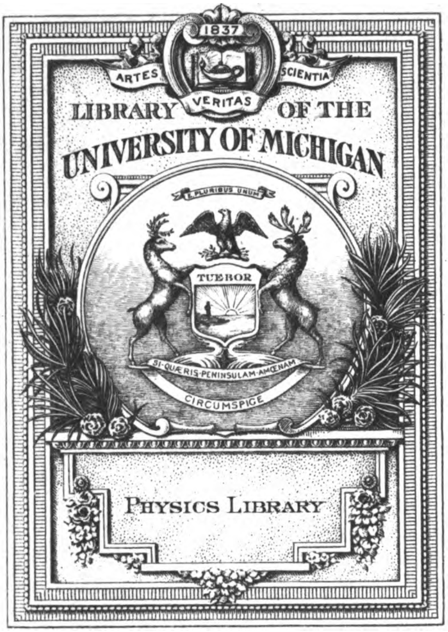
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CHAPTERS IN THE HISTORY OF SCIENCE

GENERAL EDITOR CHARLES SINGER

II

Mathematics and Physical Science in Classical Antiquity

Translated from the German

of

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by

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PREFACE BY GENERAL EDITOR

THIS little work is a translation of the second edition of *Naturwissenschaften Mathematik und Medizin im klassischen Altertum*, by Prof. J. L. Heiberg, of Copenhagen, published by Messrs. B. G. Teubner, of Leipzig and Berlin. The English rendering has been made with the consent of both author and publisher, and Prof. Heiberg has been good enough to look through it himself.

The volume gives a general survey of the science of Classical Antiquity, laying however special stress on the mathematical and physical aspects. A companion volume by the general editor of this series deals more fully with the medical and biological aspects, and the two volumes together complete the account of science in Classical Antiquity.

Further 'Chapters' in this series will gradually complete an outline of the History of Science.

CHARLES SINGER.

UNIVERSITY COLLEGE, LONDON,
1st *October* 1922.

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I

Ionian Natural Philosophy

In the primitive man, that *wonder*, which Aristotle[1] tells us is the beginning of all science, is directed to what lies outside himself. He no more reflects on his own being than a child does; even his fellow men do not attract so keen an attention as his own domestic animals. On the other hand, nature round about him excites his interest and observation. Earth and heaven and their processes stir his fancy and his awakening powers of mind. There has never been a people which has not set itself the question, what thunder and lightning and rain are, how they come to be, how heaven and earth were made, whence plants and beasts come, and how the first man was created. The earliest answers to these questions were necessarily mythological: the immature mind is readily satisfied to pass by some childish analogy from its experience of man to the explanation of all the phenomena of nature. Every mythology—the odd inventions of the Edda, the story of Genesis, or the cosmogony of Hesiod (intellectually a far greater performance)—is an attempt by man to see his way in the world outside, and to this extent contains a germ of science which will be of greater or less worth, according to the natural gifts of the people whose craving for knowledge it satisfies. No doubt in the course of time many true and valuable observations were accumulated on the basis of this mythological habit of thought; but this no more makes a *science* than do the knacks and dexterities which are called into being by the needs of practical life. When these rudimentary beginnings of science are confined to a priesthood or to practical men alone, the impulse to research does not, as a rule, develop: it is deadened by tradition or cramped by routine. The Babylonians and the Egyptians had

[1] *Metaphysics*, i. 2.

done work upon astronomy and geometry before the Greeks, and had accumulated material to hand on to them; but it was the Greeks alone who were able to create out of this dead body of stuff a science capable of development. It is not without due cause that we look up to the Ionian thinkers of the sixth century as our forerunners in the whole range of modern science.

Among all the Greek colonists who settled on the coasts of Asia Minor, the Ionians distinguished themselves by their enterprise and curiosity. They had a good climate and a rich land to ensure them the material conditions of civilization. Like Odysseus, whom they had reshaped in their own image to be their national hero, they were adventurous sailors and had wandered far. They had observed and learned in foreign lands, and brought back strange knowledge to recount at home. Notice how these points are emphasized in Odysseus:

> The cities of a world of nations,
> With all their manners, minds and fashions,
> He saw and knew.[1]

Often he puts his life in peril from mere thirst of knowledge: he goes to the Cyclopes to learn what manner of men they are:

> if of rude disdain
> Churlish and tyrannous, or minds bewrayed
> Pious and hospitable.[2]

And many an Ionian sailor on a foreign shore must have thought and acted as he did. Herodotus is the incarnation of this spirit of the *Odyssey*.

All the creations of old Ionia—the similes of Homer, the observations of the Hippocrateans, the vase-paintings, the ethnological descriptions of Herodotus—all alike display an amazing keenness of observation. And a sober sense of actuality, fortified by transplantation to a foreign soil, produced a conscious independence of tradition, ready to exercise reason boldly on each

[1] Homer, *Od.* i. 3 ff. [Chapman]. [2] *Od.* ix. 174 ff. [Chapman].

and every subject. The highly developed intellectual world of Homer brushed aside many prejudices and superstitions which the Greeks of the mainland were still long to suffer: Xenophanes waged a bitter war on popular beliefs; the physicians derided the notion of god-sent sickness; even in Herodotus, pious man, the scepticism of Ionia often enough breaks through the credulity he had learned in Athens—his knowledge of the world clarified his ideas of human standards of morals and belief.

All the conditions, then, for the birth of science were present in Ionia. Its father, that is, the first who in company with others of like mind handled scientific questions with a lasting success, was—so the Greeks held themselves from early times—Thales, the son of Examyes, a Milesian merchant of mixed blood. The little that we know of him shows him in the first place to have been a widely travelled transmitter of foreign wisdom. In Egypt he learned some problems in mensuration and their solution; and if he was able to foretell the total eclipse of the sun of the 28th of May 585 B.C., he must somewhere have become acquainted with the Babylonian astronomical tables and their use. We cannot determine whether foreign influences counted for much in his principal scientific achievements. The question he raised was the oldest of all: How did the world come to be? But his answer pushed mythology aside. Real substance took the place of the creations of fancy. The world, he held, and all that is in it, is made out of water. Recognizing justly that apparent manifoldness must be reduced as far as possible, he took at once, with the boldness of youth, the giant stride to the conception of a single primary substance. In this he was followed by the Milesian School of Natural Philosophy, though opinions varied as to the nature of this primary substance. Anaximander called it the 'Boundless'; Anaximenes discovered the properties of this Boundless in the air; for Heraclitus of Ephesus the primary substance, which created all and then destroyed it, was identical with fire.

In regard to the physical explanation of phenomena the material at their disposal for the most part permitted of nothing more than brilliant guess-work; and impatient audacity led them often enough into mistaken and adventurous hypothesis. None the less all these thinkers made real advance in this direction too. In Anaximander there are ideas which remind us of Darwin; Anaximenes explains the origin of things by the condensation and rarification of the primary substance; Heraclitus, by his assumption of the eternal movement of matter, is in touch with modern physics, and is the first to formulate the conception of the uniformity of nature. Thus the arbitrary fancies of mythology gave place to the conception of the cosmos, and the road to a rational view of the world lay open.

The natural philosophy of Ionia, thanks to the direction of its research and the statement of its problem, embraced the elements of several sciences: physics, astronomy, geography, and mathematics. Heraclitus, who throughout held a position apart and sharply condemned that 'erudition which teaches not to have understanding',[1] paid little attention to the explanation of particular phenomena of nature; but the three Milesians were particularly interested in the careful study of astronomical phenomena. The most important in this regard was Anaximander, who superseded for ever the primitive conception of the world by his hypothesis of a spherical heaven in the midst of which hangs the earth shaped like the drum of a pillar. He constructed a celestial globe to give ocular demonstration of his theory. By the side of this triumphant achievement his naïve view of the heavenly bodies seems oddly out of place; he took them to be apertures in hollow wheels, filled with fire, which rotated in space. But this juxtaposition of brilliant intuition and childish analogy is very characteristic of the confusions of the early growth of a science, still, as it were, intoxicated with its own youth.

[1] Diels, i³, p. 86, 40 [Bywater, 16].

In the history of geography, also, Anaximander has an honourable place. On the basis of information derived from Ionian sailors—who doubtless congregated in a commercial city so interested in colonization as Miletus—he produced the first map known among the Greeks. Herodotus (v. 49 ff.) gives us a lively description of the impression made in Sparta by this invention, when Aristagoras displayed it to King Cleomenes, 'a tablet of bronze on which was engraved the whole circle of the earth and every sea and river.'

The defeat of the Persians gave to Athens the intellectual headship of Ionia. The new science was carried thither by Anaxagoras of Clazomenae, who lived there for a considerable time in the circle of Pericles, and found many adherents among the more progressive Athenians; until at last the forces of reaction stirred up the old Attic piety against this foreign, godless wisdom. Anaxagoras' sober explanations of astronomical phenomena let loose a storm before which he was forced to fly. Like his Ionian forerunners he embraced the whole range of learning. He shared their weaknesses, to be sure; but the intrepid logicality of his mind led him frequently to astonishingly correct conclusions. He gave an approximately true explanation of the Nile floods. He described the sun as a mass of red-hot iron; the moon—whose phases he explained in the main correctly—was similar to the earth. The heavenly bodies, he held, were fragments of the original mass hurled out centrifugally by the rotation of the cosmos. The cause and author of this rotation he took to be mind (*nous*); but he did not allow this semi-intellectual principle to influence his explanations of phenomena, which were governed entirely by mechanical causation.

Anaxagoras' conception of an original mass divisible into infinitesimal particles influenced the growth of the greatest of all the physical systems of antiquity, that of the Atomists. His primitive particles, each possessing in itself the properties of things, were replaced by the atoms. These differed from each other only in the primary qualities of size and shape; and, in conjunction with empty space, sufficed to account for all the processes of nature. The creator of this hypothesis—an hypothesis which to this day, though greatly modified, is of inestimable service to science—was Leucippus. But even in antiquity he was overshadowed by his pupil Democritus of Abdera, a contemporary of Socrates. With colossal industry and profound acumen he elaborated in every detail the fundamental doctrines of his master, and built up a comprehensive system, which, in spite of occasional errors, not only rests throughout on the sound principles of observation and experiment, but in certain cases, notably in treating of the senses, sets forth what is an approximately correct account of the processes concerned. His hypothesis of a plurality of worlds broke down the barriers of the characteristically Greek view of the one limited cosmos—a capital hindrance to any rational science of physics. We learn from scattered notices that he found room for mathematics in the wide range of his literary productions, and published many new and fruitful ideas on this subject. In short, he is the culmination and the close of the Ionian school—not least so in respect of his versatility. In him the Ionian natural philosophy was developed into a system of physics which has played its part in the foundations of our own. Among his contemporaries his teaching found little support. In Athens,[1] where literary success or failure was determined, every receptive mind had been claimed by the new Socratic philosophy of concepts and had turned aside from the interrogation of nature. Thus it came about that physics, greatly to its own misfortune, remained in the hands of philosophers, under stepmotherly restraint, and never achieved a scientific independence or the recognition of its peculiar methods. The other special sciences had long before cut themselves loose from their mother and set up house for themselves.

[1] Diels, ii, p. 83, 116.

2

The Pythagoreans

Descriptive geography had been handled as early as 500 B.C. by the Milesian statesman Hecataeus. His book described minutely the lands of the Mediterranean, especially the coasts and coast-cities, and took account of points of interest in ethnology and natural history. The wonderland of Egypt and its peculiar fauna were described with peculiar fullness. Herodotus too, who as a keen observer had visited the greater part of the then known world, devotes a considerable part of his *Histories* to geographical descriptions of lands and peoples; and ethnography and geography remained for long a subsidiary department of history.

There are evident traces in Herodotus of the opposition of science, which had already won its independence, to the claims of philosophy. He speaks with great contempt of the Ionian attempts to account for the Nile floods; and he lets it be seen how little he thinks of speculation which cannot be tested by one's own eyes.

This distaste of the historian for the bold hypotheses of the philosophers has obscured for him the fact that the primitive view of the world, to which in the main he holds (though he dismisses the Oceanus as an invention of the poets), had for long been seriously imperilled by philosophic speculation. Herodotus knows the Pythagoreans; but he takes no account of their cosmological theories, although these are of decisive importance for geography as well.

About 530 B.C. Pythagoras of Samos, like so many of his countrymen, emigrated to South Italy. In Croton he founded a brotherhood, a narrowly exclusive society bound together by all kinds of mystical ceremony and doctrine, which pursued partly ethical and religious, partly scientific, objects. In the Ionian manner, he had travelled widely, and it was probably in Egypt that he had acquired his interest in mathematics and in numbers. It was apparently owing to the discovery of the great part played in nature by simple numerical ratios, that he arrived at the notion that number is the essence of all things. This doctrine, which degenerated into all kinds of fantastic speculation upon the properties of numbers, contains, stripped of its mystical shell, a sound kernel of truth: that the uniformity of the processes of nature finds its expression in numerical ratios; and it became the foundation of far-reaching scientific achievements. The fact that Pythagoras left nothing in writing, and the veil of mystery in which his school was enveloped until its fall, about the year 500 B. C., made it impossible even in antiquity to determine what belonged to the master and what to the pupils. But we may safely ascribe to Pythagoras himself the essential stimulus, and the applications to the early days of the school.

In astronomy the Pythagoreans were the first to maintain the sphericity of the earth and the heavenly bodies in general. No doubt they were led to this by their mathematical-mystical view of the sphere as the most perfect of the solid figures: for they could have supported their view at best only by reference to the phases of the moon. However, this hypothesis, contradicting as it did the evidence of appearance, marked a vast advance in cosmography, and opened the way not only to a scientific geography, but also to the true explanation of the phenomena of astronomy. A further step in this direction was made in the fifth century by the Pythagorean Philolaus. He abandoned the conception, inevitable in a primitive view, of the earth as the fixed middle-point of the world, and assumed a central fire, round which the earth and the other heavenly bodies revolve. With Philolaus also mystical elements play their part; thus, in order to work the sacred number 10[1] into his system of the

[1] [i.e. sun, moon, earth, the five known planets, central fire, and counter-earth.]

universe, he invented a 'counter-earth' which we cannot see, between the central fire and the ball of the earth, the inhabited half of which looks away from the central fire. But in spite of these fancies, which were soon set aside, his system none the less prepared the way for the Copernican. In these further developments the later Pythagoreans took a share. A member of the school, Ecphantus of Syracuse, was the first to teach the rotation of the earth about its axis—which disposed of the need for central fire and counter-earth. The zodiac and the obliquity of the ecliptic were known in Pythagorean circles in the fifth century; Oenopides is said to have promulgated this doctrine, as also the knowledge of the 'great year', i. e. the period after which all astronomical phenomena begin to repeat themselves.

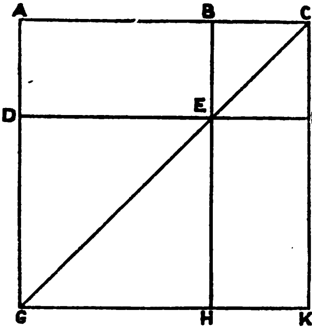
Another of the fantastic features of the Pythagorean system is their harmony of the spheres, a creation of their numerical speculations together with their interest in music. In the theory of music they made fundamental discoveries. They recognized that the pitch of musical notes depends on the length of the string, and determined the relation by simple ratios. It is not improbable that this discovery had a real influence upon their doctrine of the domination of number.

The capital achievement, however, of the Pythagoreans was the creation of mathematics as a science. There was nothing for Pythagoras to learn in Egypt except simple geometrical operations (such as a surveyor requires), and a not inconsiderable practical skill in arithmetic. Both presuppose, it is true, a certain body of theory; but the Egyptians had no mathematical science. They measured their fields by the rules which had once been laid down—whether right or wrong. The Pythagoreans, to whom mathematics owes its very name, considered the fundamental concepts of mathematics—quantity, point, line, surface, body, angle—as pure abstractions; and separated the scientific treatment of figures and numbers as such from the practical arts of geodesy and logistic. The long-known practical rules were transformed into generalized theorems and furnished with rigorous proofs. It is not surprising, if we remember the singular natural gift of the Greeks for abstract logical thinking, that once they had found the way they pursued it with unresting speed. We may observe the same development in another sphere in which a pre-eminent capacity of the people came to its own—in that of art.

The Pythagorean school quickly built up a system of plane geometry in which were formulated and proved the principal theorems of modern elementary mathematics, which concern parallels, triangles, quadrilaterals, regular polygons, and, in part, circles, along with the necessary auxiliary theorems. Further, as regards form, it is certain that they had laid the foundations of that rigorous conception of proof which was to set the standard throughout all Greek mathematics. Solid geometry had not gone far, although they dealt with the sphere and the regular solids. Their eager pre-occupation with numbers led—in spite of all its worthless mysticism—to many important theorems about prime numbers, progressions, &c. In particular, they worked out the theory of proportion, which, as the link between arithmetic and geometry, was of outstanding importance for their combined treatment of these two branches. In their investigations, which led them to equations of the second degree, the Pythagoreans were brought at once face to face with irrationals. Their proof of the existence of such quantities is preserved: it is shown that if the diameter of a square is commensurable with its side, an even number must at the same time be odd. This discovery rendered useless for geometry the older theory of proportion, which only recognized ratios of whole numbers. In order to escape from these 'inexpressible' quantities, they were driven to devise a new method, in which our algebraical expressions are replaced by lines and areas. For examples, what we express by the equation

$$(a+b)^2 = a^2 + 2ab + b^2$$

is proved by the attached figure in which, if $AD = HK = b$, and $DG = GH = a$, the squares BF, DH, AK represent the quantities b^2, a^2, and $(a+b)^2$, and the sum of the rectangles AE, EK is equal to $2ab$.



