0, 47$
$\mathfrak{L}' = -0,00062$	. .	$\mathfrak{L}' = -128$	$= -0, 2, 8$
$\mathfrak{M}' = -0,00119$	. .	$\mathfrak{M}' = -246$	$= -0, 4, 6$
$\mathfrak{N}' = +0,00020$	. .	$\mathfrak{N}' = +41$	$= +0, 0, 41$
$\mathfrak{P}' = +0,00184$	. .	$\mathfrak{P}' = +379$	$= +0, 6, 19$
$\mathfrak{Q}' = -0,00001$	. .	$\mathfrak{Q}' = -2$	$= -0, 0, 2$

XLV.

XLV.

Hinc ergo ſi ad datum tempus iam cognita ſit anomalia lunae vera v cum angulo η, longitudo lunae per aequationes in minutis ſecundis expreſſas erit

$$\phi = \text{Conſt.}$$

$$\begin{array}{rll}
+13{,}3682\omega - 22728''\ \sin v & - & 700''\ \sin 2\eta \\
- 1081\ \sin 2v & - & 3594\ \sin(2\eta - 2v) \\
- 128\ \sin 3v & + & 306\ \sin(2\eta + 2v) \\
- 3762''\ \sin(2\eta - v) & - & 246''\ \sin(4\eta - v) \\
- 115\ \sin(2\eta + v) & + & 41\ \sin(4\eta + v) \\
+ 2823\ \sin(2\eta - 3v) & + & 379\ \sin(4\eta - 3v) \\
+ 47\ \sin(2\eta + 3v) & - & 2\ \sin(4\eta + 3v)
\end{array}$$

vbi Conſt. $+ 13{,}3682\omega$ denotat longitudinem mediam; in reliquis autem terminis continentur inaequalitates periodicae pro hac hypotheſi.

XLVI.

Inde iam viciſſim anomalia vera lunae v colligitur, vt ſit

$$v =$$

$$\begin{array}{rll}
13{,}2559\omega - 33464''\ \sin v & - & 2233''\ \sin 2\eta \\
- 1530\ \sin 2v & + & 49864\ \sin(2\eta - 2v) \\
- 128\ \sin 3v & + & 770\ \sin(2\eta + 2v) \\
+ 49256\ \sin(2\eta - v) & - & 246\ \sin(4\eta - v) \\
+ 5417\ \sin(2\eta + v) & + & 41\ \sin(4\eta + v) \\
+ 2823\ \sin(2\eta - 3v) & + & 379\ \sin(4\eta - 3v) \\
+ 47\ \sin(2\eta + 3v) & - & 2\ \sin(4\eta + 3v)
\end{array}$$

vbi primus terminus $13{,}2559\omega$ deſignat anomaliam mediam lunae, quae ſit $= \zeta$: tum ex ea primum quaeratur

ano-

anomalia Kepleriana, quae scilicet a sola excentricitate pendet, sitque ea $= u$, vt sit

$$u = \zeta - 33464'' \sin u - 1530'' \sin 2u - 128'' \sin 3u$$

vnde quidem facile tabulae construentur. Tum statuatur $v = u + z$, et quia angulus z est modicus, inde is satis prope poterit definiri. Interim tamen expedire videtur aliquot operationibus iterandis istam anomaliam veram v determinari; dum scilicet primum valor non nimis a veritate abhorrens pro v aestimando assumitur, ex eoque deinceps exactior colligitur; qui si nimis ab assumto discrepare reperiatur, ex hoc denuo exactior quaeratur, donec nulla amplius correctione fuerit opus.

XLVII.

Formula denique, cui tam diameter lunae apparens geocentrica quam parallaxis horizontalis est proportionalis, ex §. 40. reperitur

$$\begin{array}{ll}
1 - 0{,}05470 \cos v & - 0{,}00120 \cos 2\eta \\
- 0{,}00142 \cos 2v & + 0{,}00700 \cos (2\eta - 2v) \\
- 0{,}00017 \cos 3v & + 0{,}00073 \cos (2\eta + 2v) \\
- 0{,}00898 \cos (2\eta - v) & - 0{,}00035 \cos (4\eta - v) \\
- 0{,}00042 \cos (2\eta + v) & + 0{,}00009 \cos (4\eta + v) \\
- 0{,}00701 \cos (2\eta - 3v) & - 0{,}00044 \cos (4\eta - 3v) \\
+ 0{,}00008 \cos (2\eta + 3v) & - 0{,}00001 \cos (4\eta + 3v)
\end{array}$$

quorum quidem terminorum plures, qui pro parallaxi infra aliquot minuta secunda subsistunt, tuto omitti poterunt. His igitur tribus formulis pro anomalia vera v, longitudine Φ et parallaxi seu diametro apparente inuentis motus lunae contineretur, si quidem tam solis parallaxis quam eius excentricitas et inclinatio orbitae lunaris ad eclipticam negligatur. Hae autem ſunt inaequalitates praecipuae, quae etiam ad reliquas eruendas adhiberi debent; vnde nunc ad inaequalitates ab excentricitate ſolis oriundas progrediamur.

INUESTIGATIO INAEQUALITATUM
LUNAE SECUNDI GENERIS SEU AB EXCENTRICITATE SOLIS PENDENTIUM.

XLVIII.

Formulae noſtrae differentiales, quatenus ab excentricitate orbitae ſolaris pendent, omiſſis terminis, quos iam conſtat eſſe minimos, erunt

$$\frac{d\xi}{d\omega} = \text{Praec.} + \frac{9e}{2m}\,\text{ſin}\,(2\eta - u) + \frac{9e}{2m}\,\text{ſin}\,(2\eta + u)$$

$$\begin{aligned}\frac{dq}{d\omega} = \text{Praec.} &+ \frac{3e}{4m}\,\text{ſin}\,(v - u) + \frac{3e}{4m}\,\text{ſin}\,(v + u)\\ &- \frac{27e}{8m}\,\text{ſin}\,(2\eta - v - u) - \frac{27e}{8m}\,\text{ſin}\,(2\eta - v + u)\\ &- \frac{9e}{8m}\,\text{ſin}\,(2\eta + v - u) - \frac{9e}{8m}\,\text{ſin}\,(2\eta + v + u)\end{aligned}$$

$$\begin{aligned}\frac{q(d\Phi - dv)}{d\omega} = \text{Pr.} &- \frac{3e}{4m}\,\text{coſ}\,(v - u) - \frac{3e}{4m}\,\text{coſ}\,(v + u)\\ &- \frac{27e}{8m}\,\text{coſ}\,(2\eta - v - u) - \frac{27e}{8m}\,\text{coſ}\,(2\eta - v + u)\\ &+ \frac{9e}{8m}\,\text{coſ}\,(2\eta + v - u) + \frac{9e}{8m}\,\text{coſ}\,(2\eta + v + u)\end{aligned}$$

Quan-

Quanquam enim nunc tam ξ quam q etiam ab excentricitate e pendeant, tamen in his formulis, in quas hae quantitates ingrediuntur, haec mutatio earum sine errore pro nihilo haberi potest; quoniam hi termini per se sunt minimi, et quia iam terminos ab e et q simul pendentes omisimus. Tum vero erit

$$\frac{d\Phi}{d\omega} = m(1+\tfrac{1}{2}qq) - 2mq\cos v + \tfrac{1}{2}mqq\cos 2v - \tfrac{1}{2}m\xi + 3mq\xi\cos v$$

et $\frac{du}{d\omega} = \frac{d\theta}{d\omega} = 1 + 2ee - 2e\cos u$

XLIX.