By means of this 'geometrical algebra' the Pythagoreans fully mastered equations of the second degree. The famous Pythagorean Theorem was doubtless known before in particular cases, but Pythagoras generalized it, and gave a formula for discovering rational numbers for the sides of a right-angled triangle; or, to put it in modern terms, he found a solution in whole numbers of the indeterminate equation $x^2 + y^2 = z^2$. If x is an odd number, $y = \frac{x^2 - 1}{2}$ and $z = \frac{x^2 + 1}{2}$ satisfy the equation.

A peculiar influence was exercised upon the early stages of mathematical development by the Eleatic philosophy, which, like the Pythagorean, had been transplanted from Ionia to South Italy. Zeno recognized the inadequacy of numerical ratios for the treatment of continuous quantities, and used the difficulties inherent in the conceptions of infinity and continuity in the famous paradoxes, in which he attempted to disprove the reality of motion. These clearly betray a familiarity with the Pythagorean mathematics and with the difficulty which had brought it to a standstill. The rigour of Zeno's logic drove the mathematicians to avoid altogether the conception of infinity, as something which could not be precisely stated. His predecessor, Parmenides, has also traces of Pythagoreanism. It was from the Pythagoreans that he took his doctrine of the sphericity of the earth; and his division of the earth's surface into zones is scarcely thinkable without a knowledge of the mathematical treatment of the sphere.

There is Pythagorean inspiration in the work of Empedocles, the poet-philosopher of Agrigentum. He made no contributions to mathematics or astronomy, but he deserves a place in the history of physics as the first to introduce into science the four elements, which, from Aristotle onwards, continued to dominate it for two thousand years. The four elements, to be sure, never deserved this distinction: they are the creatures of a purely popular point of view, not of scientific thinking. But it must not be forgotten that they form a link between the one primary substance of the Milesians and the infinitely many particles of Anaxagoras, and so mark the first step on the road which leads to modern chemistry; nor that this assumption of a minimum number of elements, capable of infinite variety of intermixture and combination, is an idea full of every kind of promise. Similar conceptions of the origins of organic life led Empedocles to views which recall Darwin: in the beginning the various parts of the body existed separately and were formed into all manner of combinations, of which only the fittest survived. In short, among all his poetical fantasies, there are many flashes of brilliant intuition.

3

Medicine in the Fifth Century. Hippocrates

Besides his other activities, Empedocles was something of a physician, though there is more than a touch of the quack about him. The Pythagoreans made important contributions to the development of medicine, the only special science which was not grounded directly upon philosophy. Their main stronghold, Croton, was famous not only for athletics but also for a flourishing school of medicine: Democedes, for some time court physician to Darius I, belonged to it. The versatile Crotoniate doctor, Alcmaeon, had already dissected animals and discovered the most important nerves, which he took to be hollow passages; he also recognized the importance of the brain in the life of the mind. He explained sickness as a disturbance of the elementary opposites in the body—the hot and the cold, the wet and the dry, &c.—and this characteristically Pythagorean doctrine had very considerable influence upon the pathology of later times.

Medicine had risen to a high level by the time of the Homeric poems. Only once is there mention of magic formulae to stay the flow of blood (*Od.* xix. 457); elsewhere the treatment of wounds is entirely rational. The ancient gods of healing, Asclepius and his sons, have become heroes who are particularly skilled in the art; but every warrior knows how to give first aid to a wounded man. The doctor is reckoned among the 'servants of the public' (*demiurgi*), like the bard, the seer, and the shipbuilder; he wanders, as these do, from town to town, at his own will or by summons; and is everywhere a welcome guest. No doubt many a prince already had a doctor attached to his person, as he had a bard; at any rate there was one such (Paieon) among the Gods in Olympus. Naturally it is mainly of war surgery that we read in these poems; but the use of 'soothing medicines' is expressly mentioned as one of the chief activities of the physician. They know of plants and roots which ease pain and of deadly poisons. Helen brought something akin to opium from Egypt

> Whose rich earth herbs of medicine do adorn
> In great abundance. Many healthful are
> And many baneful. Ev'ry man is there
> A good physician out of Nature's grace,
> For all the nation sprung of Paeon's race.[1]

[1] *Od.* iv. 219 ff. [Chapman].

The frequent descriptions of wounds—in their sober realism as far removed from bravado as from horror of bloodshed—betray not only a sound empirical knowledge of the degrees of danger in wounds, but also such fine observation and such astonishing anatomical knowledge that a German surgeon-general has hailed the author of the *Iliad* in all earnestness as a colleague.[1]

In all probability military surgery and the treatment of wounds, the practical importance of which was so obvious, continued to follow the sound lines laid down in the ninth century. No doubt their soothing herbs were borrowed, as elsewhere, from the doubtful source of old-wives' wisdom; but practice would soon drive out the useless elements, and in a department where so plainly it was a question of life and death, one would quickly learn to value the proven experience of the professional man. But in other branches the superstition, from which the Homeric world is so happily free, is by no means absent from the treatment of disease in the following generations. The art of medicine was in part confined to the temples of Asclepius and similar shrines, and was in the hands of priests. In view of the conservatism of all religion, this is in itself a hindrance to the free development of science, and is bound to lead to secrecy and fraud: for the god's failures must be hushed up at all costs. Nevertheless, we must not underrate the importance of these priestly institutions as forerunners of scientific medicine. If only for the sake of their practice the priests must have attended from the very first to the rational treatment of organic diseases which could not be dealt with by suggestion and similar means: a series of failures would in the long run have ruined their reputation. They were compelled, therefore, to observe the symptoms of disease with a view to future cases, to take notes of the treatment and drugs prescribed and of their effects, and doubtless also to record their

[1] Frölich, *Die Militärmedizin Homers*, p. 65.

failures. Thus there would accumulate in the shrines of Asclepius a not inconsiderable body of empirical observations which, combined with experience of surgery in the field, provided a valuable equipment. Moreover, professional athleticism and the daily gymnastic exercises of the youths not only gave ample opportunity for observing the naked human body, but called for skilful and swift treatment of certain injuries, especially dislocations, and for a rational system of dietetic. It is certainly no accident that two of the oldest and most famous schools of medicine are connected, the one with the island of Cos, with its cult of Asclepius, the other with Croton, the city of athletes.

Scientific medicine is the creation of the bold and critical genius of Ionia. In the fourth century the ample corpus of Ionian medical literature was fathered on Hippocrates, the chief representative of the Coan school in the second half of the fifth century. He had travelled widely as a doctor through Greek lands—his grave was shown in Thessaly—and as early as Plato appears as the founder of the scientific practice of medicine. It has not yet proved possible to determine with certainty his share in the corpus of 'Hippocratean' writings.

It is an oddly mixed collection: Coan writings side by side with writings of the rival Cnidian school; philosophical theories of health and disease and popular treatises, of the kind which is sharply criticized elsewhere in the collection, side by side with case-books never meant for publication; superstitious jugglery with numbers beside works which deride every form of supernaturalism. This much, however, is certain, that practically the whole collection belongs to the fourth or fifth century. It gives us a clear picture, not indeed of the personal achievements of Hippocrates, but of the standpoint and tendencies of scientific medicine in its adolescence.

The doctors were organized in a guild. Their oath, which has been preserved, binds the pupil to honour his master as a father; to instruct his master's descendants in the art without charge. Except these and his own sons he may instruct no one who is not a regular sworn companion of the craft. The oath is a fine monument to their high professional ideals. The new pupil promises to use his art only for the use and help of the suffering, never for their harm; never to dispense poisons or abortive medicines; never to take advantage of his position to seduce a patient; to keep under the seal of secrecy all that he may learn in the practice of his profession. Other minute instructions are given which bear the stamp of a fine humanitarianism, and display a vigorous opposition to every kind of quackery. The doctor must mark himself off from the ostentatious display of the charlatan by a quiet dignity even in his dress; he is even warned against the use of notably powerful perfumes. He must not seek to impress the layman by elaborate and imposing apparatus or by popular medical lectures tricked out with tags from the poets. He must win the confidence of his patients by attentive visiting, care, and friendliness. To bargain for the fee before the cure is completed is forbidden; for that makes the patient anxious and distrustful, and may sometimes aggravate his illness. In hard cases the doctor must lend his aid without thought of fee. In women's diseases—a speciality, it seems, of the Cnidian school—and in confinements it is assumed that the physician will have the help of a woman. In several of the writings the physician still appears as a wanderer, as in Epic times—or like Democedes, who is said to have practised, with a large yearly salary, successively in Athens, in Aegina, and with Polycrates of Samos. We read, for example, in the famous *On Airs, Waters, and Places* (a book probably older, however, than Hippocrates), '*if a man comes to a city which he does not know*, he must make a careful inquiry into its position with regard to winds and orientation'; and the author is familiar at first hand with Asia Minor, Egypt, the coasts of the Black Sea, and the Scythians of South Russia.

The practising physicians had the same vivid dislike as Herodotus for the unverifiable hypotheses of philosophical speculation. The encroachments of philosophy are repelled with especial vigour in the notable work *On Ancient Medicine*. The author pours scorn on those who laid down one arbitrary principle and explained all diseases by the warm and the cold, the moist and the dry. This play with hypotheses may pass in natural philosophy: in medicine, when life and death are at stake, it is sheer irresponsibility. It will not do to prescribe 'something warm'; the patient will ask, what? and the physician must then get to business and name some definite thing. But every warming thing has at the same time other qualities which have very various effects on the human body; and these effects also must be known in detail. Then the philosophers say, no one can treat a patient correctly unless he knows what man is and what his origin; but all these general theories, that of Empedocles, for example, belong to philosophy, and do not affect medicine in the least. Admittedly the physician should try to know 'Nature', but it is the particulars of Nature he must know—how each drug acts upon the individual, and why: and we are still far enough even from that stage. If that stage is to be reached, however, it will not be reached by idle speculation but by the proven method of experience and observation of particular cases. The man who leaves that road is lost. But the task is a hard one; and that physician deserves praise who makes only small mistakes; the majority are like unskilful steersmen who, in spite of errors, make something of it in good weather, but so soon as a storm overtakes them betray their incompetence by piling up their ship. Fortunately, the harmless ailments, where the errors of the bungler can do little harm, are far more frequent than the grave ones, where every mistake is quickly and terribly punished.

The same spirit speaks in the famous aphorism : 'Life is short, art is long'; and in the case-books which have come down to us we can see the conscientious physician at his task, observing and noting from day to day every change in the condition of his patient. However, it was no crude empiricism which these patriarchs of medicine practised; rather what has been called 'art, with reflexion'. Their spirit of observation, their hatred of rash hypothesis, their modest self-limitation—these are a wholesome antidote to the bold determination of the philosopher to explain the universe, and constitute one of the propugnacula of exact research.

They had to defend themselves on another front as well, against superstition. With all the inward satisfaction of the reformer the author of *On the Sacred Disease* (i. e. epilepsy) pours out his scorn on people who ascribe this disease now to the Mother of the Gods, now to Poseidon, now to Ares, according to the behaviour of the patient, and treat him with every kind of mystical humbug. Swindlers and false prophets, he says, discovered the name 'sacred' in order to hide their hopeless ignorance behind a mask of piety and a would-be deeper insight. Epilepsy is no more sacred than any other disease; it is due to the same causes as the others. Everything is as divine and as human as everything else: each and all is bound by the conditions of its nature; nothing is mysterious or miraculous. How far the Hippocrateans had advanced in this rationalistic view is most signally shown by the fact that they treated disorders of the mind on the same principles as other diseases—principally by diet and gymnastic.

Thus the fortifications, as it were, were built, in the security of which medicine was left free to develop; and which, in spite of many assaults of its ancient enemies, only fell at last with the general collapse of ancient civilization.

It is evident from a whole series of careful descriptions contained in the Hippocratean writings that the means at their disposal for carrying into practice their wholesome principles were very far from inconsiderable. A number of reports of actual cases has come down to us. These are almost entirely cases of grave illness, and it says much for the atmosphere of sincerity and scientific frankness in which these physicians lived that they tell us that more than half of these cases died despite their efforts at treatment.

Their weakest side, not unnaturally, was physiology. Their various writings differ greatly in this regard; but on the whole they exhibit different stages of the *pathology of humours*, which had gradually developed out of Alcmaeon's doctrine of the *opposites* in the body, until it finally crystallized into the theory of the four humours—blood, phlegm, yellow and black bile. In this form, which lent itself to many convenient analogies—the four elements, the four seasons, &c.—it remained the guiding rule for more than two thousand years. The Ionians further learned from Alcmaeon the true view of the functions and importance of the brain, knowledge which was later lost, and had to be re-conquered.

Anatomy was in better case. True, they still fought shy of dissecting the human body, and were thus driven back upon animals and such chance glimpses as were afforded by serious wounds and injuries. An exact knowledge of the internal organs of man was thus impossible; but whatever was attainable by the means to their hand was attained. The skeleton, the system of the principal blood-vessels, and the heart were in the main correctly described. That the practitioner was determined to use every opportunity for the enlargement of his knowledge is shown by a remark which occurs frequently in descriptions of blood-vessels: 'The further course of this vessel I do not *yet* know.' Dissection of animals in order to determine the causes of disease is often mentioned; there is even a first attempt at vivisection. Experiment in general was familiar enough. We may refer, for example, to the attempt to elucidate the development of the human embryo by opening day by day one of twenty fresh eggs that had been set for hatching.

The influence of the gymnasium is to be traced in the excellent works on fractures and dislocations. The descriptions of even the most infrequent cases are careful and correct. The treatment is thoroughly competent, and often carried out with the very simplest apparatus—such as might be to hand in the gymnasium. Admirable, too, is the work on head-wounds, for its careful observation and rational methods of cure. Trephining is employed with great skill. These surgical works show the Ionian school in the full blaze of its glory. They breathe a spirit of exact, critical, keen observation; they attack charlatans and speculative theorists with a vigorous and often fiercely sarcastic polemic. And one other admirable feature may be emphasized—that, for the benefit of his fellow practitioners, the author does not conceal his own mistakes.

Their lack of theoretical background was rendered harmless partly by their cautious and differentiated treatment, which consciously set itself the task of assisting the healing processes of Nature and of following her lead; partly by their unsurpassed familiarity with the normal human body—so far as that can be compassed by sight and touch. Such an *obiter dictum* as 'it is not difficult to know the state of a man's health if you see him naked in the gymnasium', gives some notion of the sharp eyes of the Greek physician and of the origin of this fine sense for form. Thanks to their eye for the details and for the whole organism alike, the Hippocrateans were in a position to make surprisingly acute and accurate diagnoses and prognoses. They rightly emphasize the value of prognosis in gaining the confidence of the patient, but at the same time deprecate any fraudulent attempts at detailed forecasts. The delicate discrimination of diseases of the lungs by auscultation may be mentioned as an example of diagnosis which still commands the respect of the modern clinical physician. In inflammation of the lungs the *succussio Hippocratica* (still so called) is employed to determine the location and extent of the diseased tissues.

The Hippocratean gift for observation is strikingly confirmed by the vivid and careful pictures of disease which we find throughout their writings. The so-called *facies Hippocratica*, an infallible sign of approaching death, is well known. The symptoms of consumption, known in antiquity to be infectious, are correctly given; similarly the consequences of an injury to the spinal cord —and so forth. The following case deserves to be mentioned as particularly significant. In the *Epidemics* there is an account of an epidemic disease of the throat, followed by paralysis, which was not identified by Littré, although he insisted that the thoroughness of the description was such as to impress any experienced physician at once. In the last volume of his edition he was able to determine that the reference was to an epidemic of diphtheria. For in the meanwhile (1860) English and French doctors had discovered that this disease is frequently accompanied by paralysis. An even more striking token of the thorough competence of this ancient observer is that he expressly points out that the brain was not diseased—as might have been supposed from the paralysis.

The Hippocrateans were reluctant to amputate, since their only method of staying the flow of blood was the application of red-hot iron. If a limb could not be saved they waited calmly till gangrene attacked it and had reached a joint: they then removed the dead part of the limb. 'Such things are worse to see than to cure' is their cold-blooded comment. However, where there was no fear of severe haemorrhage they did not hesitate to operate. They tapped the pleural cavity for matter; anal fistulas were cut out; haemorrhoids burned away—'the patient will scream: this facilitates the operation.' In the use of blood-letting they exercised a sensible moderation.

Their therapeutics is mainly dietetic. Their aim is to maintain the strength of the patient until the crisis is past, by means of suitable nourishment. In acute illness barley water is usually given. But even the diet of healthy men should be regulated by the physician. In a special work *On Diet* careful directions are given for a healthy manner of life. The food-values of various foods and their effects on the organism are noted; also the hygienic importance of the various gymnastic exercises. Warning is given against any sudden change in one's way of life. A diet is prescribed not only for those who need consider nothing but their health, but also for those whose business activities compel them to disregard the strictest demands of hygiene: a feature which deserves to be imitated.

The Coan school was sparing in the use of drugs; the Cnidian, on the other hand, used them freely, especially decoctions of herbs. A pharmacological investigation has shown that their prescriptions all contain efficacious ingredients, though these are often combined with others which are neutral; and that the doses—especially of aperients—were much stronger than we can stand nowadays. The properties of cantharides were well known; and it was handled more sensibly then than now; for the head of the insect, which does not contain the efficacious elements, was not used. Among the prescriptions are found some for tooth-powders and cosmetics.

The importance for health of drinking-water, climate, and surroundings in general was well understood. Dealing with these points, the treatise *On Airs, Waters, and Places* lays the foundations of a psychology of races. It contains, side by side with a number of immature theories, a mass of careful observation of foreign peoples and lands, and even leaves its main track to discuss the effects on racial character of freedom and despotism. The Athenians were clearly the model for the account of a people dwelling in a rocky, sterile country with abrupt climatic changes. Such peoples are brisk and energetic, conscious of self and independent, acute and active in industry. Their physique is slim, spare, and vigorous, with sharply prominent joints—exactly the ideal of the early Attic art. The author has the same contempt as Herodotus, his spiritual kinsman, for his countrymen of Asia Minor, where the temperate climate induces languor.

In general, the Ionian physicians of the fifth century brought the art of medicine, which they themselves had created, to a level which was not surpassed before the Alexandrian age.

4

Mathematics in the Fifth Century

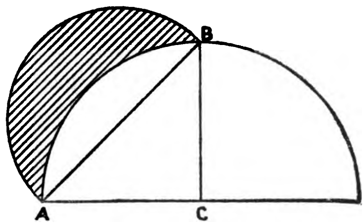
We possess immeasurably less of the mathematical literature of the fifth century than of the medical; but the scanty remains, combined with occasional historical notices and with inferences from the surviving literature of the later period, enable us to give a rough outline of the development.

Arithmetic ran out into unfruitful speculations about number, and made scarcely any real advance upon the achievements of the older Pythagoreans; geometry, on the other hand, developed swiftly and brilliantly upon the foundations which the Pythagoreans had laid. The problem of irrationals was constantly in men's minds. Plato's teacher, Theodorus of Cyrene, completed the theory and gave rigorous proofs of the incommensurability with unity of $\sqrt{3}$, $\sqrt{5} \ldots \sqrt{17}$. But the chief work was claimed by other problems, which led to the founding of the higher geometry. In its attempt to generalize and round off its results, geometry was brought face to face with three problems which could not be mastered by the elementary methods which hitherto had been exclusively employed: the quadrature of the circle, the trisection of any angle, and the duplication of the cube. The two first arose directly from the constructions and measurements of area, on which the Pythagoreans had long been engaged. The last (the 'Delian problem'), which is equivalent to the determination of $\sqrt[3]{2}$, is said to have been propounded by an oracle with reference to a cubical altar in Delos; in fact, however, the problem, which is the counter-part in solid geometry to the duplication of the square (i.e. the determination of $\sqrt{2}$), lay near enough to the range of the Pythagorean interest in the regular solids to be drawn into the widespread net of contemporary inquiry. Hippocrates of Chios, with whom we shall have to deal again, made an important advance in the treatment of the problem by recognizing that it can be reduced to the finding of two mean proportionals. If $\frac{a}{x} = \frac{x}{y} = \frac{y}{b}$, then $x^2 = ay$, $y^2 = xb$; hence $x^4 = a^2xb$, or $x^3 = a^2b$. Therefore, if $b = 2a$, $x^3 = 2a^3$. That is, x is the required side of the cube which shall be twice the cube with side a. It was in this form that the problem occupied the mathematicians of the fourth century, and led to the most important discoveries.

Hippias of Elis, the well-known sophist, invented a special curve, probably for the trisection of the angle. This was the first step to the treatment of higher geometrical figures. The same curve can also be used for the quadrature of the circle, and it is possible that it was really invented for this end.[1] This latter problem, which is said to have already engaged the attention of Anaxagoras, attracted then, as now, the ingenuity of the dilettante.

[1] [It is known as the τετραγωνίζουσα or *quadratrix*.]

Attempts by two sophists are known—Antiphon and Bryson. That of Bryson is a mere catch; Antiphon strikes a modern notion in his solution—he conceives the circle as a polygon of an infinite number of sides—but in him this was a mere fancy, and in any case could at that time attract no attention among mathematicians, who sedulously avoided the conception of the infinite. The popularity of the problem in Athens at the end of the fifth century can best be gauged by the fact that in 414 Aristophanes could bring it before his audience as the latest profundity of the mathematicians (*Birds*, 999 ff.). It led Hippocrates of Chios to an extremely acute investigation, of which we have an excellent account—a datum line of incalculable value for determining the level to which mathematics had already risen.



He discovered that the 'lune' (shaded in the figure) bounded by a semicircle and an arc of 90° is equal to the triangle *ABC* (that is, to half the isosceles right-angled triangle inscribed in the semicircle), and can therefore be squared. It is easy to understand how his interest was gripped by this example of equality between a figure bounded by arcs of circles and a rectilinear figure. He went on therefore until he discovered two other lunes which could be squared (one having its outer arc greater, and one less, than a semicircle), and finally a third, which, added to a circle, resulted in a figure which could be squared. Tradition ascribes to him the false conclusion that he could square any lune, and therefore (by simple subtraction) the circle; although, in fact, he only solved the problem for quite special cases. This ought not, however, to diminish our respect for his performance. The proofs—in part extremely difficult and complicated—are devised and carried out with brilliant ingenuity. They imply great familiarity with segments and their angles, and are based on the theorem—proved by Hippocrates himself—that circles are as the squares on their diameters.

The fifth century has thus the credit of having propounded these three very fruitful problems, and so pointing the way to the higher mathematics of the future, although its own solutions had only a partial success. In elementary mathematics this same period did notable work. Hippocrates of Chios composed the first text-book of geometry. Hitherto the knowledge that had been gained was handed down in the Pythagorean school, almost as a secret; publication meant that henceforward whoever had inclination and capacity had the groundwork to his hand on which to build higher. This first text-book, to be sure, had not, either in arrangement or in the handling of proofs, the impregnable solidity or the clean outlines of Euclid's *Elements*; but for the moment practical achievement was what mattered most for progress—perfection of form was bound to follow soon.

The exact sciences, moreover, are now found to be playing their part in the education of the young. Both Hippias and the Pythagorean Oenopides taught mathematics and astronomy in Athens.

Science was also turned to account in practical affairs. Meton made use of the progress of astronomy to bring about a very noteworthy reform in the Attic calendar; he adopted an intercalary cycle of nineteen years in order to bring the civil year into line with the solar year. The needs of the stage created the scientific treatment of perspective. Anaxagoras and Democritus are mentioned as concerned in this, but we cannot determine in detail what their contributions were. We do know, however, that the brilliant intuition of Democritus had already grasped the important theorems of the volume of pyramid and cone, although his proofs did not satisfy the strict requirements of later mathematicians; and many indications point to him as a probable forerunner of Archimedes' treatment of infinitesimals.

5

Plato. The Academy

For us at least the central figure of the intellectual life of fourth-century Greece is Plato. It is his school in the Academy which determined the direction of research.

Plato's whole turn of mind was such as to give him little taste or interest for descriptive natural science. The account of physics, which in his later years, in the *Timaeus*, he introduced into his philosophical system, is mythical and fantastic in its main features, though it contains here and there some brilliant detached ideas, and, notably in his explanation of the senses, is wholesomely influenced by Democritus. Under the influence of the Pythagorean mathematics he assumed, as the primitive forms of matter, not multiform atoms, but two kinds of triangle, the isosceles right-angled triangle and the half of the equilateral triangle. From these triangles he created the four elements, which he correlated with the four regular bodies—fire with the tetrahedron, air with the octahedron, water with the icosahedron, and earth with the cube. The dodecahedron, which is thus left over, is used by the Creator in the building of the universe—though the cosmos is spherical. Manifest traces of Pythagorean number-mysticism are to be found in Plato; and his successors in the Academy went farther astray in this direction.

But if in natural science the Platonic school marks a halt, its influence on mathematics is all the more striking. Plato himself had been introduced to mathematics by his Pythagorean teachers, and in his dialogues he frequently discusses mathematical topics. The abstractness of mathematics, its non-material figures, appealed to him. He saw in it an excellent instrument for the training of logical thought, the best, the indispensable propaedeutic to his philosophy. Mathematics really owes to him its place in higher education.

Of Plato's own achievements in mathematics we have few and rather suspicious notices. He is said to have given another solution of the Pythagorean indeterminate equation $x^2+y = z^2$, for the case when x is even (p. 15); and to have constructed a simple apparatus for finding two mean proportionals. This last, at any rate, accords very ill with Plato's conception of mathematics.

His importance for exact science lies exclusively in the stimulus he gave his pupils. In the first place it may confidently be assumed that it is to his logical teaching that the system of elementary mathematics owes the qualities which were to distinguish it for ever after: its rigour and logical finish—the whole system developing coherently and consecutively from its definitions and its few assumptions. This more urgent demand for rigour—and probably also the actual discovery of new theorems and problems—quickly rendered Hippocrates' text-book obsolete. Leo, roughly a contemporary of Plato, published a new text-book of the elements, which in turn was quickly superseded by another, which emanated from the Academy. Its author, Theudius of Magnesia, has left the reputation of being equally distinguished in philosophy and mathematics, and, in particular, of having attained to a more general conception of many of the elementary fundamental ideas. Another contemporary and fellow scholar of Plato, Theaetetus, made important contributions to the method of handling incommensurables, and to the principles of the theory of numbers. Eudoxus completely overcame the difficulties in which the Pythagoreans had been involved by the discovery of irrationals, and may almost be said to have created mathematics anew. His definition of proportion—$a : b = c : d$ if at the same time $ma \gtreqless nb$ and $mc \gtreqless nd$ (m and n being any whole numbers, and a, b, c, d any quantities whatever)—extended the Pythagorean conception of proportion so that it included irrationals and thus could be strictly applied to geometry as well.

Eudoxus came from Cnidus and had studied with the Pythagoreans; it was only in his later years that he came into permanent relation with Plato. He never became a member of the school; but although he preserved his independence, much of his great epoch-making work was undoubtedly due to Plato's inspiration.

In close connexion with his theory of proportion he developed, and rigorously based, the so-called *method of exhaustion*, which played so large a part throughout Greek mathematics as a means of avoiding the forbidden conception of infinity. It is based on the theorem propounded by Eudoxus himself, that, if from any quantity the half or more be taken, and this process be repeated with the remainder and so continued indefinitely, then it is possible to arrive at a quantity which shall be smaller than any assigned quantity. Take as example the proof (presumably Eudoxus' own) of the theorem that the volume of a cone is one-third the volume of the cylinder with the same base and height. The proof is indirect. Suppose the cylinder (C) to be greater than three times the cone (c); let a regular polygon be inscribed in the base, and the number of its sides be doubled until the difference between the prism (P), erected on the polygon and having the same height as C, is less than the difference between the cylinder and three times the cone. Then from the inequality $C-P < C-3c$, it follows that $P > 3c$. Now if p be a pyramid with the same base and height as P, $p = \frac{1}{3}P$ (see below); that is, $p > c$, which is impossible since p is contained in c. Similarly $C < 3c$ is shown to lead to an impossibility. Hence $c = \frac{1}{3}C$.

The theorem used above—that the volume of a pyramid is one-third the volume of the prism with the same base and height—was proved by the same method; which also supplied the difficult proof of the theorem that spheres are as the cubes of their diameters. Thus the way was opened to the hitherto unapproachable problem of calculating the volume of curved bodies. Eudoxus also studied the construction of the regular solids, which led him to deal systematically with the 'golden section', i.e. the problem of dividing a given straight line a into two parts, b and c, so that $a : b = b : c$.

The problem of two mean proportionals also engaged his attention; and he produced a curve, of which nothing more is known, which yielded a solution. His teacher and Plato's friend, the Pythagorean Archytas, gave an exceedingly ingenious and elegant solution of the problem, using the curve of intersection of a cylinder and a cone, which displays an amazing mastery of three-dimensional relations, and shows how far the mathematics of the time had gone in the handling of geometrical loci. We can see from this how the same problem gave rise to the study of conic sections, which, to begin with, were treated as geometrical loci. The proof that these loci are produced by sections of the cone was adduced by a pupil of Eudoxus and the Academy, Menaechmus. He used conic sections for the solution of this problem, and discovered the asymptotes of the hyperbola. His discovery put a new instrument into the hands of mathematicians, which soon proved itself serviceable in mastering the most difficult problems.

On the formal side Plato rendered a further service to mathematics by his creation of the analytical method. This consists in assuming the required problem to be solved, and working backwards step by step through the presumptions involved, until a premise is reached the truth or falsehood of which is known. In this way it is discovered whether the problem is soluble or not, whether there are any limitations (conditions of possibility) to the solution, and what path the solution must follow. The method did great service not only in discovering new theorems but in the carrying out of constructions. In complicated problems analysis as well as synthesis is necessary, in order to be certain that all solutions, and none but genuine solutions, have been found.

Many examples of the complete application of this method survive in the written works of the great period of mathematics; but in presenting a solution it was more usual to give the synthesis alone. We are expressly told that not only Eudoxus, but another mathematician of the Academy, Leodamas, employed this method, on Plato's advice, to guide them to new discoveries.

The inspiration of Plato was scarcely less important in the field of astronomy. His view of the cosmos impelled him to refuse to acknowledge the reality of the visible, irregular movements of the planets. He therefore propounded to the Academy the problem of determining by what combinations of simple, i. e. circular, motion the apparent movements of the planets could be explained. The axiom that the heavenly bodies can only move in circular orbits is due to the Pythagoreans, who had already explained the annual course of the sun by the theory, later universally held, that the sun and the planets move in circles from West to East, the fixed stars from East to West. In other regards also Plato's cosmic system betrays Pythagorean influence, notably in his conception of the harmonic distances of the heavenly bodies and his doctrine of 'the great year'. He held firmly to the central position of the earth, but appears ultimately to have accepted the later Pythagorean doctrine of the earth's rotation about its axis.

Plato's call to astronomical studies was obeyed by Eudoxus, in his brilliant system of homocentric spheres. He conceived each of the heavenly bodies as situated on the surface of a sphere which turned about the earth as centre; the poles of this sphere lie on the surface of a similar sphere, rotating however in another direction, so that besides its own motion, it is involved in the rotation of the second; this second sphere in turn is carried round by a third rotating in another direction, and so forth. Eudoxus was able, by assuming three such concentric spheres, to give a satisfactory explanation of the movements of the sun and of the moon; for each of the five planets then known he was forced to assume four such spheres. He was able also to determine the curve traced out by a planet under these very complicated conditions of motion; from its double-looped shape it was known as the *hippopede*, or horse-fetter. (It was used by Archytas for the curve mentioned on p. 34.) This purely theoretical solution of Plato's problem is, in fact, a mathematical achievement of the first rank. Eudoxus probably also founded the science of spherical geometry. Fragments of an astronomical text-book in verse are preserved.

He was, moreover, a practical astronomer as well. He drew up a catalogue of stars which was the basis of Aratus' poetical description of the constellations (third century). He appended meteorological observations to his calendar. These activities belong probably to his earlier days, when he was head of a school in his native city of Cnidus, and to his apprenticeship with the Pythagoreans in South Italy. In Plato's circle he is the single representative of Ionian science. His many-sidedness is Ionian too. He was trained as a doctor, he studied geography. There are good grounds for believing him to have made the estimate, mentioned by Aristotle, of the earth's circumference: 400,000 stades;[1] he held the diameter of the sun to be nine times that of the moon. It is no matter for surprise that so hard-headed an astronomer sharply condemned the fallacies of astrology, which at that time were beginning to filter through from Babylon.

The example of Archytas shows that there was still much to be learned from the Pythagoreans. He is said to be the founder of scientific mechanics, but we are not in a position to give any details of his achievement in this direction.

Nor can we determine the progress of medicine in this period. We still find the travelling doctor of the old type; but more and more the settled physician, sometimes practising on his own account, sometimes retained by a city. Charlatans, such as the

[1] [The Attic stade is 177·6 metres, rather less than a furlong.]

Hippocrateans described and combated, were not wanting; but for the most part physicians were in high repute, although they worked for payment, and were often men of liberal education—like Eryximachus, who delivers a philosophical speech in Plato's *Symposium.* The miracles of Asclepius and his priest excited a good deal of scepticism; and Aristophanes in his last play (*Plutus*, 653 ff.) was not afraid to give a far from reverent account of their on-goings. The inscriptions from the famous and much frequented temple of Asclepius at Epidaurus show to what extent these institutions were given up to pious fraud and trickery. The votive inscriptions of the cured are full of the absurdest miracles. We read of a woman who had been pregnant for five years and, after incubation in the temple, was delivered of a son who immediately bathed himself in the spring and trotted about round his mother. Another woman had her head cut off in error by the sons of Asclepius, who was absent at the time. They were unable to replace the head. The following night Asclepius returned from his journey, put the head to rights, and relieved the woman of a tape-worm to boot. A man had defrauded the god of a fee he was to pay for another who had been freed of a brand upon his forehead; when he in turn approached the god to be cured of some complaint, he received the brand of his friend—by way of warning to other such defrauders of temples. The god even mends some broken pots for a poor slave who applied to him in faith. But such an attractive trait of kindliness in the god cannot outweigh the crude superstition of the bulk of these records; and one longs to recover something of that robust Ionian criticism which the Hippocrateans directed at such miraculous cures.