Ad formulas has integrandas seu tantum ad eas integralium partes inueniendas, quae ab excentricitate solis e pendent, opus est vt formularum $\frac{d\eta}{d\omega}$, $\frac{dv}{d\omega}$ et $\frac{du}{d\omega}$ primum habeamus partes principales, tum vero etiam eas quae a simplici solis excentricitate e pendent: habebimus ergo primo

$$\frac{d\eta}{d\omega} = m(1+\tfrac{1}{2}gg) - 1 - 2ee - 2mg\cos v + 2e\cos u$$

$$\frac{dv}{d\omega} = m(1+\tfrac{1}{2}gg) - 2mg\cos v + \frac{3e}{4mg}\cos(v-u) + \frac{3e}{4mg}\cos(v+u)$$

$$-\frac{1}{4m}\left(1-\frac{9A+3B-2C}{2gg}\right) + \frac{27e}{8mg}\cos(2\eta-v-u) + \frac{27e}{8mg}\cos(2\eta-v+u)$$

$$-\frac{9e}{8mg}\cos(2\eta+v-u) - \frac{9e}{8mg}\cos(2\eta+v+u)$$

seu introducendis, vt supra §. 27. breuitatis gratia, litteris

$\alpha = m(1+\tfrac{1}{2}gg - C) - 2ee$; $\gamma = 2mg$

$\beta = m(1+\tfrac{1}{2}gg - C) - \frac{1}{4m}\left(1-\frac{9A+3B-2C}{2gg}\right)$; $\delta = 2mg + \frac{1}{2mg} + \frac{3g}{8m}$

 erit:

erit: $\frac{d\eta}{d\omega} \alpha - \gamma \cos v + 2e \cos u$

$$\frac{dv}{d\omega} = \beta - \delta \cos v - \frac{9}{4mg} \cos(2\eta - v) + \frac{3}{4mg} \cos(2\eta + v)$$

$$+ \frac{3e}{4mg} \cos(v-u) + \frac{3e}{4mg} \cos(v+u) + \frac{27e}{8mg} \cos(2\eta - v - u)$$

$$+ \frac{27e}{8mg} \cos(2\eta - v + u) - \frac{9e}{8mg} \cos(2\eta + v - u) - \frac{9e}{8mg} \cos(2\eta + v + u)$$

et $\frac{du}{d\omega} = 1 - 2e \cos u$.

L.

Fingamus nunc primo:

$$\xi = \mathfrak{A} \cos 2\eta + \mathfrak{B} \cos(2\eta - v) + \mathfrak{C} \cos(2\eta + v) + \mathfrak{P} \cos(2\eta - u) + \mathfrak{Q} \cos(2\eta + u)$$

ac differentiando eos tantum ſumamus terminos, qui formulae differentiali reſpondent, quandoquidem reliquos iam inuenimus: eritque

$$\frac{d\xi}{d\omega} = - 2e \mathfrak{A} \sin(2\eta - u) - 2e \mathfrak{A} \sin(2\eta + u)$$
$$- (2\alpha - 1) \mathfrak{P} \quad - (2\alpha + 1) \mathfrak{Q}$$

vnde colligitur:

$$(2\alpha - 1) \mathfrak{P} = -\frac{9e}{2m} - 2e \mathfrak{A}; \; (2\alpha + 1) \mathfrak{Q} = -\frac{9e}{2m} - 2e \mathfrak{A}$$

Cum igitur ſit $e = 0,0168$, erit in numeris:

$$\mathfrak{P} = -0,000247 \quad \text{et} \quad \mathfrak{Q} = -0,000227.$$

LI.

Fingatur porro: $q = g$

$$+ A \cos(2\eta - v) + B \cos(2\eta + v) \, C \cos v + M \cos(v - u) + N \cos(v + u)$$
$$+ P \cos(2\eta - v - u) + Q \cos(2\eta - v + u) + R \cos(2\eta + v - u) + S \cos(2\eta + v + u)$$

ac

ac differentiando obtinebitur: $\frac{dq}{d\omega}=$

$$-2eA\sin(2\eta-v-u)-2eA\sin(2\eta-v+u)-2eB\sin(2\eta+v-u)-2eB\sin(2\eta+v+u)$$
$$-(2\alpha-\beta-1)P \quad -(2\alpha-\beta+1)Q \quad -(2\alpha+\beta-1)R \quad -(2\alpha+\beta+1)S$$
$$-(\beta-1)M\sin(v-u)-(\beta+1)H\sin(v+u)$$

Comparatione ergo instituta reperietur:

$$(\beta-1)M=-\frac{3e}{4m} \quad ; \quad (\beta+1)N=-\frac{3e}{4m}$$
$$(2\alpha-\beta-1)P=\frac{27e}{8m}-2eA \quad ; \quad (2\alpha-\beta+1)Q=\frac{27e}{8m}-2eA$$
$$(2\alpha+\beta-1)R=\frac{9e}{8m}-2eB \quad ; \quad (2\alpha+\beta+1)S=\frac{9e}{8m}-2eB$$

et in numeris

$$M=-0{,}00008 \quad ; \quad P=+0{,}00042 \quad ; \quad R=+0{,}00004$$
$$N=-0{,}00006 \quad ; \quad Q=+0{,}00036 \quad ; \quad S=+0{,}00004$$

LII.

Hic autem in differentiatione neglеximus partes ipsius $\frac{dv}{d\omega}$ ab excentricitate e pendentes, quarum tamen eodem iure ratio haberi debuisset, atque partis in differentiali $\frac{d\eta}{d\omega}$; inde autem multo plures termini accedent ad valorum ipsius q, ponatur ergo ob hos terminos:

$$q=g$$
$$+A\cos(2\eta-v)+B\cos(2\eta+v)+C\cos v+M\cos(v-u)$$
$$+N\cos(v+u)+D\cos(2\eta-u)+E\cos(2\eta+u)$$
$$+P\cos(2\eta-v-u)+Q\cos(2\eta-v+u)+R\cos(2\eta+v-u)$$
$$+S\cos(2\eta+v+u)+K\cos(2v-u)+L\cos(2v+u)$$

 +

$$+F\cos(2\eta-2v-u)+G\cos(2\eta-2v+u)+H\cos(2\eta+2v-u)$$
$$+J\cos(2\eta+2v+u)+T\cos(4\eta-u)+V\cos(4\eta+u)$$
$$+W\cos(4\eta-2v-u)+X\cos(4\eta-2v+u)+Y\cos(4\eta+2v-u)$$
$$+Z\cos(4\eta+2v+u)$$

et fumto differentiali pleno reperiotur:

$$\frac{dq}{d\omega}=+\sin(2\eta-v-u)\ [-2eA-(2\alpha-\beta-1)P]$$
$$+\sin(2\eta-v+u)\ [-2eA-(2\alpha-\beta+1)Q]-(\beta-1)M\sin(v-u)$$
$$+\sin(2\eta\mp v-u)\ [-2eB-(2\alpha+\beta-1)R]$$
$$+\sin(2\eta+v+u)\ [-2eB-(2\alpha+\beta+1)S]-(\beta+1)N\sin(v+u)$$
$$+\sin(2\eta-u)\left(+\frac{3eA}{8mg}-\frac{3eB}{8mg}-\frac{27eC}{16mg}-\frac{9eC}{16mg}-(2\alpha-1)D\right)$$
$$+\sin(2\eta+u)\left(+\frac{3eA}{8mg}-\frac{8eB}{8mg}-\frac{27eC}{16mg}-\frac{9eC}{16mg}-(2\alpha+1)E\right)$$
$$+\sin(2\eta-2v+u)\left(+\frac{3eA}{8mg}+\frac{27eC}{16mg}-(2\alpha-2\beta+1)G\right)$$
$$+\sin(2\eta-2v-u)\left(+\frac{3eA}{8mg}+\frac{27eC}{16mg}-(2\alpha-2\beta-1)F\right)$$
$$+\sin(2\eta+2v-u)\left(-\frac{3eB}{8mg}+\frac{9eC}{16mg}-(2\alpha+2\beta-1)H\right)$$
$$+\sin(2\eta+2v+u)\left(-\frac{3eB}{8mg}+\frac{9eC}{16mg}-(2\alpha+2\beta+1)J\right)$$
$$+\sin u\left(+\frac{27eA}{16mg}-\frac{27eA}{16mg}+\frac{9eB}{16mg}-\frac{9eB}{16mg}-\frac{3eC}{8mg}+\frac{3eC}{8mg}\right)$$
$$+\sin(4\eta-2v-u)\left(+\frac{27eA}{16mg}-(4\alpha-2\beta-1)W\right)$$
$$+\sin(4\eta-2v+u)\left(+\frac{27eA}{16mg}-(4\alpha-2\beta+1)X\right)$$

+

$$+\sin(4\eta+2v-u)\left(+\frac{9eB}{16mg}-(4\alpha+2\beta-1)\,Y\right)$$
$$+\sin(4\eta+2v+u)\left(+\frac{9eB}{16mg}-(4\alpha+2\beta+1)\,Z\right)$$
$$+\sin(2v-u)\left(+\frac{9eA}{16mg}-\frac{27eB}{16mg}-\frac{3eC}{8mg}-(2\beta-1)\,K\right)$$
$$+\sin(2v+u)\left(+\frac{9eA}{16mg}-\frac{27eB}{16mg}-\frac{3eC}{8mg}-(2\beta+1)\,L\right)$$
$$+\sin(4\eta-u)\left(-\frac{9eA}{16mg}-\frac{27eB}{16mg}-(4\alpha-1)\,T\right)$$
$$+\sin(4\eta+u)\left(-\frac{9eA}{16mg}-\frac{27eB}{16mg}-(4\alpha+1)\,V\right)$$

vnde reperitur:

$D = -0{,}000010$	;	$H = +0{,}000001$	
$E = -0{,}000010$	;	$J = +0{,}000001$	
$F = +0{,}000005$	;	$K = -0{,}000006$	
$G = +0{,}000065$	;	$L = -0{,}000006$	
$T = +0{,}000004$	;	$X = -0{,}000022$	
$V = +0{,}000004$	;	$Y = -0{,}000000$	
$W = -0{,}000023$	;	$Z = -0{,}000000$	

LIII.

Ponamus nunc etiam pro motu apogei

$$\phi - v = \text{Const.} + \Delta\omega$$
$$+A'\sin(2\eta-v)+B'\sin(2\eta+v)+C'\sin v+M'\sin(v-u)+N'\sin(v+u)$$
$$+P'\sin(2\eta-v-u)+Q'\sin(2\eta-v+u)+R'\sin(2\eta+v-u)+S'\sin(2\eta+v+u)$$
$$+D'\sin(2\eta-u)+E'\sin(2\eta+u)+K'\sin(2v-u)+L'\sin(2v+u)+O'\sin u$$
$$+F'\sin(2\eta-2v-u)+G'\sin(2\eta-2v+u)+H'\sin(2\eta+2v-u)+J'\sin(2\eta+2v+u)$$
$$+W'\sin(4\eta-2v-u)+X'\sin(4\eta-2v+u)+Y'\sin(4\eta+2v-u)+Z'\sin(4\eta+2v+u)$$
$$+T'\sin(4\eta-u)+V'\sin(4\eta+u)$$

et

et sumto differentiali pleno reperietur:

$$\frac{d\Phi - dv}{d\omega} = \Delta + \cos(2\eta - v - u)\ [2eA' + (2\alpha - \beta - 1)P']$$

$$+ \cos(2\eta - v + u)\ [2eA' + (2\alpha - \beta + 1)Q'] + (\beta - 1)M'\cos(v - u)$$

$$+ \cos(2\eta + v - u)\ [2eB' + (2\alpha + \beta - 1)R']$$

$$+ \cos(2\eta + v + u)\ [2eB' + (2\alpha + \beta + 1)S'] + (\beta + 1)N'\cos(v + u)$$

$$+ \cos(2\eta - u)\left(-\frac{3e}{8mg}A' + \frac{3e}{8mg}B' + \frac{27e}{16mg}C' - \frac{9e}{16mg}C' + (2\alpha - 1)D'\right)$$

$$+ \cos(2\eta + u)\left(-\frac{3e}{8mg}A' + \frac{3e}{8mg}B' + \frac{27e}{16mg}C' - \frac{9e}{16mg}C' + (2\alpha + 1)E'\right)$$

$$+ \cos(2\eta - 2v - u)\left(-\frac{3e}{8mg}A' + \frac{27e}{16mg}C' + (2\alpha - 2\beta - 1)F'\right)$$

$$+ \cos(2\eta - 2v + u)\left(-\frac{3e}{8mg}A' + \frac{27e}{16mg}C' + (2\alpha - 2\beta + 1)G'\right)$$

$$+ \cos(2\eta + 2v - u)\left(+\frac{3e}{8mg}B' + \frac{9e}{16mg}C' + (2\alpha + 2\beta - 1)H'\right)$$

$$+ \cos(2\eta + 2v + u)\left(+\frac{3e}{8mg}B' + \frac{9e}{16mg}C' + (2\alpha + 2\beta + 1)J'\right)$$

$$+ \cos u\left(-\frac{27e}{16mg}A' - \frac{27e}{16mg}A' - \frac{9e}{16mg}B' - \frac{9e}{16mg}B' + \frac{3e}{8mg}C' + \frac{3e}{8mg}C' + O'\right)$$

$$+ \cos(4\eta - 2v - u)\left(-\frac{27e}{16mg}A' + (4\alpha - 2\beta - 1)W'\right)$$

$$+ \cos(4\eta - 2v + u)\left(-\frac{27e}{16mg}A' + (4\alpha - 2\beta + 1)X'\right)$$

$$+ \cos(4\eta + 2v - u)\left(-\frac{9e}{16mg}B' + (4\alpha + 2\beta - 1)Y'\right)$$

$$+ \cos(4\eta + 2v + u)\left(-\frac{9e}{16mg}B' + (4\alpha + 2\beta + 1)Z'\right)$$

+

$$+\operatorname{cos}(2v-u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'\mp\frac{3e}{8mg}C'+(2\beta-1)K'\right)$$

$$+\operatorname{cos}(2v+u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'+\frac{3e}{8mg}C'+(2\beta+1)L'\right)$$

$$+\operatorname{cos}(4\eta-u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'+(2\alpha-1)T'\right)$$

$$+\operatorname{cos}(4\eta+u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'+(2\alpha+1)V'\right)$$

LIV.