A healthy contrast is a fragment of the most famous physician of the time, Diocles of Carystus. It contains the minutest directions for a healthy way of life from morning to night in the different seasons of the year. On awakening he orders rubbing, bending and stretching of the limbs, careful washing of the face, teeth, and head; then work, or a short walk; and then gymnastics. Then, and not before, a slight meal—mainly bread and vegetables with light wine. After a midday siesta, work once more—until it is time for the gymnasium. The principal meal comes at sunset. Careful accounts are given of the various foods and their effects. Even rules for sleeping are not forgotten: on the side, not on the back. The gymnastic instructors, who had considerable medical knowledge, attended to the hygiene of the physical exercises.

6

Aristotle. The Lyceum

A CHANGE in the tendencies of science took place in the second half of the century, when Aristotle took over the intellectual leadership of Greece. He was the son of a Macedonian court physician, Nicomachus, but came to Athens in early life and joined the school of Plato. In spite of all differences, his philosophy betrays the influence of his master; but at the same time he possesses an interest for the empirical knowledge of nature and an understanding for inductive research which he undoubtedly brought with him from his home. His father, who doubtless instructed him, according to the ancient custom, in his own profession, was a highly educated man and had published works on natural science. The tendencies of the later Platonic philosophy could not appeal to Aristotle. He went his own way; and when under Plato's successors, Speusippus and Xenocrates, mystical speculation gained the upper hand, he deserted the school entirely. His own lines followed those of the Academy, but he demanded a higher standard of strictness and systematic organization. He has always exercised the greatest influence on the organization of scientific work; his own doctrine is the only fully worked out and comprehensive philosophical system, and as such dominated the whole world of thought for more than fifteen hundred years, with the consequences, good and evil, which inevitably follow such authority. That the evil consequences outweigh the good is the fault neither of the system nor its maker, who toiled till his dying day at the enlargement and completion of the structure of his teaching. But even the immediately following generations were oppressed by its weight and its logical coherence. Here and there an attempt was made to add another wing, as it were, to the structure; but the ground-plan could never be revised. And when in the Middle Ages the Church took over the ancient building and established itself within its walls, no single stone might be moved: even its weakest and most crumbling masonry was sacred.

In contrast to the unhampered intercourse of old and young in the search for truth, which was the ideal of the Academy, the teaching of Aristotle in the grounds of the Lyceum (the *peripatus*) was more like a school in the modern sense. Many of his writings are regular lecture-notes, with the corrections and additions which come from frequent repetition of the same lectures. Aristotle (whom Plato is said to have called the 'reader') is distinguished for his own immense book-learning. He had collected a large library, probably the first to deserve the name by modern standards; and he was accustomed to preface the discussion of every problem by a survey of the older literature on the subject. It is significant that he induced his students to work up the older literature and collect the materials for the history of the sciences in short handbooks. Theophrastus collected the views of the ancients on the chief questions of natural philosophy, Meno made extracts from the literature of medicine, Eudemus wrote the history of mathematics and astronomy, and Aristoxenus the history of music. The master's own position varies in regard to the various branches. In medicine he is the well-informed layman, and readily makes use of medical experience and conclusions—but at that time every educated man had some measure of medical knowledge, a result of their deliberate pursuit of gymnastics and of their highly developed dietary rules. He had a complete mastery of elementary mathematics and assumes it in his hearers. He likes to use mathematical examples; and minutely discusses the nature and task of the mathematical disciplines, their system and their methods of proof. His own system of formal logic is unquestionably constructed on the model of mathematics, which has even determined the form of his logical proofs. He made no independent contribution to mathematics; and appears to have no knowledge of its higher ranges. Indeed the subject had developed so quickly that only the specialist could keep pace. Thus he never really understood the nature of such a problem as the squaring of the circle.

His position in regard to astronomy is similar. He occasionally records astronomical observations, but his chief interest is in the cosmic system of astronomy. In order to meet the requirements of more exact observation, a contemporary astronomer, Calippus, had extended the system of Eudoxus by assuming seven more concentric spheres (two each for the sun and moon, one each for Mars, Venus, and Mercury). Aristotle followed him; but as he misunderstood the mathematical and theoretical nature of the problem as proposed by Plato and the solution offered by the specialists, and wished to produce a real *mechanism* of the heavens, he found himself obliged to interpolate twenty-two more spheres, with contrary motion, in order to get rid of the disturbing influence of the outer upon the inner spheres. Apart from the fact that his purpose could have been more simply achieved, this system of fifty-five massive spheres was so complicated that it soon fell into discredit, and other methods of explanation were adopted. Aristotle took the earth to be the centre of the cosmos, and repudiated its axial rotation. There is much in his *Meteorology* which concerns astronomy—comets and falling-stars, for example. In this work he gives a substantially correct explanation of the rainbow, and even attempts a mathematical account of refraction, which he recognizes to be the cause of the phenomenon. Optical problems in general frequently occupied his attention.

Aristotle's physics is mainly speculative. He elucidates acutely the fundamental principles such as motion, position, becoming and ceasing to be; but he only occasionally applies observation or experiment, and seldom with success. He retains the four elements as the primitive constituents of matter; ascribing to earth absolute weight and to fire absolute lightness. This assumption, and his view of a limited cosmos, barred him out from the correct view of the most important physical processes; for example, he could never reach the notion of specific gravity, although he frequently gets near to it. As substratum for the circular motion of the heavenly bodies he assumes a fifth element, the ether, which under the name of *quinta essentia* became the object of endless speculation among later philosophers. His system was a hindrance more than a help to the rise of empirical physics; but it preserved in men's memories many doctrines of the Atomists whom in spite of all his polemics he valued highly, and these were to prove a fruitful nucleus. His theory of a common undifferentiated primitive matter behind the elements and of the continual passage of the elements into each other is the origin of chemistry.

We can form some notion of the comprehensive teaching of his school from his collection of *Problems*. These deal with medical, physiological, mathematical, optical, musical, and other questions. The book contains a mass of observations intended to stimulate the search for explanation, and obviously dealt with in viva voce instruction. Of similar origin is the interesting little collection *Mechanical Problems*, which shows that the Aristotelian school, though still feeling its way and making many mistakes, was on the track of the principal mechanical laws. The lever, the balance, and the pulley are discussed; and the principles of statics, the principle of virtual velocities, the parallelogram of forces, the law of inertia, are all more or less clearly set out. No doubt we are entitled to see in this the influence of the researches instituted by Archytas.

Aristotle's attitude towards the exact sciences which Plato had brought to a position of dignity was essentially receptive; his creative work was in the field of descriptive natural history and biology. Here only are his real powers displayed, his admirable empirical method, his colossal knowledge, the systematic grasp which brings order into everything, the gift for finding analogies and similarities. The teleological outlook, which often led him astray in physics, provides in the treatment of organic nature a most fruitful hypothesis. Early students, especially Democritus and the physicians, had no doubt collected many observations, and attempts had been made to reduce them to system; nevertheless the whole of the magnificent structure bears unmistakably the stamp of Aristotle's genius: he may be regarded as the creator of scientific zoology and comparative anatomy. The systematic account of the animal kingdom appears in his *History of Animals*. He has an astonishing eye for the significant characteristics, which yield a natural classification (he includes whales, for example, among the mammals); he points out the dangers of being misled by striking but essentially unimportant differences, and of employing dichotomy as the main principle of classification. No advance was made on the Aristotelian system until Linnaeus; and in its main outlines and in method the zoology of to-day still follows the track which Aristotle pointed out. Still more striking are the two books *On the Parts of Animals* and *On the Generation of Animals*. The former gives a comparative description of the organs of animals and their functions. It is a wonderful revelation of Aristotle's *flair* for relationship of forms. The latter shows to what a surprising degree he was able, by means of his own acuteness, backed by a considerable accumulation of observed instances, to anticipate modern ideas without the help of modern technique.

A very small part of this overwhelming multitude of facts and observations, relating to the whole animal world, is drawn from literature. Much is derived from the author's inquiries among fishermen, huntsmen, cattle-breeders, and herdsmen. He justly remarks that the observations of such men may be trusted, but seldom their explanations. A large part especially of the anatomical data is his own; he dismembered animals and got drawings made of what he found. His method is consciously empirical and inductive; in these departments of study he will have nothing to do with premature theories and abstract deductions, and insists on painstaking observation. He makes a fine defence of empirical research against the contempt of the philosophers: its subject-matter is not so sublime, he says, as that of metaphysics; but to make up for that, it lies nearer to our grasp; things that are small and ugly, yes, and even repulsive, can still bring joy to the researcher, if he is able to recognize their causes; to shrink back with disgust from the study of the lower animals is merely childish: remember the utterance of Heraclitus—'Enter, for here too are gods.'

Aristotle put the study of nature on the right road; and after that achievement it can matter little to our judgement of him that even his clearly defined method did not always save him from mistakes. (That his guidance in matters of principle was little heeded was not his fault.) Without microscopes and instruments of precision his task naturally seemed simpler than it was; and his accumulation of material was so enormous that he was bound to overestimate it and judge it to be sufficient. So that, in spite of his cautious theory, he was bound in practice to draw too hasty conclusions and generalize on insufficient evidence. Even his powers of work did not suffice to check all the statements of his authorities; and side by side with the shrewdest of observations we find surprisingly incorrect statements, even on points which he could easily have checked for himself; and often—though he warns others against this error—he has manifestly allowed a preconceived theory to blind an otherwise penetrating eye. Sometimes his doctrine marks a retrogression; for example, he completely mistook the functions of the brain, which had for long been understood by the physicians. But in spite of all these shortcomings, which modern research has perhaps tended unduly to ignore, Aristotle has earned an honourable place in the history of science as well as in philosophy.

His work was continued and completed in his own spirit by his successor as head of the school, Theophrastus of Lesbos. This industrious and thoughtful inquirer belongs to the tradition of Aristotle, but is ready to exercise an independent criticism. He repudiated the teleological view of nature—which even Aristotle in his later works did not pursue to its extreme conclusions—and is much more ready than his master ever was to content himself with a *non liquet* pending further evidence. His collection of the views of the ancients on natural philosophy and physiology was provided with critical notes—as a surviving fragment on the senses shows. This work was, in its own subject, the model and source of later historians of philosophy.

The choice of subject—Aristotle selected the task himself for his pupil—indicates Theophrastus' bent for natural science. Like his master, his chief importance is in the field of descriptive natural history. We possess, besides some meteorological fragments (on the winds and weather forecasts), part of a mineralogy, with descriptions of stones and kinds of soil; and, especially, two larger works on botany, which in every respect are worthy to stand beside the zoological works of Aristotle. In the usual Aristotelian way, the first contains the empirical material, a system of flora, on the basis of which the second constructs the physiological and biological explanation of the phenomena, normal and abnormal alike. The mass of observations is overwhelming. The author had to hand a wide literature on the subject, which is now lost; but in addition to this he made observations himself in a variety of places, and prefers to use the results of his own experience; he knows how to avail himself of the empirical knowledge of farmers and gardeners, druggists and 'rhizotomi'; and we learn from his works with astonishment the vast progress made in scientific agriculture and the amount of attention devoted, for example, to the care of the vine.

The organs of plants and their functions, which form the basis of his system, are on the whole rightly understood. The most striking proof of the high level of botanical science attained in those days is that it proved competent to face the task, imposed by Alexander's expeditions, of understanding and describing an entirely new world of flora. With that high sense for science which distinguished him, and which he owed in the main to Aristotle's teaching, Alexander saw to it that everything of interest encountered by him or his generals in the new worlds of Asia, whether animals or plants, should be carefully described by trained observers. Their reports were probably accessible to Aristotle and certainly to Theophrastus. From these he drew his exact and vivid descriptions of the Indian flora, then so utterly strange to the Greeks. That he and his sources were competent to grasp with full morphological correctness something so new to them as the giant banyan-tree of India with its rooted branches, or the vegetation of the mangrove swamps, and were able to describe these and similar phenomena in a way which comes up to the requirements of modern science, is perhaps the greatest testimony to the teaching of Aristotle, which had developed in them the gift of segregating the significant and the insignificant, and of fastening upon the really essential and characteristic. They used an extremely simple method of overcoming the difficulty of giving their readers a vivid picture of an entirely strange vegetation without the use of illustrations: they invariably adduced native plants for comparison; and in their choice of these they again displayed that keen eye for the essential and the characteristic.

In Theophrastus' will, which has come down to us,[1] instructions are given for placing the 'maps' in a colonnade near the school; and naturally enough geography was part of the studies of the Lyceum. Theophrastus himself is interested in the geographical distribution of plants. The expedition of Alexander, with its abundance of new material, must have quickened the interest in descriptive geography—in the sense of the old Ionian *perihegesis*. Geography plays a large part in the historical accounts of the Asiatic campaigns, as earlier in Herodotus; Aristobulus, for example, has many excellent ethnological and scientific descriptions, and Megasthenes has interesting information on India. The *bematistae* of Alexander, who paced out distances on the main roads, were of service to science, besides fulfilling their administrative and military purposes; so too the voyages of discovery undertaken by his admirals: Nearchus gives a most trustworthy description of the south coast of Asia. However, there were few who could resist the temptation to use the free exercise of their fancy still further to embellish the fabulous elements of their experience and the wonders of the new world; even Megasthenes is full of incredible information. These fabrications brought travel-books into general disrepute; so that the first accounts of northern Europe, written by the adventurous Massiliot sailor Pytheas—a worthy descendant of the Ionian sea-captains—were received by the learned with unmerited distrust.

Naturally enough this large accretion of material gave rise to the desire to recast the science of geography on a systematic basis. This task was undertaken by Dicaearchus, a fellow student

[1] Diogenes Laertius, v. 51.

of Theophrastus; but although his more popular works on the history of civilization in general enjoyed for long the approval of the public—they were among the favourite reading of Cicero—his scientific achievements were soon overshadowed by Eratosthenes, and forgotten. Scattered notices, however, show that they contained valuable preliminary work. He wrote a description of the earth, which doubtless was accompanied by a map; and to him is probably due the measurement of the earth's diameter (300,000 stades) which came into use about this time. He further calculated the heights of various mountains, and employed himself on physical geography.

Work was done in the sciences outside of the Peripatetic school. Two small works of Autolycus, a contemporary of Aristotle, in which the geometry of the sphere is expounded for astronomical purposes, are particularly interesting as the oldest specimens of exact scientific literature which survive in their entirety. In astronomy, Heraclides Ponticus, a friend of Aristotle, but more than he a follower of the scientific traditions of the Academy, earned a considerable but short-lived reputation. He solved the Platonic problem by formulating the so-called Tychonic planetary system—Mercury and Venus moving round the sun, the sun and the other planets round the earth—and perhaps even foreshadowed, as a possibility, the Copernican system. He maintained, against Aristotle, the infinitude of the universe, and gave a completely correct explanation of the nature of musical notes.

As Heraclides, in combating Aristotle's doctrine of the limited universe, harked back to Democritus, so, even within the school, Strato of Lampsacus, Theophrastus' successor, shows the influence of the Atomists in an important point of physics. Aristotle had denied the existence of empty space; Strato, while repudiating the assumption of Democritus that there exists a continuous empty space, maintains, on the basis of experiment, the existence of a vacuum distributed between the particles of bodies. Whenever we are able to reconstruct his views we can discern a vigorous attempt to set free the science of physics from all *a priori* speculation, and rebuild it on a basis of experiment. In this respect he had next to no following; although in other respects he had a great and various influence on Alexandrian science.

7

The Alexandrians

The period immediately after Alexander's death is rightly enough known by the name of the new capital city of Egypt which he had founded. Athens had first of all lost her political importance; she now lost also the intellectual headship of Greece. Philosophy, it is true, maintained its connexion with the city of Plato—mainly for the purely material reason that the philosophical schools had their local habitation there—but in all the sciences Alexandria assumed the leadership.

In the new kingdoms which were built up from the ruins of Alexander's world empire the external conditions of scientific activity were very different from what they had been in the purely Greek democratic city-states. Greek had become a universal language. For the mass of the inhabitants of the most important of the new kingdoms it was a foreign tongue which had to be acquired; and on barbarian lips it lost much of its purity and subtlety. Above the lower stratum of non-Greek elements there was a thin but widely distributed stratum of higher culture (more or less closely connected with the courts); and here, by a reaction from the ignorance of the masses and the negligent speech of official and daily life, there grew up a cult of the literature and language of the golden age, which, however, soon degenerated into an unhealthy archaism. Such conditions were little favourable to literature, which now made its appeal no longer to the whole people but to a narrow cultured audience. There was room for elaboration and refinement, not for vigour or freshness. Literature was stifled by erudition and artificiality.

For science, on the other hand, these new social conditions—which are not unlike our own—had many advantages. The sciences had advanced so far by this time that they could no longer appeal to the general public, and could only command the interest of specialists: and with the Hellenization of the East the number of these was bound to increase considerably. The new universal language secured easy intercourse among the learned all over the world. Its aesthetic shortcomings did not trouble them; and out of its various elements they quickly built up a fixed scientific terminology. The book-trade of Alexandria ensured a vastly wider circulation of learned works. From the point of view of international science the obliteration of the local peculiarities and characteristics of the small states was of no moment. Add to this the encouragement given by the princes, who interested themselves in science—or at least felt called upon in virtue of their position to support it. The patronage of princes is not always a gain to poetry or history, oratory or philosophy; but science in certain of its branches needs large sums of money for the purchase of materials and instruments, and money it now had at its disposal in greater abundance than before. And more important even than that, the new form of government, with its regular official class, secured to the student of science vastly more leisure than the claims of a democracy upon its citizens had ever permitted; and the generosity of the princely courts made it possible for many men to devote their lives exclusively to science. True, this dependency had its dangers; and when the source of royal benefaction dried up, science was only too apt to wither away.