Singuli iam hi termini multiplicentur per q, cuius valor quidem erit $=g$, quoniam hi termini in ſuo genere iam ſunt minimi: ſed quoniam valor $\frac{d\Phi-dv}{d\omega}$ adhuc hos terminos praecipuos continet:

$$(2\alpha-\beta)A'\operatorname{cos}(2\eta-v)+(2\alpha+\beta)B'\operatorname{cos}(2\eta+v)+\beta C'\operatorname{cos}v$$

ſi et hi per q multiplicentur, inde naſcentur quoque termini angulum u inuoluentem, erit autem pro his, ſumtis partibus tantum praecipuis:

$$q=\text{Praec.}+P\operatorname{cos}(2\eta-v-u)+Q\operatorname{cos}(2\eta-v+u)$$

Ergo ad illos terminos per q multiplicatos inſuper accedent iſti:

$\operatorname{cos}u\ [\tfrac{1}{2}(2\alpha-\beta)PA'+\tfrac{1}{2}(2\alpha-\beta QA']$ $\quad+\tfrac{1}{2}\beta PC'\operatorname{cos}(2\eta-u)$
$\operatorname{cos}(4\eta-2v-u)\ [\tfrac{1}{2}(2\alpha-\beta)PA'+\tfrac{1}{2}\beta PC']$ $\quad+\tfrac{1}{2}\beta QC'\operatorname{cos}(2\eta+u)$
$\operatorname{cos}(4\eta-2v+u)\ [\tfrac{1}{2}(2\alpha-\beta)QA'+\tfrac{1}{2}\beta QC']$
$\operatorname{cos}(2v+u)\ [\tfrac{1}{2}(2\alpha+\beta)PB'$ $\quad+\operatorname{cos}(4\eta-u)\ [\tfrac{1}{2}(2\alpha+\beta)PB'$
$\operatorname{cos}(2v-u)\ [\tfrac{1}{2}(2\alpha+\beta)QB'$ $\quad+\operatorname{cos}(4\eta+u)\ [\tfrac{1}{2}(2\alpha+\beta)QB'$

LV.

Hinc ergo obtinentur sequentes determinationes:

$$2eg\,A' + (2\alpha-\beta-1)\,g\,P' = -\frac{27e}{8m}$$

$$2eg\,A' + (2\alpha-\beta+1)\,g\,Q' = -\frac{27e}{8m}$$

$$2eg\,B' + (2\alpha+\beta-1)\,g\,R' = +\frac{9e}{8m}$$

$$2eg\,B' + (2\alpha+\beta+1)\,g\,S' = +\frac{9e}{8m}$$

$$(\beta-1)\,g\,M' = -\frac{3e}{4m} \quad ; \quad (\beta+1)\,g\,N' = -\frac{3e}{4m}$$

$$-\frac{3e}{8m}(A'-B') + \frac{9e}{8m}C' + (2\alpha-1)\,g\,D' + \tfrac{1}{2}\beta PC' = 0$$

$$-\frac{3e}{8m}(A'-B') + \frac{9e}{8m}C' + (2\alpha+1)\,g\,E' + \tfrac{1}{2}\beta QC' = 0$$

$$-\frac{3e}{8m}A' + \frac{27e}{16m}C' + (2\alpha-2\beta-1)\,g\,F' = 0$$

$$-\frac{3e}{8m}A' + \frac{27e}{16m}C' + (2\alpha-2\beta+1)\,g\,G' = 0$$

$$+\frac{3e}{8m}B' + \frac{9e}{16m}C' + (2\alpha+2\beta-1)\,g\,H' = 0$$

$$+\frac{3e}{8m}B' + \frac{9e}{16m}C' + (2\alpha+2\beta+1)\,g\,J' = 0$$

$$-\frac{27e}{8m}A' - \frac{9e}{8m}B' + \frac{3e}{8m}C' + g\,O' + \tfrac{1}{2}(2\alpha-\beta)(P+Q)A' = 0$$

$$-\frac{27e}{8m}A' + (4\alpha-2\beta-1)\,g\,W' + \tfrac{1}{2}(2\alpha-\beta)PA' + \tfrac{1}{2}\beta PC' = 0$$

$$-\frac{27e}{8m}A' + (4\alpha-2\beta+1)\,g\,X' + \tfrac{1}{2}(2\alpha-\beta)QA' + \tfrac{1}{2}\beta QC' = 0$$

$$-\frac{9e}{16m}B' + (4\alpha+2\beta-1)gY' = 0 \quad ; \quad -\frac{9e}{16m}B' + (4\alpha+2\beta+1)gZ' = 0$$

+

$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+\frac{3e}{8m}C'+(2\beta-1)gK'+\tfrac{1}{2}(2\alpha+\beta)QB'=0$$

$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+\frac{3e}{8m}C'+(2\beta+1)gL'+\tfrac{1}{2}(2\alpha+\beta)PB'=0$$

$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+(4\alpha-1)gT'+\tfrac{1}{2}(2\alpha+\beta)PB'=0$$

$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+(4\alpha+1)gV'+\tfrac{1}{2}(2\alpha+\beta)QB'=0$$

LVI.

Valores ergo horum coefficientium iam ad minuta secunda reductorum erunt: $O' = +310''$

$P' = -1285''$; $M' = -293''$; $F' = +401''$
$Q' = -1087$; $N' = -251$; $G' = +5412$
$R' = +148$; $D' = -52$; $H' = -2$
$S' = +141$; $E' = -45$; $J' = -2$
$W' = -91''$; $K' = +61''$
$X' = -95$; $L' = +61$
$Y' = -1$; $T' = +35$
$Z' = -1$; $V' = +31$

Vnde ob excentricitatem orbitae solaris erit:

$$\xi = +0{,}008931\cos 2\eta -0{,}000247\cos(2\eta-u) +0{,}002090\cos(2\eta-v) -0{,}000227\cos(2\eta+u) +0{,}000640\cos(2\eta+v)$$

$$\begin{aligned}q = g &-0{,}013995\cos(2\eta-v) -0{,}000280\cos 2\eta\\ &-0{,}001460\cos(2\eta+v) +0{,}014012\cos(2\eta-2v)\\ &+0{,}002834\cos v -0{,}000162\cos(2\eta+2v)\\ &+0{,}000340\cos 2v\\ &-0{,}000184\cos 4\eta +0{,}000420\cos(2\eta-v-u)\\ &+0{,}000682\cos(4\eta-2v) +0{,}000360\cos(2\eta-v+u)\\ &+0{,}000009\cos(4\eta+2v)\end{aligned}$$

$$\Phi - v = \text{Const.} + 0,1123\,\omega$$

$-53018''\sin(2\eta-v) + 1533''\sin 2\eta \quad -1285''\sin(2\eta-v-u)$
$-5532\ \sin(2\eta+v) - 53549\ \sin(2\eta-2v) - 1087\ \sin(2\eta-v-u)$
$+10736\ \sin v \quad - 464\ \sin(2\eta+2v) + 148\ \sin(2\eta+v-u)$
$\quad + 449\ \sin 2v \quad + 141\ \sin(2\eta+v+u)$

$-293''\sin(v-u) \quad + 401''\sin(2\eta+2v-u) + 61''\sin(2v-u)$
$-251\ \sin(v+u) \quad + 5412\ \sin(2\eta-2v+u) + 61\ \sin(2v+u)$
$-52\ \sin(2\eta-u) \quad - 91\ \sin(4\eta-2v-u) + 35\ \sin(4\eta-u)$
$-45\ \sin(2\eta+u) - 95\ \sin(4\eta-2v+u) + 31\ \sin(4\eta+u)$
$+310\ \sin u$

neglectis scilicet terminis minimis.

LVII.

Denique pro longitudine lunae vera Φ inuenienda, cum sit $\frac{d\Phi}{d\omega} =$ Praec.

$+mg.\ 0,00005\cos(2\eta-v-u) \quad -m.\ 0,00005\cos(2\eta-u)$
$+mg.\ 0,00002\cos(2\eta-v+u) \quad -m.\ 0,00002\cos(2\eta+u)$
$+mg.\ 0,00005\cos(2\eta+v-u) \quad -m.\ 0,00042\cos(2\eta-2v-u)$
$+mg.\ 0,00002\cos(2\eta+v+u) \quad -m.\ 0,00036\cos(2\eta-2v+u)$
$+mg.\ 0,00042\cos(2\eta-3v-u)$
$+mg.\ 0,00036\cos(2\eta-3v+u)$

ponatur $\Phi = \text{Const.}$

$+\mathfrak{A}'\omega + \mathfrak{B}'\sin v + \mathfrak{D}'\sin(2\eta-2v) + \mathfrak{G}'\sin(2\eta-v) + \mathfrak{J}'\sin(2\eta-3v)$

vna cum nouis terminis

$+\mathfrak{a}'\sin(2\eta-v-u) + \mathfrak{e}'\sin(2\eta-u) + \mathfrak{g}'\sin(2\eta-2v-u) + \mathfrak{l}'\sin u$
$+\mathfrak{b}'\sin(2\eta-v+u) + \mathfrak{f}'\sin(2\eta+u) + \mathfrak{h}'\sin(2\eta-2v+u) + \mathfrak{m}'\sin(2v-u)$
$+\mathfrak{c}'\sin(2\eta+v-u) \quad + \mathfrak{j}'\sin(2\eta-3v-u) + \mathfrak{n}'\sin(2v+u)$
$+\mathfrak{d}'\sin(2\eta+v+u) \quad + \mathfrak{k}'\sin(2\eta-3v+u)$

$+\mathfrak{o}'$

$$+\mathfrak{o}'\sin(2\eta+2v-u) + \mathfrak{q}'\sin(v-u) + \mathfrak{s}'\sin(4v-u)$$
$$+\mathfrak{p}'\sin(2\eta+2v+u) + \mathfrak{r}'\sin(v+u) + \mathfrak{t}'\sin(4v+u)$$

pro reliquorum terminorum, quos forma differentialis requirit, coefficientibus ponamus litteram I.

LVIII.

Differentiatione iam per regulas praecedentes instituta erit: $\frac{d\Phi}{d\omega} = \text{Praec.}$

$$+\cos u\left(\frac{3e}{8mg}\mathfrak{B}' + \frac{3e}{8mg}\mathfrak{B}' - \frac{27e}{16mg}\mathfrak{G}' - \frac{27e}{16mg}\mathfrak{G}' + \mathfrak{l}'\right)$$
$$+(2\alpha+\beta-1)\mathfrak{c}'\cos(2\eta+v-u)$$

$$+\cos(2v-u)\left(\frac{3e}{8mg}\mathfrak{B}' + \frac{9e}{16mg}\mathfrak{G}' - \frac{81e}{16mg}\mathfrak{J}' + (2\beta-1)\mathfrak{m}'\right)$$
$$+(2\alpha+\beta+1)\mathfrak{d}'\cos(2\eta+v+u)$$

$$+\cos(2v+u)\left(\frac{3e}{8mg}\mathfrak{B}' + \frac{9e}{16mg}\mathfrak{G}' - \frac{18e}{16mg}\mathfrak{J}' + (2\beta+1)\mathfrak{n}'\right)$$

$$+\cos(2\eta-u)\left(+\frac{27e}{16mg}\mathfrak{B}' - \frac{9e}{16mg}\mathfrak{B}' - \frac{3e}{8mg}\mathfrak{G}' + (2\alpha-1)\mathfrak{e}'\right)$$

$$+\cos(2\eta+u)\left(+\frac{27e}{16mg}\mathfrak{B}' - \frac{9e}{16mg}\mathfrak{B}' - \frac{3e}{8mg}\mathfrak{G}' + (2\alpha+1)\mathfrak{f}'\right)$$

$$+\cos(2\eta-2v-u)\left(+\frac{27e}{16mg}\mathfrak{B}' + 2e\mathfrak{D}' - \frac{9e}{8mg}\mathfrak{G}' - \frac{9e}{8mg}\mathfrak{J}' + (2\alpha-2\beta-1)\mathfrak{g}'\right)$$

$$+\cos(2\eta-2v+u)\left(+\frac{27e}{16mg}\mathfrak{B}' + 2e\mathfrak{D}' - \frac{3e}{8mg}\mathfrak{G}' - \frac{9e}{8mg}\mathfrak{J}' + (2\alpha-2\beta+1)\mathfrak{h}'\right)$$

$$+\cos(4v-u)\left(+\frac{27e}{16mg}\mathfrak{J}'+(4\beta-1)\,\mathfrak{s}'\right)$$

$$+\cos(4v+u)\left(+\frac{27e}{16mg}\mathfrak{J}'+(4\beta+1)\,\mathfrak{t}'\right)$$

$$+\cos(2\eta+2v-u)\left(-\frac{9e}{16mg}\mathfrak{B}'+(2\alpha+2\beta-1)\,\mathfrak{v}'\right)$$

$$+\cos(2\eta+2v+u)\left(-\frac{9e}{16mg}\mathfrak{B}'+(2\alpha+2\beta+1)\,\mathfrak{p}'\right)$$

$$+\cos(2\eta-v-u)\left(-\frac{3e}{4mg}\mathfrak{D}'+2e\mathfrak{G}'+(2\alpha-\beta-1)\,\mathfrak{a}'\right)$$

$$+\cos(2\eta-3v+u)\left(-\frac{3e}{8mg}\mathfrak{D}'+2e\mathfrak{J}'+(2\alpha-3\beta+1)\,\mathfrak{f}'\right)$$

$$+\cos(2\eta-v+u)\left(-\frac{3e}{4mg}\mathfrak{D}'+2e\mathfrak{G}'+(2\alpha-\beta+1)\,\mathfrak{b}'\right)$$

$$+\cos(2\eta-3v-u)\left(-\frac{3e}{4mg}\mathfrak{D}'+2e\mathfrak{J}'+(2\alpha-3\beta-1)\,\mathfrak{j}'\right)$$

$$+\cos(v-u)\left(-\frac{27e}{8mg}\mathfrak{D}'+(\beta-1)\,\mathfrak{q}'\right)$$

$$+\cos(v+u)\left(-\frac{27e}{8mg}\mathfrak{D}'+(\beta+1)\,\mathfrak{r}'\right)$$

$$+\cos(4\eta-3v-u)\left(-\frac{27e}{8mg}\mathfrak{D}'+(4\alpha-3\beta-1)\,\mathfrak{l}\right)$$

$$+\cos(4\eta-3v+u)\left(-\frac{27e}{8mg}\mathfrak{D}'+(4\alpha-3\beta+1)\,\mathfrak{l}\right)$$

$$+\cos(3v-u)\left(+\frac{9e}{8mg}\mathfrak{D}'+(3\beta-1)\,\mathfrak{l}\right)$$

$$+\cos(3v+u)\left(+\frac{9e}{8mg}\mathfrak{D}'+(3\beta+1)\,\mathfrak{l}\right)$$