The model of all those literary courts, and the only one of permanent importance, was that of the Ptolemies in Alexandria.

The founder of the dynasty, himself a man of learned inclination, had laid the foundations of the intellectual supremacy of his capital. Demetrius of Phalerum, a pupil of Aristotle, was received by him when expelled from Athens, and may be considered as the link between Alexandria and the Lyceum. For there can be no doubt that the school of Aristotle, with its library, its collections, its organized system of co-operative effort, served as model for the scientific institute of Alexandria; and Strato, the pupil of Theophrastus, was invited to undertake the tuition of the heir to the throne. Ptolemy II (Philadelphus) was the real founder of the Museum—where the *savants* lived together for study at the public expense—and of the two great libraries, where the whole range of existing literature was collected and catalogued: the nucleus appears to have been the library of Aristotle. Hand in hand with the collecting of books went the editing of books—in the first instance of early poetry, but of scientific works as well. It was in Alexandria that philology, in the sense of the study of language and literature, grew up out of the preliminary data of Aristotelian research. It was there that the book-trade had its head-quarters: the *papyrus* gave Egypt a natural monopoly in paper-making. These favouring conditions attracted distinguished representatives of every branch of science to Alexandria: and around them there grew up scientific schools of thought which maintained themselves, with very varying success, throughout the whole course of antiquity.

Descriptive natural science remained curiously in the background, although Ptolemy II was himself a lover of strange and rare animals. All manner of observations were recorded with praiseworthy diligence in specialist works, mainly on gardening, agriculture, bee-keeping, cattle-raising, and such-like practical subjects; but scientifically no advance was made upon Aristotle and Theophrastus. The catalogue of birds by Callimachus, the poet and literary historian, and the zoological compilation of the philologer Aristophanes are both based upon Aristotle; although in both can be discerned that inclination to the strange and the fabulous which proved fatal to the later development of natural history.

The scientific spirit of the time, aided no doubt by the ancient Egyptian custom of embalming, overcame the old prejudice against dissection of the human body; the Ptolemies are even said to have placed condemned criminals at the disposal of the physicians. It was now possible to place the anatomy of the human body upon a basis of exact and systematic observation; and so keenly were the new opportunities of knowledge embraced, that one after the other, within a short space of time, the most important anatomical and physiological discoveries were made.

The real creator of human anatomy, and the founder of the Alexandrian school of medicine, was Herophilus of Chalcedon. This amazing man devoted himself with great success to every branch of medicine. He was the pupil of the Coan physician Praxagoras, but cut himself loose from dogma and determined to build on observation and experience alone. His chief discovery related to the nerves, whose nature and function he was the first to recognize. He made exhaustive researches into the anatomy of the eye, the liver, the genital organs, and, above all, the brain. In medical practice his services were scarcely less considerable. He brought diagnosis and prognosis to a high degree of perfection. He understood fully the importance of the pulse as an aid to diagnosis, and is the real founder of the extremely elaborate (and later over-elaborated) theory of the pulse in ancient medicine. He wrote also on obstetrics. He laid great stress on drugs, mainly vegetable drugs, and in his therapeutics constantly advises their use; he was able on the other hand to appreciate the importance of rational diet and the virtues of gymnastics. In pathology he held firmly by the Hippocratean doctrine of the four humours. Himself an off-shoot of the Coan school, he had studied the Hippocratean writings in general and on some had written explanatory commentaries, not without criticisms.

Something of a contrast to Herophilus is the other great physician of the time, Erasistratus of Ceos, the personal physician of Seleucus. He repudiated the humours of the Hippocrateans and recommended, as against Herophilus, simple curative measures. He was sceptical of drugs in general and preferred dietetic treatment; believing that improper nourishment was the principal cause of all diseases. Blood-letting too, a great stand-by of ancient medicine, he fought shy of. He was a distinguished anatomist. His tireless labours corrected and completed the discoveries of Herophilus. He was the first to distinguish the sensory and the motor nerves; gave the first exact anatomical description of the heart; completed the discovery, begun by Herophilus, of the lacteal vessels; and enriched the anatomy of the brain by a more exact description of its convolutions, the high development of which he recognized to be characteristic of man. He laid the foundations of pathological anatomy by systematic post-mortem examinations; and distinguished himself as a surgeon by his boldness and dexterity. His physiology suffered by his accepting the fatal error of Praxagoras—that the arteries conveyed not blood but air. He tried to meet the objection that in a wound blood flows from the arteries as well as from the veins, by postulating minute channels of communication between the veins and the arteries. In the further development of this theory he supported himself by Strato's physical doctrine of *horror vacui*.

Besides those two leaders, many others were active in this new field. One of the most important seems to have been Eudemus, who did useful work especially on the nervous system, and in the discovery and description of the glands. From the school of Herophilus there broke off shortly afterwards that of the Empiricists who, pardonably over rating the results achieved by sober observation, flung aside all theory and appealed, not unreasonably, to certain Hippocratean writings in support of their purely empirical methods of treatment.

The total disappearance of the rigorously scientific and fundamental medical literature of this period is a damning testimony against the physicians of the latter days of antiquity, who contented themselves with compendia and extracts at third or fourth hand. It seriously hindered the rebirth of medical science; and many facts of anatomy had laboriously to be re-discovered.

Very different was the fate of mathematics, another science which in this period attained a level not to be surpassed until quite recent times. The mathematical literature of the earlier periods of its development has disappeared entirely; but the principal works of the leading researchers of this period survived; and the scholars of the Renaissance could draw upon them directly, and could use these results of the ancients as a starting-point for further research, for new problems, and for new discoveries. The swift advance of mathematics in the sixteenth century was only possible because the Greek masters had provided an absolutely trustworthy and rigorous basis, which could be accepted as it was. In no other branch of science is a text-book of antiquity in regular and honoured use to-day for its original purpose, as is the *Elements* of Euclid in England.[1]

Of Euclid's life and personality we know nothing. His many surviving works are text-books, and consequently attend to the subject in hand and tell nothing of their author. All that can be asserted is that he was teaching mathematics in Alexandria as early as the reign of Ptolemy I. His five remaining books were written with a view to his teaching and enable us to form some conception of it.

His chief work, the *Elements* (Stoicheia) of Geometry, takes up the thread where the Academy had let it fall. He worked up the discoveries in elementary mathematics of Eudoxus and Theaetetus

[1] [Euclid is unfortunately no longer in ordinary use in England.]

into a system of complete logical rigour and perfection of form. The *Elements* is henceforth the recognized, usually tacitly assumed, foundation of all further mathematical research. It is the final completion of the systematization of mathematics which Plato had inspired; and, in spite of the discoveries of Archimedes, which are now included in modern elementary instruction, no need was felt by the ancients for any far-reaching reconstruction.

The work consists of thirteen books. To each book are prefixed the necessary definitions, mere verbal interpretations of the *termini technici*. These are particularly comprehensive in Book I; for there Euclid also explains such terms as do not enter into his own system—e. g. rhombus and trapezium—but had established themselves in mathematical terminology and were necessary for a complete classification of figures. In Book I are collected also the postulates and the axioms ('common notions'), five of each, which, combined with the definitions, exhaust the assumptions necessary for the logical construction of the system. This division into these three categories of the assumptions, the differentiation of postulates and axioms and the choice of those which are to be accepted, are certainly results of the Platonic investigation into the logical principles of mathematics; in any case they are the result of mature reflection and a subtle mental discipline. Even in antiquity many changes were made in Euclid's arrangement, but modern investigation has shown that he has admitted nothing that could be dispensed with, and that, judged even by the exacting standard of modern mathematics, he has left out little that matters. Even the much disputed fifth postulate, the falsely so-called axiom of parallels (that two straight lines will intersect if the sum of the interior angles, which they make with a third straight line, is less than two right angles), justly maintains its place as the test of the existence of a point of intersection of two straight lines. Similarly the four others assert the existence and uniqueness of the straight line between two points and its prolongations, and of the circle with given centre and radius, and serve as the assumption for all the constructions which follow. These have again the purely theoretical purpose of demonstrating the existence of the figures constructed, and without such demonstration nothing may be used in the proof of a theorem. The axioms give a short and sufficient statement of the concepts of equality and inequality in their application to geometrical quantities. On this basis, with the same logical rigour and economy, the whole system is built up, theorem upon theorem; there is no appeal to intuition or to genetic development of propositions, but only to that strict logic which delighted the hearts of the best of the Greeks.

Book I contains the principal theorems on perpendicular and parallel straight lines, and on triangles and parallelograms; and closes with the Pythagorean theorem. Book II deals with the geometrical algebra, which—operating with areas—provided the solution of quadratic equations (p. 14). Book III treats of the circle and the straight lines and angles connected with it; Book IV of the inscribed and circumscribed regular polygons. So far this is all Pythagorean matter, handled on the lines of the text-books of the Academy. What is new is that throughout Euclid has entirely avoided the use of proportion. This is not introduced until Book V has set forth Eudoxus' generalized theory of proportion. Its application to geometry (similar figures) and geometrical algebra is given in Book VI, most of the theorems of which were doubtless known in substance to the Pythagoreans, but only now received exact demonstration. Books VII–IX contain the theory of rational numbers, leading up to important theorems on progressions and continuous proportion. Here, too, there is much that is old, but the extremely ingenious basis of the theory is apparently due to Theaetetus. To him, too, is due part of Book X, which deals minutely with irrational quantities; but here Euclid himself seems to have contributed more than usual that is positively new. The finely perfected system of irrationals and their nomenclature seem to be mainly his own work. He uses it for the complete determination of the number of regular ('Platonic') polyhedra, for which constructions are given in Book XIII. (Books XI and XII contain the necessary elementary theorems in solid geometry.) The Pythagoreans themselves were able to construct the simplest of the Platonic solids; and the fact that there are only five, with which Euclid closes his book, is doubtless derived from them. Other important theorems—measurements of volume—are due to Eudoxus. Euclid's own contribution to solid geometry, as to the other branches, seems to have been in the elaboration of the system. This is perhaps less complete than in his plane geometry—we miss a sharp distinction between symmetry and congruence, for example, and in some of the proofs there are more abrupt leaps than are permitted in the earlier books—and this indicates that we have here a first attempt to reduce solid geometry, with the state of which Plato[1] was still dissatisfied, to an exact system. The merit of the *Elements* consists not so much in new theorems and proofs, though these are by no means wanting (the proof given of the Pythagorean theorem is Euclid's own), as in improvements in form—not only in the systematic construction of the whole, but in the handling and arrangement of the individual demonstrations. The clarity and rigour of terminology and technical language, which distinguished Greek mathematics to the last and became the model for all time, is the work of Euclid.

The *Data*, a work intended to facilitate the analytical treatment of theorems, achieved the same classic authority as the *Elements*. It deals with the same subject-matter as Books I–VI of the *Elements*. Each theorem demonstrates under what conditions a certain geometrical figure is 'given', that is to say, determined. Another work, the *Porisms*, which performed an analogous service for problems, determining what under certain conditions can be

[1] *Rep.* vii. 528 D, E; *Laws* vii. 819 B ff.

constructed, has unfortunately been lost; and the various attempts at reconstruction have not succeeded in giving us any clear notion of its contents.

Two other works, dealing with higher geometry, are also lost. One of them dealt with 'surfaces as geometrical loci'; the other, in four books, contained the elements of conics. The latter, which used the works of Menaechmus and of Aristaeus, an older contemporary, was soon driven out of use by the new treatment of conics by Apollonius of Perga; we can, however, partly from remarks of Apóllonius, partly from Archimedes, arrive at a fairly accurate notion of its contents: it corresponded to the first three books of Apollonius.

Much to be regretted is the loss of the *Pseudaria*, a treatise on fallacious solutions; for the great systematist must here have had much to record that would be of interest both historically and methodologically. It was presumably inspired by Aristotle's *De Sophisticis Elenchis*, a systematic work on fallacies. Independent investigations are contained in the work *On the Division of Figures*, which survives in an Arabic edition; it deals with elementary methods of dividing triangles, quadrilaterals, and circles into two or more parts which shall either be equal or in a given ratio.

For purposes of teaching, Euclid also wrote books on optics (i. e. perspective), on mathematical astronomy, and on the mathematical theory of pitch (on Pythagorean lines); these remained in use in spite of certain shortcomings, and have survived to this day.

Euclid, then, is in the main a summarizer of the achievements of earlier mathematical research. The great creator and discoverer in all branches of the study was Archimedes, the greatest mathematical genius of antiquity and the equal of the greatest of the moderns. He was born in Syracuse, and is said to have been related to the king, Hiero; he was at any rate in his service and a friend of the royal house: he dedicated to Gelo, the son and co-regent of Hiero, a brilliant popular treatise. Of his mechanical achievements—not only in defence of Syracuse against the Romans, but for the private purposes of the king—we have numerous notices, which in part at least are more credible than such stories usually are. It appears that his behaviour was as original as his scientific work; even the use of his native dialect in his writings displays the same independence of school tradition which distinguishes him as a researcher. He was killed at the taking of Syracuse in 212 B. C., by accident it seems, and against the wishes of the Roman commander, Marcellus. The well-known story of his death while at work corresponds not so ill to what we know of him, and is at least *ben trovato.*

He was the son of an otherwise unknown astronomer Phidias, whose measurement of the sun's diameter (twelve times that of the moon) he mentions. He probably began life himself as an astronomer; his friend and fellow student was the astronomer Conon; he was familiar with the literature and the observational methods of astronomy, conducted investigations and observations on the length of the year, constructed, and described in one of his own works, an ingenious planetarium,[1] which Marcellus carried off to Rome, where it was later admired by Cicero. A celestial globe of Archimedes was carried off at the same time and placed by Marcellus in the temple of Virtus in Rome. It was probably his astronomical labours which led Archimedes to construct his ingenious system for expressing numbers of any magnitude, and to study catoptrics, in the course of which he proved the fundamental theorem of refraction.

Presumably he was taught by his father at first; but there is certain testimony that he studied for a long period in Alexandria. All his life he maintained friendly scientific relations with Alexandrian *savants*, such as Eratosthenes and the astronomers, Conon and Dositheus; he published his books in Alexandria; and it was to his Alexandrian fellow workers that he communicated his new

[1] Cic., *De Rep.* i. 21.

discoveries and results. He must have had some unpleasant experiences in the course of this; for he mocks in one place at the worthy professors who behave as though nothing were new or surprising, and even submits some false propositions 'to lead them astray into proving the impossible'.

In Egypt he is said to have invented the water-screw, with which he furnished the giant-ship of Hiero. This shows that he early devoted himself to mechanics, and indeed without considerable mechanical knowledge his planetarium would not have been possible. Given his scientific mind, it is not surprising that these practical tasks led him to consider the exact principles of mechanical laws. His theoretical formulation of the problem of mechanics, 'to move a given weight with a given mechanical power' (pithily expressed in his own alleged utterance—'Give me a place to stand, and I will move the earth'), is connected in tradition with the giant-ship of Hiero, the launching of which he achieved by means of toothed wheels and an endless screw, or of the pulley, which he is said to have invented. It is therefore probable that his works on theoretical mechanics—of which he may be considered to be the founder—belong to an early period of his life. Of these there only survives the treatise which he himself quotes as the *Elements of Mechanics*, in which an exact proof is given of the principal theory of moments, and the centre of gravity of triangles, parallelograms, and parallel trapezia is determined. But beyond this he treated of the centre of gravity in various works, and determined it for the solids of elementary geometry, such as the cylinder and the cone. As he proceeded to extend these investigations to figures of higher order, such as the surfaces bounded by conics or their solids of revolution (which he called conoids and spheroids), he discovered the value for mathematics of this mechanical treatment, and worked out a method foı the provisional determination of areas and volumes, which corresponds to the infinitesimal calculus of modern mathematics. The first result of this method was the quadrature of a segment of the parabola, which he demonstrates, in a surviving treatise, both mechanically and by pure mathematics. Having thus proved the utility of his new method, he proceeds to give more detailed information about it in a treatise (only recently discovered) which he dedicates to Eratosthenes. He has previously submitted to Eratosthenes two intricate problems in the calculation of volumes, to both of which he now gives a solution by the new method, adding rigorous proofs by the method of exhaustion. He further communicates a series of new results (determinations of volume and of centre of gravity) which he had arrived at by the mechanical method. As this method operates with the notion of infinity it cannot, as Archimedes expressly intimates, provide rigorous proof in accordance with the stringent demands of ancient mathematics, but only provisional results. Strict proofs, by the method of exhaustion, are given in the two great works *On the Sphere and the Cylinder* and *On Conoids and Spheroids*. The former culminates in the famous theorems that the surface of a sphere is four times that of its greatest circle, and that the volume of the sphere is two-thirds the volume of its circumscribed cylinder. Archimedes must have considered this latter proposition to be his greatest discovery; the figure for it was on his tomb, which Cicero [1] discovered when quaestor in Syracuse. The work also contains the solution of several difficult problems relating to the sphere. In the book *On Conoids and Spheroids* these solids of revolutions are exhaustively dealt with and their volumes determined; the area of the ellipse is also discovered. The theorems as to the centre of gravity of these bodies, which are given in the above-mentioned treatise on the method, were given exact proofs; there survives only the determination of the centre of gravity of a segment of a parabola, which is derived from the quadrature of the parabola. Archimedes finally connected these mathematical investigations with his discovery of

[1] Cic., *Tusc.* v. 64.

specific gravity, to which he was led, according to the story, in the course of testing for alloy in Hiero's crown. His results are given in the amazing work *On Floating Bodies*, which contains an exact statement of the fundamental principles of hydrostatics.