+

$$+\cos(4\eta-v-u)\left(+\frac{9e}{8mg}\mathfrak{D}'+(4\alpha-\beta-1)\,I\right)$$

$$+\cos(4\eta-v+u)\left(+\frac{9e}{8mg}\mathfrak{D}'+(4\alpha-\beta+1)\,I\right)$$

$$+\cos(4\eta-2v-u)\left(-\frac{27e}{16mg}\mathfrak{G}'+\frac{27e}{16mg}\mathfrak{J}'+(4\alpha-2\beta-1)\,I\right)$$

$$+\cos(4\eta-2v+u)\left(-\frac{27e}{16mg}\mathfrak{G}'+\frac{27e}{16mg}\mathfrak{J}'+(4\alpha-2\beta+1)\,I\right)$$

$$+\cos(4\eta-u)\left(+\frac{9e}{16mg}\mathfrak{G}'+(4\alpha-1)\,I\right)$$

$$+\cos(4\eta+u)\left(+\frac{9e}{16mg}\mathfrak{G}'+(4\alpha+1)\,I\right)$$

$$+\cos(2\eta-4v-u)\left(-\frac{9e}{8mg}\mathfrak{J}'+(2\alpha-4\beta-1)\,I\right)$$

$$+\cos(2\eta-4v+u)\left(-\frac{9e}{8mg}\mathfrak{J}'+(2\alpha-4\beta+1)\,I\right)$$

$$+\cos(4\eta-4v-u)\left(-\frac{81e}{3mg}\mathfrak{J}'+(4\alpha-4\beta-1)\,I\right)$$

$$+\cos(4\eta-4v+u)\left(-\frac{81e}{8mg}\mathfrak{J}'+(4\alpha-4\beta+1)\,I\right)$$

LIX.

Collectis hinc valoribus coefficientium assumtorum, obtinebitur longitudo lunae vt sequitur:

$\Phi =$

$$\varphi = C + 13,3682\,\omega$$

$$\begin{array}{lll}
-22728'' \sin v & +2823'' \sin(2\eta-3v) & +100'' \sin u \\
-1081 \sin 2v & +47 \sin(2\eta+3v) & -23 \sin(v-u) \\
-128 \sin 3v & -246 \sin(4\eta-v) & -20 \sin(v+u) \\
-700 \sin 2\eta & +41 \sin(4\eta+v) & +22 \sin(2v-u) \\
-3594 \sin(2\eta-2v) & +379 \sin(4\eta-3v) & +21 \sin(2v+u) \\
+306 \sin(2\eta+2v) & -2 \sin(4\eta+3v) & +2 \sin(3v-u) \\
-3762 \sin(2\eta-v) & & +2 \sin(3v+u) \\
-115 \sin(2\eta+v) & & -2 \sin(4v-u) \\
 & & -2 \sin(4v+u)
\end{array}$$

$$\begin{array}{ll}
+17'' \sin(2\eta-u) & -1' \sin(2\eta-4v-u) \\
+19 \sin(2\eta+u) & -1 \sin(2\eta-4v+u) \\
+1 \sin(4\eta-u) & -11 \sin(4\eta-2v-u) \\
+1 \sin(4\eta+u) & -10 \sin(4\eta-2v+u) \\
+6 \sin(2\eta-v-u) & -28 \sin(4\eta-3v-u) \\
+5 \sin(2\eta-v+v) & -23 \sin(4\eta-3v+u) \\
+60 \sin(2\eta-2v-u) & -98 \sin(4\eta-4v-u) \\
-194 \sin(2\eta-2v+u) & -245 \sin(4\eta-4v+u) \\
-6 \sin(2\eta+2v-u) & +2 \sin(4\eta-v-u) \\
-6 \sin(2\eta+2v+u) & +2 \sin(4\eta-v+u) \\
+6 \sin(2\eta-3v-u) & \\
+8 \sin(2\eta-3v+u) &
\end{array}$$

LX.

Plurimae igitur prodeunt inaequalitates ab excentricitate solis pendentes, quarum nonnullae ita sunt magnae,

gnae, vt fine notabili errore omitti nequeant; cuiusmodi funt imprimis, quae ab angulis $2\eta-2v\mp u$ et $4\eta-4v\mp u$ pendent. Sed in his fere idem incommodum vſu venit, quo methodus praecedens premebatur, quod magnitudo harum inaequalitatum per Theoriam non ſatis accurate definiri queat. Cum enim pro his terminis inueniendis diuiſores $2\alpha-2\beta\mp 1$ et $4\alpha-4\beta\mp 1$ fiant perquam exigui, manifeſtum eſt in diuidendis terminos minimos neglectos non exigui fore momenti; praecipue cum pro litteris g' et h' termini maiores fere ſe mutuo deſtruxiſſent. Vnde cum ex valore Φ tantum termini maiores $\mathfrak{B}'$, $\mathfrak{D}'$, $\mathfrak{G}'$ et $\mathfrak{J}'$ eſſent adhibiti, perſpicuum eſt ſi etiam reliqui minores fuiſſent introducti, ex iis inſignem mutationem in valore coefficientium g' et h' orituram fuiſſe.

LXI.

De his autem inaequalitatibus tenendum eſt, eas per ſatis notabile temporis ſpatium vix immutari; nam inaequalitates ab angulo $2\eta-2v+u$, ob quantitatem $2\alpha-2\beta+1=-0,1594$ periodum habent annorum circiter $6\frac{1}{3}$ annorum, et interuallo 19 annorum ter tantum reuoluuntur: et inaequalitas ab angulo $4\eta-4v+u$ pendens ſpatio 29 annorum 19 periodos abſoluit. Ex quo cum iſtae inaequalitates per theoriam ſaltem propemodum fuerint definitae, eas deinceps per obſeruationes accuratius definiri conueniet: niſi forte quis laborem in ſe ſuſcipere voluerit, calculum hic adumbratum multo accuratius inſtituendi terminorumque hic omiſſorum rationem habendi; tum vero etiam valores ξ, q et $\Phi-v$

 multo

multo maiori ſtudio, quam hic feci, euolui oporteret, quoniam in horum determinatione multa neglexi, quae in calculo tandem ad notabilem quantitatem excreſcere potuiſſent.

LXII.

Interim tamen hic notari conuenit, hac methodo eas tantum inaequalitates prodire incertas, quae ſatis longis periodis abſoluuntur; quae incertitudo minus officit, cum per obſeruationes facilius emendari poſſit: praecedente vero methodo etiam aliae inaequalitates minoribus periodis circumſcriptae aliquantum incertae prodierunt, quod ſane ingens erat incommodum. Vnde ex hac parte haec methodus poſterior priori anteferenda videtur: verum ſi ingentem inaequalitatum numerum ſpectemus, quibus non ſolum lunae longitudo afficitur, ſed etiam longitudo apogei, calculus tantopere fit operoſus, vt etiamſi has formulas accuratiſſime euoluerem, tamen in praxi difficillimi foret vſus. Quin etiam plurimae inaequalitates in motum apogei ingredi videntur, quarum effectum deinceps per alias longitudinis inaequalitates iterum deſtrui oportet, ita vt ſatius fuiſſet illas penitus omittere.

LXIII.

Multitudo autem harum inaequalitatum, quibus tam apogei, quam ipſius lunae longitudo turbatur, inde potiſſimum originem trahit, quod inaequalitates excentricitatis prae eius quantitate media admodum ſint notabiles, atque adeo quadrantem mediae quantitatis ſuperent; ita vt prae ea negligi minime queant. Multo plures autem

tem

tem adhuc inaequalitates essent accessurae, si excentricitas lunae media adhuc esset minor, quo certe casu calculi difficultates insuperabiles euasissent: hoc vero ipso casu methodus prior multo tractabilior redderetur, tum enim pleraeque inaequalitates ibi multo minores prodirent. Atque ob hanc causam minus expedire videtur, anomaliam lunae ita constituere, vt eius sinus tam pro maximis quam pro minimis distantiis lunae a terra plane euanescat, etiamsi haec ratio naturae rei maxime consentanea videatur.

LXIV.

Cum igitur numerus inaequalitatum iam tantopere increuerit, facile perspicitur eum adhuc multo magis auctum iri, si eas inaequalitates, quae cum a parallaxi solis, tum ab eius inclinatione ad eclipticam essem euoluturus, quo labore propterea, cum eius vsus fere nullus futurus esset, supersedebo. Interim tamen hinc tantum colligere licet, inaequalitates ab angulis $2\eta - 2v \mp u$ et $4\eta - 4v \pm u$ ortas, minime esse contemnendas; quae cum methodo praecedente sint vel omissae vel non satis accurate determinatae, sine dubio causam in se continent, quod etiam accuratissimae tabulae per obseruationes emendatae adhuc vltra 4′ saepe a veritate aberrent.

LXV.

Sufficiat igitur methodum exposuisse, cuius ope inaequalitates lunae tam ratione apogei, quam longitudinis ac latitudinis verae ex anomalia hic adhibita determinari queant; neque propterea laborem calculi reliquarum inaequalitatum, quae vel ex folis parallaxi vel ex inclinatione orbitae lunaris ad eclipticam oriuntur, fufcipio; quippe quarum numerus, fiquidem omnes, quae alicuius momenti effent futurae, perfequi vellem, in immenfum excrefceret. Non folum autem multitudo inaequalitatum hanc methodum omni vtilitate in praxi priuabit, fed etiam ingentes aequationes, quas determinatio apogei, atque anomaliae inde pendentis requirit, ita funt comparatae, vt ipfae iam fatis exactam tam longitudinis quam anomaliae cognitionem requirant; quae res etfi initio fupponi poffent, deinceps iterata eadem operatione accuratius definiendae, tamen quia correctio apogei vltra 30° gradus affurgere poteft, calculus ob inaequalitatum multitudinem per fe taediofus, nimis crebro repeti deberet, antequam de conclufione certi effe poffemus.

APPLICATIO FORMULARUM INUENTARUM AD ALIOS CALCULOS LUNARES.

LXVI.

Cum igitur calculus inaequalitatum motus lunae hactenus duplici modo fit inftitutus, dum priori anomalia vera regulis Keplerianis conformis eft affumta, pofteriori vero ita conftituta, vt eius finus tam pro maximis lunae a terra diftantiis quam pro minimis prorfus euanefceret, quorum vterque vti vidimus incommodis non caret: ita etiam infinitis aliis modis lunae inaequalitates repraefentari poterunt, quos breuiter expofuiffe haud abs

re

re fore arbitror. Nullum enim eſt dubium, quin inter hos infinitos modos quidam reperiantur, qui ipſi naturae rei magis ſint conſentanei, neque iis incommodis laborent, quibus vtrumque expoſitum non mediocriter impediri comperimus; etiamſi adhuc difficillimum videatur, inter hanc infinitam multitudinem modum conuenientiſſimum eligere.

LXVII.

Poſtquam autem inueſtigationem ab aequationibus differentio-differentialibus ad aequationes ſimpliciter differentiales produximus, etiamſi ad hoc anomalia vera v cuius ſinus in maximis ac minimis lunae a terra diſtantiis euaneſcat, ſimus vſi, tamen haec conditio iam iterum exui poteſt. Cum enim tam ſinus quam coſinus ipſius v vbique per quantitatem q ſit multiplicatus, loco harum duarum variabilium q et v iam alias duas variabiles in calculum introducere poterimus, quod commodiſſime fiet ponendo $q \cos v = r$ et $q \sin v = s$, vt ſit $qq = rr + ss$ et $\text{tang}\, v = \frac{s}{r}$, tum enim vi formularum §. IX. exhibitarum habebimus iſtas aequationes:

$$dr = -s d\Phi + \frac{2M}{A} d\omega \sqrt{Ap};$$

$$ds = r d\Phi + \left(\frac{N}{A} - \frac{Ms}{A(1-r)}\right) d\omega \sqrt{Ap}.$$

 LXVIII.

LXVIII.

Hinc autem porro erit: $x = \frac{p}{1-r}$;

$$dp = -\frac{2Mp\,d\omega}{A(1-r)}\sqrt{Ap} \quad \text{et} \quad d\Phi = \frac{x\,d\omega(1-r)^2}{pp}\sqrt{Ap},$$

tum vero vt ante $du = d\theta = \frac{d\omega(1-e\cos u)^2}{(1-ee)\sqrt{(1-ee)}}$.

Deinde vero si statuamus $p = b(1+\xi)$, reliquasque denominationes in §§. 11, 12, 13. factas adhibeamus, obtinebimus:

$$\frac{M}{A}\sqrt{Ap} = \frac{3(1+3ee)}{2m}\,\frac{(1-e\cos u)^3}{1-r}\,(1+\tfrac{3}{2}\xi)\sin 2\eta$$
$$+\frac{3n}{8m}\cdot\frac{(1-e\cos u)^4}{(1-r)^2}(1+\tfrac{5}{2}\xi)(\sin\eta + 5\sin 3\eta)$$

$$\frac{N}{A}\sqrt{Ap} = -\frac{(1+3ee)}{2m}\cdot\frac{(1-e\cos u)^3}{1-r}(1+\tfrac{3}{2}\xi)(1+3\cos 2\eta)$$
$$-\frac{3n}{8m}\cdot\frac{(1-e\cos u)^4}{(1-r)^2}(1+\tfrac{5}{2}\xi)(3\cos\eta + 5\cos 3\eta)$$
$$+m(1-r)^2(1-\tfrac{3}{2}\xi)\Pi - i$$

atque

$$x = \frac{b(1+\xi)}{1-r}; \quad d\xi = -\frac{2(1+\xi)\,d\omega}{1-r}\cdot\frac{M}{A}\sqrt{Ap};$$

ac tandem:

$$d\Phi = m\,d\omega(1-\tfrac{3}{2}\xi + \tfrac{15}{8}\xi\xi)(1-r)^2$$

LXIX.

LXIX.

Hinc iam omnium differentialium rationes ad $d\omega$ habentur, erit enim

$$\frac{d\xi}{d\omega} = -\frac{2(1+\xi)}{1-r}\cdot\frac{M}{A}\sqrt{Ap}:$$

$$\frac{d\Phi}{d\omega} = m\left(1-\tfrac{3}{2}\xi+\tfrac{15}{8}\xi\xi\right)(1-r)^2$$

$$\frac{du}{d\omega} = \frac{d\theta}{d\omega} = \frac{(1-e\cos u)^2}{(1-ee)\sqrt{(1-ee)}} \quad \text{et} \quad \frac{d\eta}{d\omega} = \frac{d\Phi - d\theta}{d\omega}.$$

$$\frac{dr}{d\omega} = -ms\left(1-\tfrac{3}{2}\xi+\tfrac{15}{8}\xi\xi\right)(1-r)^2 + 2.\frac{M}{A}\sqrt{Ap}$$

$$\frac{ds}{d\omega} = mr\left(1-\tfrac{3}{2}\xi+\tfrac{15}{8}\xi\xi\right)(1-r)^2 + \frac{N}{A}\sqrt{Ap} - \frac{s}{1-r}\cdot\frac{M}{A}\sqrt{Ap}$$