The fact that Archimedes immediately applied his new method to determining areas and volumes shows that from the start he was influenced by the contemporary problems of pure mathematics, which had brought elementary mathematics for the time being to a halt. Both the problems submitted to Eratosthenes involve the determination of the volume of curved bodies, and Archimedes expressly emphasizes as their peculiarity that, in contrast to all other such determinations, they assert the equality of bodies bounded partly by curved surfaces with such as are bounded by plane surfaces only. Whether the lost treatise on the semi-regular polyhedra belongs to this group of investigations, we cannot say; but certainly his interest in the old problem of squaring the circle belongs here. He fully realized that the problem was not elementary, and solved it in the first place by approximation. In *The Measurement of the Circle* he gives a method by which π can be shown to lie between two limits which may be taken as close together as is desired. In this treatise, which survives in a mangled condition, he contents himself with the approximation $3\frac{10}{70} > \pi > 3\frac{10}{71}$, but in another work, whose mysterious title *On Parallelepipeds and Cylinders* gives no clue to its contents, he gave an approximation to five or six places. In these works he overcame the old prejudice of the schools against introducing concrete numbers into exact geometry. He attacked the problem in another way in his excellent work *On Spirals*, in which he finds, by means of the spiral, a straight line equal to the circumference of a circle. The spiral is traced out by a point traversing a straight line while the straight line rotates about one end. Starting with two theorems in the theory of motion (on the proportionality of the space traversed and the time of continuous motion) Archimedes gives a masterly account of the properties of this curve.

Unique in the whole range of Greek literature is the popular treatise (see above) addressed to Gelo, and entitled the *Sand-reckoner*. Alluding to the proverbial 'as the sands of the sea-shore in multitude' Archimedes illustrates the endlessness of the series of numbers, showing that even if the whole universe were filled with sand, the number of grains of sand would still fall within the range of the numbers which can be named by means of his system. This system consists in taking the highest number which can be expressed in the ordinary Greek numerals, namely, $10{,}000 \times 10{,}000$, as the unit of a new series, which thus goes up to 10^{16}—and the process is repeated *ad libitum*. The demonstration is most carefully carried out; in the course of it Archimedes undertakes a mechanical determination of the apparent diameter of the sun; gives an interesting trigonometrical theorem; and sets out rules for the multiplication of the members of a geometrical series. Since Archimedes was interested in this way in large numbers and moreover shows a considerable dexterity in the extraction of roots necessary for the measurement of the circle, it is not impossible that an interesting arithmetical problem, which survives under his name in an epigram addressed to Eratosthenes, was really propounded by him. It concerns the solution of an indeterminate equation, which quickly involves such large numbers that it is not practically possible to carry it out. His complete mastery of the higher algebra is proved by the arithmetical theorems, mainly concerned with series, which he occasionally uses as lemmas.

Archimedes' presentation of his subject is worthy of its epoch-making character, throughout clear and elegant, without a superfluous word. He handles with complete *maestria*, and in the classical forms of Euclid, the mathematical instruments of his day—the method of exhaustion and the conic sections.

It was Archimedes' skill in the handling of conics in the solution of problems of a higher order which led his friend and biographer Heraclides to bring a charge of plagiarism against the third Greek mathematician of the period, Apollonius of Perga, an accusation which rests upon a mere misunderstanding.

Of Apollonius' chief work, on conics, we possess only the first four books in Greek; the following three are preserved in an Arabic version; the eighth is lost. The first three books are dedicated to a Pergamene friend, Eudemus; the later books, after his death, to one Attalus, possibly Attalus I of Pergamum. We learn from the interesting dedicatory prefaces that the book was based upon lectures delivered partly in Alexandria, where Apollonius had studied, partly in Pergamum, and was intended to give the final shape to his material and so replace the faulty copies of his pupils. He gives an exact account of his relations to his predecessors: the first four books contain the elements of conics, generalized and expanded, as compared with the existing text-books; in particular the third book contains many new theorems which are useful for the problems of higher geometry, and make possible the complete solution of one problem of this kind which had been inadequately treated by Euclid; the fourth book corrects the hitherto unsatisfactory investigation by Conon and others into the points of intersection and of contact of conics; the remaining books contain new and far-reaching investigations into the properties of conics and their applications.

Apollonius' contribution—by reason of which his work, in the elementary parts, superseded all the earlier works, and which was to determine the future of the subject—was his new definition of conics. It is true that Archimedes, and perhaps Euclid, was aware that all three could be regarded as sections of one and the same cone; but in accordance with the definitions, the plane of section had always been conceived as perpendicular to a generator of a circular cone; so that the parabola was produced in a right-angled, the ellipse in an acute-angled, and the hyperbola (i. e. one of its branches) in an obtuse-angled cone. Apollonius put forward a new definition in accordance with which all three sections can be produced on any conical surface having a circular base. This generalization made possible the important methodical advance of treating the two branches of the hyperbola as one curve, and so discussing the theorems in a more general way; for it is only in this way that the correspondence of hyperbola and ellipse can be fully seen. The geometrical property which underlies this definition is the same which in modern mathematics is expressed by the equations referred to the vertex as origin. In accordance with this he gave to the sections the now familiar names. Throughout, his procedure corresponds more or less to that of modern analytical geometry except that algebraic equations are replaced by geometrical operations with areas.

The range of his results still impresses the expert. He established their applications to the intricate problems of contemporary higher geometry in an imposing series of special treatises, in which his generalized view of the problems and his careful discussion of the conditions of possibility have quite the modern tone. They partly served the purposes of instruction, as model examples to be used in learning how to handle problems exhaustively, and long continued to be so used in the Alexandrian school. Later they disappeared with one exception, which survives in an Arabic translation. For the others we are reduced to short (but expert) notices of their contents. Apollonius also gave a solution of the old problem of the duplication of the cube.

Akin too to modern ideas is an interesting treatise by him, unfortunately lost, on the principles of mathematics. The few remaining fragments indicate a desire to link up the fundamental concepts of mathematics with the sensible world; for example, he explains a line by reference to the boundary of light and shadow.

He endeavoured to reduce the number of assumptions and accordingly to re-handle the earlier proofs of Euclid's *Elements*. But in spite of the reputation of the 'great geometer', as he came to be called, we cannot find that these endeavours had any results.

His other writings, which have disappeared but for a few notices, are partly written under the influence of Archimedes, partly as continuations of Euclid. The latter applies especially to his treatise *On Unordered Irrationals*, in which he extends the classification of Euclid Book X; but his treatment of the dodecahedron and the icosahedron in the sphere seems also to be connected with the Euclidean solid geometry. On the other hand, his work on the cylindrical helix (*On the Cochlias*) uses Archimedean ideas; as does his *Ocytocium* (lit. 'swift delivery'), in which he expounds a system, akin to Archimedes', for the expression of large numbers, and probably gave an approximation for π. He playfully connects this investigation with a verse, the letters of which he added up according to their numerical value—and here he is clearly following after the popular form of Archimedes' *Sand-reckoner*. His works on catoptrics—where he discusses burning-glasses *inter alia*—were doubtless inspired by Archimedes. For the legend of Archimedes setting fire to the Roman ships before Syracuse with burning-glasses arose no doubt from theorems which he had proved in his *Catoptrics*. Apollonius' activities included astronomy; the theory of epicycles is his—a device of great mathematical ingenuity to explain the apparently irregular course of the planets.

The classical period of mathematics includes Nicomedes, the inventor of the conchoid, a curve which provided a very elegant solution for the trisection of the angle.

Practical mechanics, too, made immense strides in this period. It was not only Archimedes who was able to hamper the Roman operations at Syracuse by his cunningly constructed and powerful engines of war; Marcellus also had similar engines at his command; and in the many wars among Alexander's successors catapults and similar devices played a hitherto unheard-of part. In this regard the siege of Rhodes by Demetrius marks an epoch; it was here that Demetrius earned his name of Poliorcetes. Archimedes' mechanical writings deal with statics only; but in Alexandria, even under Strato, the theory of pneumatics had been developed and applied to practice. The founder of this branch of technology is supposed to have been Ctesibius, who was active in Alexandria about the middle of the third century. He constructed heavy 'guns'—worked partly by compressed air—and all kinds of mechanical devices; he also wrote on the theory of mechanics. His works have been lost, but we can get a fair notion of his activities from the comprehensive *Mechanics* of his successor Philo of Byzantium, which survives in part—though some of it only in Arabic translation. The nine books of this capital work handle the whole field of technical mechanics. After a general introduction he describes all the varieties of catapults and other engines. These have recently been reconstructed with the help of the drawings which accompany the descriptions, and their range and accuracy is astonishing. Philo further describes other practical devices of poliorcetics which strictly have nothing to do with mechanics; so that his work is at the same time a welcome commentary on ancient siege-operations. The theory of the lever is also discussed at length. He explains the construction of automata and an automatic theatre; and in a section devoted to pneumatics, which begins with an experimental determination of the density of the air, he describes with figures all manner of charming mechanical toys intended to entertain the guests in the gardens and at the festivities of the capital—puzzle-glasses, cans which pour out various fluids at will, fountains with drinking animals and singing birds, an ink-pot in the so-called suspension of Cardan, a censer worked by steam, and other artifices of the kind, not to mention useful inventions—water-wheels, water-engines, and an automatic slot-machine for providing lustral water at the entrance of a temple. In most of these machines he makes use of atmospheric pressure, and throughout displays a complete familiarity with the laws of the siphon.

The vast accessions of geographical material, due to the expeditions of Alexander and the voyages of discovery which he encouraged, were already to hand in a very considerable literature; they were further increased by the Seleucids in Asia and the Ptolemies in Egypt. Ethiopia, and the Caspian Sea—the real nature of which had long been disputed—were now explored. Now that mathematics had advanced so far, it was an obvious step to continue the Aristotelian work on the mathematical aspects of physical geography and to complete the labours of Dicaearchus. This was done by the many-sided and erudite librarian of Alexandria, Eratosthenes of Cyrene, whose work, in spite of all attacks, set the standard for the scientific geography of antiquity and has earned him not unjustly the name of the founder of the science. In the first place, he gives, like Aristotle, a history of geography from Homer onwards. With real historical understanding he emphasizes the limits of Homer's geographical knowledge—as against the fantastic views of Homeric commentators, who credited him with omniscience in this regard. Then follows a mathematical account of the inhabited surface of the earth, which he calculates to be 78,000 stades in length and 38,000 stades in breadth. He divides it by a line parallel to the equator and passing through the Straits of Gibraltar into a northern and a southern half; and further, by six other parallels and seven meridians, into unequal quadrilaterals which he describes in turn. The most northerly parallel passed through 'Thule'; here he used the data of Pytheas. Finally he gives a detailed elucidation of his own map. This is based on measurements of the earth which he had previously carried out and described. Using as his base the distance between Alexandria and Syene (which he took to be on the same meridian and 5,000 stades apart: neither is quite exact), he computed the earth's circumference at 250,000 (or 252,000) stades—a very creditable approximation. Besides this, he drew upon existing data—number of days' march from place to place, lengths of coastline, &c.—and possibly upon official surveys undertaken specially for him; but for the most part he depended on books, and the data he got out of them were naturally inadequate for topographical exactitude. Where, however, he had exact astronomical observations to go upon he used them with care and knowledge, and to all appearance he achieved all that was to be achieved with the material he had. And moreover he fully recognized the inadequacy of many of his statements. The attacks of later critics upon his untrustworthiness and errors are only to a very limited extent deserved.

On the other hand, it cannot be denied that his versatility, which impressed even his contemporaries, has a certain element of dilettantism about it. Archimedes praises his interest and understanding in mathematical questions, communicated his discoveries to him, and invited his collaboration; but what we know of his mathematical achievements amounts to very little. He devised a practical method of discovering the prime numbers, the so-called 'sieve of Eratosthenes'; and invented a very competent instrument, the *mesolabium*, for finding two mean proportionals, which he dedicated in a temple of Alexandria. (He composed an epigram on the instrument, addressed to Ptolemy II.) The contents of his mathematical treatise *On Means* are entirely unknown. He expounded his views on physics in a kind of commentary on Plato's *Timaeus*, in the course of which he treats of the Delian problem (this doubtless led him to his *mesolabium*), discusses proportion, and adumbrates, in opposition to the Pythagoreans, a theory of the mathematical relations of musical notes.

It has already been pointed out that astronomy had been greatly advanced by the amazing progress of mathematics, and partly by the work of the leading mathematicians themselves. The development of mechanics also contributed. The improvement in technical skill not only provided the surveyor with complicated instruments with delicate screw adjustments, but gave the astronomer accurate apparatus for observation and measurement, trustworthy sundials, and so forth. In his *Sand-reckoner* Archimedes describes an instrument invented by himself which gives an approximation to the sun's diameter in angular measure, and speaks of the limited accuracy of instruments as a familiar topic of discussion; from which we can safely infer the activities of the time in this direction. His movable planetarium must have called for a very high degree of mechanical technique. In the observatory of Alexandria a series of systematic observations was begun with the deliberate purpose of solving the fundamental problems of astronomy; and the specialists were in a position to make scientific use of the observations of the ancient Chaldeans, which the expedition of Alexander had rendered accessible, and to estimate their value correctly. For the more exigent demands of astronomy the ordinary rough division of the day into three, or four, parts was useless. The astronomers therefore introduced the Babylonian division of hours (known already to Herodotus [1]) which later was extended to the uses of ordinary life; the word *ὥρα* (hora), originally a time or season, has acquired the meaning of *hour*. They also borrowed the Babylonian sexagesimal system. In practical affairs and in the other sciences the old Egyptian fractions (with 1 as numerator) were retained; but the astronomers worked with sexagesimal fractions and divided the circle into 360 degrees, each containing 60 minutes of 60 seconds each. This system, which prevailed in astronomy ever afterwards, first appears in a small treatise by Hypsicles (second century) on the rising of the signs of the zodiac, but is not presented there as a novelty. The foundations of trigonometry were laid, to assist the science of

[1] ii. 109.

astronomy. Considerations of a trigonometrical nature are found as early as the surviving work of Aristarchus of Samos (third century), in which, following Eudoxus, he attempts mathematical computations of the size and distance of the sun and the moon. His method gives for the moon a fairly satisfactory approximation, but for the sun the results are inevitably inadequate.

In this book Aristarchus follows the traditional geocentric theory of the universe; but we know from trustworthy sources (among others a casual remark of Archimedes) that in another work he gives reasons for the view that the earth and the planets move round the sun as centre—in fact, for the regular Copernican system. Aristarchus was a pupil of Strato; and it is possible that in this matter he was influenced by that daring innovator; kindred ideas were familiar not only to the Pythagoreans, but to philosophical circles in Athens. But this break—not only with popular views, but with the fundamental conceptions of philosophy—was too abrupt. Aristarchus' theory was rejected by the astronomers: its sole supporter was the gifted and original researcher Seleucus of Seleucia (*c.* 150); and Cleanthes, the Stoic, goes so far as to describe it as blasphemous.

Seleucus is credited with the correct explanation of the tides; by means of observations he determined their dependence upon the moon and its position. He, like Heraclides Ponticus, maintained the infinity of space.

Conon and Dositheus, the friends of Archimedes, were pre-eminent among the Alexandrian astronomers as observers. They both, like Eudoxus, compiled calendars with meteorological observations.

Conon, who was also a considerable mathematician, gave the name of 'the Lock of Berenice' to a hitherto unknown constellation—out of compliment to the consort of Ptolemy Euergetes; an event celebrated by Callimachus in a famous poem. Altogether, the constellations aroused general interest. Eratosthenes uses the star-myths in his poems, and Aratus of Soli (third century) in his

Phenomena gives a poetical description of all the constellations in Eudoxus' chart. The poem had an immense success although its poetical merits are not conspicuous. Throughout the whole of antiquity it was commented on by experts, was translated more than once into Latin (e.g. by Cicero), and kept alive a certain amount of astronomical knowledge far into the Middle Ages. There are mediaeval manuscripts of the Latin editions with pictures which undoubtedly derive from ancient originals.

Hipparchus, the most accurate astronomer of antiquity, wrote in his youth a commentary on Aratus, in which he pointed out his mistakes. It is a significant testimony to the decay of science in the later period of antiquity that this unimportant work of his youth is the sole survival of the great astronomer's vast output—and it has only survived as an appendage to the poem of the gifted dilettante. Of his strictly scientific works, of which he gives a list in one of his writings, only scanty fragments have come down to us; they are sufficient, however, to let us recognize his importance.