$$\frac{d\pi}{d\omega} = -\frac{1}{m}\sin(\theta-\pi)\sin(\Phi-\pi)\left(\frac{3(1+3ee)(1+\tfrac{3}{2}\xi)}{(1-r)^2}\cos\eta \right.$$
$$\left. + \tfrac{3}{4}n(3+5\cos 2\eta)\right)$$

$$\frac{d.ltg\,\rho}{d\omega} = -\frac{1}{m}\sin(\theta-\pi)\cos(\Phi-\pi)\left(\frac{3(1+3ee)(1+\tfrac{3}{2}\xi)}{(1-r)^2}\cos\eta \right.$$
$$\left. + \tfrac{3}{4}n(3+5\cos 2\eta)\right)$$

at est $\Pi = 1 - \lambda - \tfrac{1}{4}\,\mathrm{tang}\,\rho^2 - \tfrac{3}{4}\,\mathrm{tang}\,\rho^2 \cos 2(\Phi-\pi)$, vbi pro λ assumi potest $1 - \tfrac{1}{4}\,\mathrm{tang}\,\epsilon^2$, denotante ϵ inclinationem mediam orbitae lunaris ad eclipticam, vt fit $\Pi = \tfrac{1}{4}\,\mathrm{tang}\,\epsilon^2 - \tfrac{1}{4}\,\mathrm{tang}\,\rho^2\,\{1+\cos 2(\Phi-\pi)\}$

LXX.

LXX.

Quodſi iam pro r ſtatueretur iſte valor $k \cos v$, ita vt k eſſet quantitas conſtans, orirețur modus initio traditus inaequalitates lunae repraeſentandi, foret enim tum v anomalia vera Kepleri et k denotaret excentricitatem orbitae lunaris. Vnde patet etiam inaequalitates lunae per methodum primam erutas, ex his formulis inueniri poſſe, neque ad hoc aequationibus ſecundi gradus eſſe opus. Reduceretur autem hoc caſu indoles differentio-differentialium ad inuentionem quantitatis s, quem in finem pro s aſſumi deberet ſeries quaedam ſinuum angulorum η, v, u et $\Phi - \pi$ formatorum cum indefinitis coefficientibus, quos deinceps determinare liceret: hoc autem modo ſolutio primum tradita eſſet proditura.

LXXI.

Cum ſit $p = b(1+\xi)$ et $\sqrt{A} = m\sqrt{b^3}$, ob $dx = -\frac{s\,d\omega}{p}\sqrt{Ap}$, fiat $\frac{dx}{d\omega} = -\frac{mbs}{\sqrt{(1+\xi)}} = -mbs(1 - \frac{1}{2}\xi + \frac{3}{8}\xi\xi)$, vnde patet ſi eiusmodi anomalia v introducatur, vt ſit $s = q \sin v$, ſiue q ſit quantitas conſtans ſiue variabilis, tum hanc anomaliam tam in maximis quam minimis diſtantiis ſinum euaneſcentem eſſe habituram. Ac ſi pro q quantitas vel conſtans vel ex angulis cognitis compoſita aſſumatur, tum inde coefficientes aſſumti ac praeterea valor litterae r determinabitur. Sin autem pro q eiusmodi quantitas incognita aſſumatur, vt ſit praeterea $r = q \cos v$, tum ſolutio ante expoſita reſultabit.

LXXII.

LXXII.

Semper autem vſus aſtronomicus exigit, vt anomalia vera quaedam angulo v contenta introducatur, id quod infinitis modis fieri poteſt. Quo autem quantitas r variabilitatem diſtantiarum lunae a terra accuratius exprimat, et valor ipſius ξ quam minimas mutationes ſubeat, neceſſe eſt, vt quantitas r huiusmodi contineat terminum $k\cos v$, vbi k excentricitatem deſignet, qui ſit quaſi eius pars praecipua; hocque etiam locum habet, ſi pro r ſumatur $q\cos v$, denotante q quantitatem variabilem, quippe cuius pars potior excentricitatem k praebere debet. Verum praeterea quantitas r alios terminos continere poteſt, qui ab angulo v vel pendeant vel non pendeant: ita poni poſſet: $r = k.\cos v + A\cos 2\eta + B\cos 4\eta + C\cos(2\eta - v)$ etc. quo valore aſſumto litterae quoque s, ξ cum reliquis ſuos valores debitos obtinerent.

LXXIII.

Hoc modo illud incommodum euitari poteſt, quo methodum in hoc additamento traditam laborare vidimus, ſi excentricitas orbitae lunaris eſſet nimis parua, vel adeo euaneſcens; tum enim diſtantiae maximae et minimae non amplius ab anomalia penderent, ſed potius ab angulo η, atque imprimis quidem a coſinu dupli anguli 2η. Caſu ergo quo excentricitas plane euaneſcit, pro variabili r, cuius loco vtique noua variabilis introduci debet, non conueniet anomaliam v introducere, ſed praeſtabit aſſumi ſeriem coſinuum ex ſolis angulis 2η, et et $\varphi - \pi$ constantem, quorum coefficientes etsi sunt constantes, tamen quia terminorum numerus in infinitum excurrit, vicem novae variabilis sustinebunt. Tum autem valor ipsius s ex simili serie sinuum eorundem angulorum constabit.

LXXIV.

Quodsi ergo rem generatim pro quacunque excentricitate expedire velimus, poterimus ad hos terminos, qui ex hypothesi excentricitatis euanescentis prodeunt, adhuc adiungere terminos ex anomalia v formatos. Ita neglectis tam inaequalitatibus parallacticis, quam iis quae tum ab excentricitate orbitae solaris, tum ab inclinatione orbitae lunaris ad eclipticam pendent, poni conueniet:

$$r = k\cos v + A\cos 2\eta + B\cos(2\eta - v) + C\cos(2\eta + v) \\ + D\cos 4\eta + E\cos(4\eta - v) \text{ etc.}$$

$$s = \Delta k \sin v + \mathfrak{A}\sin 2\eta + \mathfrak{B}\sin(2\eta - v) + \mathfrak{C}\sin(2\eta + v) \\ + \mathfrak{D}\sin 4\eta + \mathfrak{E}\sin(4\eta - v) \text{ etc.}$$

$$\xi = ..\cos v + ..\cos 2\eta + ..\cos(2\eta - v) + ..\cos(2\eta + v) \\ + ..\cos 4\eta + ..\cos(4\eta - v) \text{ etc.}$$

Atque si hoc modo omnes angulorum 2η et v combinationes adhibeantur, hique valores in aequationibus supra datis substituantur, primo inde elicietur ratio $dv : d\omega$, ac deinceps coefficientes determinationes suas nancifcentur.

LXXV.

Manebunt autem coefficientes vnius seriei veluti ipsius r indeterminati, propterea quod ipsa series haec

ab

ab arbitrio nostro pendet, dum pro r vel solum primum terminum k cos v, vel quotquot lubuerit, assumere potuissemus. Hinc autem id commodi consequemur, vt istos coefficientes ad scopum quam conuenientissime definire valeamus: scilicet eos ita definiri conueniet, vt primo nullius reliquorum coefficientium determinatio lubrica et incerta euadat, vti in vtraque methodo exposita vsu venit: deinde vero vt nulli coefficientes fiant nimis magni praeter necessitatem, ita vt eorum effectus per alios terminos iterum destrui necesse sit. Fateri quidem cogor calculum hoc modo instituendum admodum futurum esse prolixum, verum fortasse in ipsa operatione non contemnenda se offerent compendia; vnde confido hanc speculationem, etiamsi mihi ipsi eam suscipere non vacet, vsu non esse cariturum.

BEROLINI, EX OFFICINA MICHAELIS.