He was born in Nicea, in Bithynia, in the first half of the second century, and it was in Bithynia that he made the great part of his astronomical and meteorological observations, though some belong to Rhodes and others probably to Alexandria. He was fully aware that it was only by accurate and continuous observation that a firm foundation for astronomical theory could be laid, and that even trifling errors of observation must endanger the results. He therefore strove throughout for absolute accuracy. With the instruments of the time this was quite impracticable; but this must not deter us from recognizing his merits: he laid down the necessity of this accuracy and did all that in him lay to achieve it. In accordance with his principles he attended primarily to the accumulation of materials and is hesitant in theory. He improved the existing instruments and invented others; and was thus able to attain to a greater accuracy of observation than had been possible before him. He made wide use of the observations of the Babylonians, the Athenians, and the Alexandrians, partly in order to test them, partly to demonstrate certain changes in the heavens. He was thus able to discover the precession of the equinoxes, to collect valuable material for the movements of the planets, and to correct in many important points the statements of his predecessors. He determined, for example, with greater exactitude the length of the solar year, the equinoctial and solstitial points, and the orbits and distances of the sun and the moon. The discovery of a new star led him to compile a catalogue of fixed stars which superseded all earlier attempts in respect both of completeness—it contained 800–900 fixed stars—and of systematic arrangement. This colossal work had the avowed purpose of enabling the astronomers of the future to determine with certainty whether or not the fixed stars change their position, size, and brilliance with the course of time. And in other ways too Hipparchus was at the pains of perfecting the equipment of his subject; he was the first to elaborate a system of trigonometry, and worked out a table of chords.

We have unfortunately only the vaguest information about his physical works (on weight) and his mathematics. A little more is known of his astronomical-geographical work *Against Eratosthenes*, in which he submits the work of Eratosthenes to a harsh and not always justifiable criticism, and condemns his whole attempt as precipitate. True to his own principles he derides the data of Eratosthenes as untrustworthy, and demands as a basis of a map of the world absolutely accurate astronomical data for latitude and longitude—little knowing that he was demanding what the next thousand years and more could not supply, and displaying that lack of understanding for the necessary weaknesses of a first great attempt which is characteristic of rigid exactitude of mind. But his severe criticism did not prevent him from using Eratosthenes (he accepts his measurement of the earth, for example) or from resting content himself from time to time with equally inadequate material.

8

The Epigoni

For all his greatness, Hipparchus shows unmistakable signs of the decadence which characterizes the second and the first centuries before Christ—that barren spirit of criticism, for example, which kept him from accepting the hypothesis of Aristarchus with all the wealth of new problems with which it teemed. The fresh spirit of adventure, the creative genius of the golden age, have passed. Industrious and rigorous work is done, but only on the ground already broken: no new fields are opened up. This is partly the fault of the times. The Greek states were wearing each other out. The princes no longer gave to science that material support which the indifference of the public rendered indispensable. Under the misrule of Ptolemy Physcon (145–116), Alexandria, the capital city of science, lost its headship; and the growing power of Rhodes could provide no lasting substitute. The burning of its library under Caesar was a loss to Alexandria which could never be made good. And apart altogether from these external conditions, so tremendous an advance in science could not be expected to continue uninterrupted; a period of assimilation, of revision of the new results, must necessarily follow; and before a new outburst of activity could come, the cold breath of Rome had blown across the world.

Medicine shows more signs of life and activity than the other sciences. With the spread of civilization increased the need for doctors. As public officials or as personal attendants of princes they rose to wealth and position. Rome, where the elder Cato [1] warned his son against the Greek 'poison-mongers' and preferred to cure even fractures with some childish abracadabra—even Rome was soon to offer the Greek physicians a new field for

[1] Pliny, *Hist. Nat.* xxix. 14; Cato, *De Agri Cult.* 160.

lucrative public and private practice. There, for example, was the most famous physician of the time, Asclepiades (first century). He hailed from Asia Minor, as did most of the physicians of this and the following period. He had been originally a *rhetor*, and even as a doctor he retained from his old profession a penchant for attracting attention; and indeed the struggle for existence in Rome must have brought the danger of charlatanism very near. He found himself so well off in Rome that he refused the invitation of King Mithridates, a man who was keenly interested in medicine and natural science in general, though mainly on the superstitious and mystical side. Asclepiades has the credit of having opposed the rapidly increasing use of medicaments, emetics, and purges. He stoutly maintained the importance of diet, and his own methods were as simple as could be: water-cures, baths, rubbings, moderation in eating, and the like. He had no originality, and lacked any profound medical knowledge, especially in anatomy; but his sensible and skilful use of his own simple methods had valuable results, and his school long maintained its influence.

On the whole, medicine followed the tendency of the age. It complied with the demand, and attended more to actual practice than to the purely scientific research which the Alexandrians had so brilliantly advanced. Nevertheless, under the influence of philosophy, the physiological principles of medicine were eagerly discussed. Asclepiades abandoned the Hippocratean theory of humours and built up his physiology on the atomic theory of Epicurus, with certain modifications. The followers of Erasistratus vigorously attacked the school of Herophilus, from which, as has been mentioned, the Empiricists had broken off. Faced with the contradictions of physiological hypotheses, these latter renounced all theory, and denied validity to anything except practical experience and a differentiated handling of individual cases. Their leader, Heraclides of Tarentum, gained a great reputation in materia medica.

In spite of all attacks, Hippocrates maintained his rank as a classic, and as such continued to be read and elucidated. Following the Alexandrian tradition most of the well-known doctors of this period, both Asclepiades and Heraclides, for example, wrote commentaries on Hippocratean works. The commentary of Apollonius of Citium on the treatise *Of Dislocations* still survives. It is particularly interesting because the methods of reduction are illustrated by figures.

Doctors were also apothecaries; and the importance of vegetable drugs for their practice was indirectly a stimulus to botany. Crateuas, the body-physician of Mithridates, compiled an admirably illustrated herbal, with pharmacological text; the beauty and correctness of his drawings of plants can be seen from the later copies which still survive.

Nicander of Colophon (beginning of second century) may be mentioned as a curiosity. He was a prolific but incompetent poet who made a collection of antidotes in verse, which in spite of dullness found readers and commentators, and has consequently come down to us. His last poem on agriculture contained descriptions of plants; and in general the abundant agricultural literature with its accounts of vegetables and foods for cattle made some additions to botanical knowledge.

Scientific zoology, on the other hand, got no support from medicine or agriculture, and fell to pieces entirely. Possibly veterinary surgery, which made great progress in antiquity, dates its origin to this period; but in all other branches the fashionable delight in the fabulous had free play. The principal zoological work of the time, the work on animals of Alexander of Myndus (first century)—it had some vogue as a convenient handbook—uses the materials of Aristotle interspersed with all kinds of uncritical and fabulous nonsense. It is the direct ancestor of Aelian's *Historia Animalium* (third century A.D.) and the fable-books of the early Middle Ages.

In mineralogy we find scientific treatises—mainly on the precious stones which were much sought after since the conquest of Asia, and were consequently often imitated—side by side with whole works devoted to the mystical properties of minerals.

Superstition found its way into astronomy as well. Under the aegis of the fashionable Stoic philosophy, Babylonian astrology penetrated into Greece; it can be traced back almost to Hipparchus. A standard astrological work named after an Egyptian king, Nechepso, and a priest, Petosiris, was current as early as the first century.

Scientific astronomy was not dead, for observations continued to be taken in Alexandria; but the literature is inconsiderable. The *Spherics* of Theodosius, which presumably belongs to this period, is a revision of an old text-book on the geometry of spherical surfaces, which probably goes back to Eudoxus. There are two small astronomical works of the same Theodosius which were later used by teachers in Alexandria: these have not yet been edited.

Mathematics wears a more encouraging aspect, though our information is fragmentary. In certain branches of the subject at least the work of the great geometers was continued and carried farther. Zenodorus follows Archimedes in a not unskilful treatment of isoperimetric figures, and an elegant treatise of Hypsicles, preserved as an appendix to the *Elements*, continues the investigations of Apollonius on the regular solids. The discovery of new curves was an object of special interest: an otherwise unknown Perseus treated exhaustively of spirals, to which Eudoxus had already drawn attention; and Diocles invented the so-called *cissoid*, which he employed to tackle the old problem of the duplication of the cube. It appears that he also wrote a work on burning-glasses, in which he solved by conics a difficult problem proposed by Archimedes. Investigations into spirals on a spherical surface were also undertaken at this time in continuation of the work of

Archimedes and Apollonius. An almost unknown scholar, Geminus, dealt with the systematic side of the mathematical disciplines in an interesting work which is full of historical material.

In geography the strict demands of Hipparchus produced a reaction. No further work was done in that spirit, and the old Ionian *perihegesis*, or descriptive geography, came again into fashion. Agatharchides, a somewhat older contemporary of Hipparchus, had already given excellent ethnographical descriptions, especially of Africa and Arabia, in his historical and geographical works. The most eminent historian of the time, Polybius, turned definitively away from mathematical geography; he devotes a whole book of his history to a description of the Roman world. The description of the Mediterranean countries by Artemidorus of Ephesus (*c.* 100) is based partly on Agatharchides; but he seems to have approximated to the older literature of the *periplus*. A change shortly took place, however, and the mathematical and astronomical side of geography attracted more attention. The contrast is most clearly to be seen in the *Geography* of Strabo, the most important work on the subject which has survived from antiquity. Strabo was born in Apamea and had the ordinary education in letters and philosophy; he had no specialist knowledge of mathematics or astronomy. He wrote an historical work as well, and inclines to the view of Polybius that the main function of geography is to serve the needs of rulers and generals. The geographer must possess some understanding of mathematics and astronomy and be able to assume it in his readers; but the exact science is for him only an ancillary discipline, the results of which he uses without troubling about its methods and demonstrations in detail. His subject is the *oecumene*, the known, inhabited world; general theoretical questions about the whole earth—whether other parts of it are habitable and so forth—do not affect him. Even the mythological excursuses, which Strabo loves—especially where Homer is concerned—are cut down in deference to practical people who have no taste for learned digression.

Strabo saw a creditable amount with his own eyes—he travelled the whole Roman world from Armenia to Sardinia, from the Black Sea to Ethiopia—and his work is invaluable for its wealth of information and its lively descriptions of contemporary conditions. Among the best is his account of Italy. That of Greece is disappointing in comparison, for his interest in Homer had led him astray into basing it upon a learned Alexandrian commentary on the catalogue of ships in the *Iliad*.

For Greece we possess a few fragments of an anonymous *periegesis*, some one hundred and fifty years older, which has some acute and shrewd observations of detail. There is a delightful account of the luxurious city of Thebes, with its many springs and gardens; with its brutal men, who made the streets unsafe, and its fascinating women, dressed like the Turkish ladies of to-day. These few pages are all that is left of a form of literature once diligently cultivated. It traces its descent to Dicaearchus, and one of its chief exponents was Polemo (second century), who was specially interested in the description of works of art. He also copied inscriptions and used them as evidence for the history of art and civilization.

The fact that Strabo, in spite of an inward distaste, does consider to some extent the mathematical and astronomical aspect of geography is probably due to the influence of Posidonius, whom he used as his source for the description of Spain and Gaul.

Posidonius was a native of Syria who settled in Rhodes, where his school was visited even by Romans—Cicero and Pompey, for example. He was a Stoic; but contrary to the stern habit of that sect he combined with philosophy an interest in mathematics and natural science, and skill in letters. In his great work *On the Ocean*, which contains the results of his travels to western Europe, there was, Strabo complains, more mathematics and astronomy than befits the geographer who is writing for the ordinary educated public. He had written on geometry, and his commentary on Plato's *Timaeus* gave the impetus to a renaissance of Pythagorean number-mysticism. Like the early Stoics—and unlike his master Panaetius—he was given up to all manner of superstition, and was a stout defender of divination and astrology. He used his astronomical knowledge to construct a planetarium after the model of Archimedes, and wrote some astronomical treatises. He composed a large work on meteorology and a treatise on the size and distance of the sun. Of his astronomical works we possess an expert epitome by Geminus, and in the second century A. D. Cleomedes pillages them for his astronomical compendium.

Posidonius has been somewhat extravagantly called the last independent researcher of antiquity. He did conduct research of his own, especially in geography and ethnology; but in the exact sciences he was a dilettante. His chief claim to importance is that he offered the general public what it was capable of digesting. As Panaetius' modified and ennobled Stoicism became the universal religion of the educated, so Posidonius' easily assimilable writings captured the world of readers—in which he was greatly aided, as was Panaetius, by his close connexion with the leading circles in Rome. It is significant that his patron Cicero, wishing to write himself on geography, turned in the first instance to Eratosthenes: after the new tendencies which Posidonius had introduced, it was hardly decent to pass by the founder of the scientific study of the subject. But it is no less significant that Cicero had scarcely embarked upon his reading when he found himself involved in technical discussions of which he could make neither head nor tail, and abandoned his wild project altogether. There is no doubt that Posidonius disseminated a great amount of useful knowledge, especially in the Roman world; but the curse of the popularizer is upon him. He gave no impetus to original work—rather he helped to bring about the ever-increasing neglect of the fundamental scientific works: everything worth knowing could be absorbed much more comfortably from him. And his influence was so great that results which he accepted—for example, Seleucus' explanation of the tides—became at once the common property of the educated world, and those he rejected, like the heliocentric system of Aristarchus, passed into oblivion.

9

The Romans

From the time when Polybius, deeply impressed by Rome—which he had come to know in the best Roman society—had described its political greatness to his politically degenerate compatriots, the influence of Roman tastes and Roman needs began to make itself felt in letters: we have seen this already, in Posidonius and Strabo. For science this could only be harmful. The Romans, with their narrow, rustic horizon, their short-sighted, practical sobriety, had always in their heart of hearts that mixture of suspicion and contempt for pure science which is still the mark of the half-educated—and sometimes bragged of it. Cicero,[1] the arch-dilettante, boasts that his countrymen, God be thanked, are not as these Greeks are, but restrict the study of mathematics to what is useful and practically applicable. So the Romans have produced no original work in these fields; what they needed they borrowed from the Greeks.

They were at their worst in the exact sciences. Here and there a Roman dabbled in them to the amazement of his contemporaries: Gallus, for example, who, Cicero[2] tells us, would spend whole days and nights on astronomical calculations in order to have the pleasure of surprising his friends by predictions

[1] *Tusc.* i. 5. [2] *De Senect.* 49.

of eclipses. Marcus Terentius Varro, the friend of Cicero, who had more of the scientific spirit than any other Roman, occasionally touched upon these sciences; but we are in no position to form any clear notion of his writings on the subject. The little mathematics which surveyors needed was translated from the Greek, and so arranged that it could be applied in practice without any theoretical knowledge. In the course of their private and public duties they must have had considerable practice in the ordinary tasks—serious errors on their part were punishable—but they were not able to cope with new and greater problems. When Agrippa undertook his survey of the Empire he had to bring in Alexandrian specialists, though the nominal superintendent was a Roman. Even more pitiful are the scraps of mathematics which appear in the encyclopaedias of later times. Martianus Capella (*c.* 400) has an exceedingly dull work, *The Wedding of Mercury and Philogia*, the oracle of the Middle Ages, in which he introduces a few fragments of the *Elements*; he betrays his entire lack of mathematical understanding by translating the first definition (a point is that which has no parts), by *a point is that whose part is nothing*.[1] It is not until the real Roman age is over that Boethius' (died 524) translations of Euclid's *Elements* and of works on arithmetic and musical theory gave to the West some knowledge of mathematics, to serve as pabulum for the Middle Ages. Sporadically better work was done, but only where practical ends demanded a greater readiness. The expert treatise *On Aqueducts*, for example, written by Julius Frontinus (first century A. D.), displays both skill in calculation and geometrical understanding of the problem.

This unscientific spirit went hand in hand with superstition and mysticism. Cicero's queer friend, Nigidius Figulus, busied himself with Pythagorean numerical speculation, and introduced astrology to Roman literature. During the Empire astrology

[1] vi. 708.

played an important part in the highest circles, and was even treated in verse by the otherwise unknown poet Manilius. We possess an exhaustive fourth-century text-book by a zealous adept, Firmicus Maternus.

To scientific astronomy the Romans contributed no more than to mathematics. The constellations and the myths connected with them excited some interest; there were two translations besides Cicero's of Aratus' poem, and the essay on star-mythology, ascribed to Eratosthenes, was also translated. Varro occupied himself with systems of time reckoning, and from him is probably drawn the work of Censorinus, *On the Birthday*, which contains some interesting information.

Seneca gives a dexterous popular account of astronomical and physical science—mainly after Posidonius—in his *Natural Questions*, an important source-book in the Middle Ages. Vitruvius' book on architecture, on the other hand, is enigmatic and unreadable; he piles up all kinds of excerpts from the Greek on mechanics and kindred topics, in so silly a form and so strange a style that it has been seriously questioned whether he really was a master-builder under Augustus as he claims to be.

Descriptive natural science in the widest sense is represented by the *Natural History* of the elder Pliny, who died in A.D. 79, during an attempt to study at close quarters the eruption of Vesuvius which destroyed Pompeii. In this vast compilation he has collected with astounding industry an overwhelming mass of information, from all manner of sources, mainly Greek, on geography, anthropology, zoology, botany, medicine, mineralogy, and art. The book is like an old curiosity shop—precious early information side by side with all the rubbish which lay so readily to the hand of the tireless excerpter. The book was frequently excerpted itself, and in particular the medical sections were collected together for practical use.

The art of medicine was an offence to the genuine Roman like Cato—he advocated colewort in various preparations, as a panacea.[1] But Varro recognized its value; and in the first century A. D. we find a really gladdening book, the small handbook of Cornelius Celsus, far and away the best work in the whole range of Roman scientific literature. It is part of one of those encyclopaedias which the Romans had long admired (the rest of it is lost). The author is not a professional man; but he reproduces his Greek original with understanding, and in clear and simple language. He has rescued from oblivion many valuable bits of information, notably about the admirable surgery of the Alexandrians. But for this one happy exception, the same desolation has overtaken medicine too. After Pliny we find only books of prescriptions, one of them in verse by Quintus Serenus; most of them are very late, and written in a barbarous Latin of great linguistic interest. It is only at the very close of antiquity that we meet with fuller works on medicine and veterinary surgery, all of them from Greek originals. The most important is a translation by Caelius Aurelianus of the *Therapeutics* of Soranus (p. 96). This translated medical literature continues without interruption far down into the Middle Ages. Even in the darkest periods of Western civilization it was never forgotten that the Greeks were the masters of medicine; and the necessities of health—only too painfully urgent in the Middle Ages—impelled men to overcome the linguistic difficulties, which, with this one exception, closed the way to the sources of Greek learning. Even when later on the Arabian medicine came in, it was still only an echo of the Greek.

Though scientific geography lay outside the beat of the Romans, they had both the capacity and the opportunity for ethnography and topography. The better Roman historians never hesitated to introduce geographical insertions, as indeed had been the habit of Greek historians from the earliest times. Even Cato had

[1] *De Agri Cult.* 157.

included in his historical work such ethnographical curiosities as he encountered on his campaigns, especially those which were bound to interest a landowner. Writing of the Jugurthine War, Sallust gives several descriptions of North Africa, with which he was acquainted. Caesar includes in his *Commentaries* some short but accurate accounts of Germany and Britain. Tacitus, in his life of his father-in-law, Agricola, the distinguished governor of Britain, writes an account of the hitherto unknown parts of the island; and in an essay on Germany throws much valuable light on periods and districts otherwise dark (including Scandinavia). Unfortunately, he is always trying to point the contrast between the noble simplicity of the Germans and the luxury and over-civilization of Rome. In the only place where Tacitus [1] discusses astronomical geography he betrays the depths to which these sciences had sunk: he explains the light nights of the extreme north by the flatness of the remoter regions of the earth, forgetting what had been for centuries common knowledge to the Greeks—that the earth is round. Geography, as an independent subject, was little studied by the Romans. The best work is the very modest little text-book of Pomponius Mela (first century A.D.). They were not even in a position to make a full use of the excellent statistical materials which Agrippa collected, and the road-maps based on these are very moderate performances.

10

Greek Scientific Literature of the Empire. Byzantium

In spite of all unfavourable conditions, Greek scientific activity never came to a complete standstill: tradition—even when it came to be little more than routine—was too strong for that. What this tradition was worth, and what vigour the failing spirit of the Greeks still possessed, was quickly shown whenever an active

[1] *Agricola*, 12.

and cultured emperor, such as Hadrian or the Antonines, succeeded to the throne. Friends alike to science and to the Greeks, they rescued the higher education from neglect; and in the general revival of Greek letters scientific literature had its share. In fact it is in scientific work that we find the most pleasing features of the revival; for it clung fast to facts and would have no traffic with that *rhetoric* which, with very few exceptions, bedizened and befogged every other subject. The fashionable cult of the ancients, which in literature and art led almost always to pedantry and artificiality, could do nothing but good for science. The great researchers of the golden age had really something to teach, and the study of their works was necessarily an inspiration for further work. And work in fact was done in most departments of science; not of striking originality indeed—the time for that was long past—but industriously and in the true scientific spirit. The classics were eagerly collected, systematically studied and elucidated, and many minor additions to their results were made.

The revival of Pythagoreanism, begun by Posidonius, bore fruit in the field of mathematics. The text-books of the Syrian Nicomachus (*c.* 150), contains a short survey of the theory of numbers and the mathematical theory of music of the Pythagoreans. His arithmetic, in particular, remained the regular text-book; it was twice translated into Latin (the second time by Boethius), and called into being a series of commentaries which last on into the Byzantine period. He also gave a full account of the Pythagorean number-mysticism in a special treatise, of which only excerpts survive.

The influence of Posidonius can be detected here and there in the writings of Hero of Alexandria, who probably belongs to the first or second century A. D. This date admirably fits the collection of definitions which survives under his name, and his commentary on Euclid's *Elements*, of which enough remains (partly in Arabic) to enable us to recognize that it was intended originally for instructional purposes. We possess besides these a series of writings by Hero which show a systematic endeavour to present mathematics and mechanics in a form convenient for practical purposes. These played an important part both among the Byzantines and Arabians, and even in the Renaissance. The best is the recently discovered *Metrica*, in which he gives rules and formulae for the measurement and division of the more important geometrical figures, plane and solid; these are accompanied by theoretical demonstrations, and presented in the form of problems with numerical examples. *Inter alia* he gives the ancient methods for extraction of squares and cube-roots. The subject-matter derives in the main from Euclid and Archimedes, and makes no claim to originality; even the well-known formula ascribed to him (for the area of a triangle in terms of its sides), and its elegant proof, is not given as his own discovery; whether the form, which for us is new in Greek mathematics, is really an innovation, is doubtful but not improbable. It is adapted to practical use, especially to surveying, which had early been of importance in Egypt. The Byzantines left out the theoretical parts, and re-handled the work in the form of arithmetic-books and collections of problems. The surveyors refer to Hero's account of his new levelling instrument. It is a fine and complicated instrument of precision, described with such minuteness and knowledge that one can hardly avoid concluding that the writer had had something to do with the training of surveyors. His book on vaults, which has been lost except for a few uncertain fragments, must have had a similar practical aim. One of the architects of S. Sophia, Isidorus of Miletus, studied it and published a commentary. His *Belopoeica* (on throwing-machines) also displays practical knowledge, but is mainly drawn from Greek sources. The books on mechanics, on the other hand, which are admittedly and demonstrably based on Archimedes and Philo, display very curious weaknesses, especially the *Pneumatics*, in which the theory of atmospheric pressure is applied to several kinds of apparatus. Most of these are already in Philo; and the descriptions of these, as well as the few additions, betray a transcriber who has paid little attention to experiments or to the practical construction of apparatus. His directions for setting up an automatic theatre, which reproduce and develop a work of Philo's, have similar shortcomings; for example, the force which sets the whole thing in motion is insufficient. Nevertheless, these very books were the greatest favourites of the Renaissance and of the Arabians; they are responsible for all the fountains with automatic moving figures and such-like devices which were set up in the gardens of great houses, and served to surprise and delight the guests at banquets and festivities. Even the old cathedral clock at Strassburg, which has so often been imitated, is a direct descendant of Hero's automatic theatre. Similar toys, puzzle-glasses, and the like, occupy most of a *Catoptrica*, preserved in Latin only, but probably derived from Hero. It has some theoretical matter in it as well. The descriptions of apparatus leave much to be desired. The *Mechanics* (unfortunately only in Arabic) is a greater success. Here he expounds, mainly after Archimedes, the principles of statics and kinetics, including the parallelogram of forces; he describes the use of machines, such as the cog-wheel, the lever, the pulley, the wedge, and the screw. He further devotes a special treatise to the windlass, in which he thoroughly investigates the Archimedean problem—to move a given weight with a given power.

So many of the original writings have been lost that Hero's works, with all their faults, are one of our main sources for the history of Greek mechanics; the other works which survive from this period have little that is new. They are mainly concerned with engines of war.

Hero undoubtedly worked in Alexandria, and to judge from his commentary on Euclid was a teacher in the school. Mathematics and astronomy, in particular, were successfully studied in the Alexandrian school down to the close of antiquity. In the first century we have an excellent text-book on spherical geometry (not, however, preserved in the Greek) by Menelaus of Alexandria; it closely follows the *Elements*, but deals as well with spherical trigonometry. The same scholar improved the table of chords of Hipparchus, and extended his catalogue of fixed stars. His two surviving astronomical observations were both, oddly enough, made in Rome (in the year 98). Theo of Smyrna (who made a collection of all the mathematics, astronomy, and music necessary in his opinion for the reading of Plato) also conducted observations, doubtless in Alexandria. His observations of the planets belong to the year 130. He is the immediate forerunner of Ptolemy, to whom he handed over his material for redaction.

Claudius Ptolemaeus, like his great predecessors, embraced, in his vast output of books, all the exact sciences. He further occupied himself with philosophy, as appears from occasional passages in his other works and from a special treatise on the theory of knowledge, an eclectic work based on the Peripatetics. His lost works on physics suggest an Aristotelian origin in their treatment of weight and material substance, though they are partly polemical. In his book on weight he is said to have maintained that divers perceived no pressure of the water above them, and that a skin filled with air is lighter than an empty one. His *Optics* (which survives, all but the first book, in an unattractive Latin translation from the Arabic) is not confined, like Euclid's, to the theory of perspective, but treats of the physical processes of sight and the optical illusions conditioned by them. Here he abides by the Platonic theory, according to which sight is due to the union of sight-rays proceeding from the eye with the light which comes from without. The work includes the *Catoptrics*, in which he deals with mirrors of every kind, and attempts to prove by measurements the fundamental law that the angle of incidence is equal to the angle of reflection. He also makes experimental researches in refraction through water and glass at various angles of incidence, and applies his results in astronomy to the determination of the refraction of star-light in passing from the ether into the atmosphere. Neither his method nor the figures he arrives at meets modern requirements; but the attempt to lay an experimental foundation deserves all credit, especially if the idea came from Ptolemy himself, as it certainly appears to do; but we know too little of his predecessors—and his literary methods elsewhere justify a certain doubt. In any case, however, the *Optics* is the most complete treatment of the subject which antiquity has left to us. Compared with it, a small book by the otherwise unknown Damianus of Larissa is of very small account.

Much as the *Optics* of Ptolemy was used, it cannot compare in historical importance with his great astronomical work, which determined the astronomy of East and West alike for more than fifteen centuries. It is known by the name of the *Almagest*, an Arabic corruption of the Greek designation ἡ μεγίστη (*he megiste*, 'the greatest'—*sc.* book). Oddly enough this name is not found in Greek. The Greeks cite it as 'the great book' or simply as 'the book'; and the author seems to have called it 'mathematics'. The title 'greatest book' must have arisen quite late in Alexandria, at a time when a collection of small astronomical writings, known by the Arabians as 'the middle book', was inserted in the curriculum between Euclid and Ptolemy, by way of preparation for the latter. This vast work of thirteen books sums up the whole previous development of astronomy. It owes what is best in it to its forerunners, especially to Hipparchus. The historically very important catalogue of fixed stars—which gives 1,022 stars, with their latitude, longitude, and luminosity—is simply Menelaus' extended reproduction of Hipparchus' original catalogue, worked out afresh for Ptolemy's own time. But at the same time, in order to solve the problems which Hipparchus raised, he conducted observations of his own; he handled spherical trigonometry with great skill, and his powers of calculation are considerable. His own contributions are sometimes for the worse; but he was a scientific worker, and no mere armchair student or transcriber. He developed his own hypothesis of eccentric circles to account for the orbits of the planets, and claimed for it the same authority as Apollonius' epicyclic theory; he entirely rejected the heliocentric system of Aristarchus, and attacked the axial rotation of the earth on fallacious Aristotelian grounds.

His very complicated system which, although it rejected the Aristotelian spheres, still suffers from a multiplicity of eccentric spherical surfaces, is handled in a special work *On Planetary Hypotheses*. In his book on the rising of stars, he gives for thirty stars of the first and second magnitudes a list, for five different latitudes, of the days on which they first and last are visible above the morning and evening horizon; he couples with this calendar extracts from the daily weather-forecasts of the older astronomers from Democritus to Caesar. His *Hand-tables*, in which the chronological and other tables required by astronomers are collected, is a useful instrument for daily astronomical work, and remained for long in use. A special book was devoted to an explanation of their construction and use.

Two large works (for the most part extant in translation only) dealt with two different methods of projecting a spherical surface on a plane. In these books—following the lead of his predecessors—he attacks, and works out with full mathematical understanding, a problem of great importance for the other subject in which he was long an authority—geography. His geographical work, after an introduction on method and sources, gives a list of 8,000 places with their latitudes and longitude. Here, too, most of the material is borrowed, principally from the similar work of Marinus. He falls into a good many errors, especially when he is excerpting Latin sources. Nevertheless, the book deserves respect; his method is critical, and where he can he introduces his own corrections. A long time was to pass before the maps which accompanied the book were superseded.

Ancient tradition had attached the theory of music to the exact sciences; and, true to tradition, Ptolemy dealt with this subject too, in a comprehensive and important work, the *Harmonies*. There are not a few ancient works which give satisfactory information on the mathematical basis of music and on the Greek notation; but unfortunately they do not suffice to give us any adequate conception of the practical side of Greek music.

Finally, Ptolemy wrote what was to be the standard work on astrology, which at that time ranked as a science. This was the so-called *Tetrabiblos* (four books), which was at one time quite mistakenly assumed (out of reverence for the great astronomer) to be spurious. It is a well-arranged, systematic survey, introduced by a defence of astrology. The second book is particularly interesting, giving as it does a psychology of races based on astrological principles. The book is held of little account nowadays; but it not only called into being numerous commentaries in antiquity, but actually engaged the attention of men like Melanchthon and his circle. It compares very favourably with the nearly contemporary work of Vettius Valens, a book only interesting for its language, which is as plebeian as the mind of its author: it is not through any merit of his own that the careers he describes in confirmation of his horoscopes are sometimes quite amusing reading. The other astrological writings, which only survive in fragments, are for the most part only valuable for the history of the constellations. The little dialogue, *Hermippus*, by an unknown writer, is interesting as containing a defence of astrology from the Christian point of view.

Alexandria was also the home of the other black art which interested the last days of Greece, namely, alchemy. It was principally fostered by the highly developed Egyptian technique in dying stuffs and colouring metals, which soon led to fraudulent imitations. About the third century the belief was evolved that it was actually possible by means of various juggleries to transmute metals. Hence the attempts at making gold which, for all their native fraud, allured so many minds—and these not always the feeblest. There grew up an alchemistic literature, for the most falsely attributed to great names of the past, such as Democritus. In the extant specimens there is more mysticism and superstition than genuine learning. The most famous work of which anything remains is the text-book of Zosimus (*c.* 300), which gives us also an insight into the intestine quarrels of these wonder-workers, male and female. Books on alchemy were banned by Diocletian, but without success.

Ptolemy lived and worked in Alexandria, and following the ancient custom he dedicated in the Egyptian city of Canobus an inscription, which gives a tabulated survey of his astronomical system. His chief astronomical work became the basis of instruction in the school at Alexandria. In the third century a full commentary on it was written by Pappus, whose work was continued in the next century by Theo. Pappus has also left a large compilation on the mathematical disciplines, which gives an interesting picture of the teaching in Alexandria. The chief works of the great days were still extant and were systematically used in the curriculum. Pappus gives succinct summaries of their contents, adding explanations and auxiliary theorems to fill such gaps in the proofs as might offer difficulties to the beginner. He is concerned rather to understand and explain than to continue the researches of the ancients, but he does not exclude some critical emendations and minor additions. For the history of Greek mathematics the book is a chief source; and many valuable works of the golden age are known to us only through its references to them.

Of the same type are the two probably contemporary treatises by Serenus of Antinoeia, a city of Egypt founded by Hadrian. The first proves at length that the ellipse can be derived from a cylinder as well as from a cone—a fact which Archimedes had used long before without wasting words on discussing it. The other treats with laborious care the triangles formed by a section through the vertex of a cone. These triangles are of very minor interest, and the author may well be right when he claims to have been the first to handle them.

The single pre-eminent exception among all this laborious and unoriginal mathematical work is the *Arithmetic* of Diophantus (probably third century). This work makes such an impression of strangeness that it was at one time seriously thought that Indian influence might have something to do with it; but since historical research has come to recognize the nature and importance of the early geometrical algebra, beneath its purely geometrical presentation, it has become more probable that the novelty of Diophantus' point of view and methods of proof is only apparent, and is due to the loss of earlier works. No doubt he contributed a dexterity in calculation which was his own and devices which he had invented; but so large a collection of problems could not be the work of one man, and the author nowhere gives the impression of being an innovator. The more likely supposition is that his terminology and symbols are an independent contribution to the systematization of the subject: this would accord with the spirit of the time. The book contains a multitude of very various solutions of equations, which display an amazing skill in the treatment of numbers and the handling of special devices, by means of which the shortcomings of a still incomplete system of symbols are overcome. The problems—contrary to the old tradition—are always stated in concrete numbers, and solved case by case: no general rules are formulated. Although the terminology of geometrical algebra—e.g.

'rectangle' for 'product'—is retained, the treatment throughout is purely arithmetical; the solutions are always rational. Particularly important is his comprehensive and extraordinarily dexterous handling of indeterminate equations. The work has had great importance in the development of the modern theory of numbers; no less a man than Fermat edited it and commented upon it. Apart from this book (which unfortunately is not completely preserved) we possess a treatise by Diophantus on polygonal numbers, which follows the Pythagorean teaching without adding much that is new.

The requirements of teaching—the curriculum began with Euclid and ended with Ptolemy—brought about the publication of commentaries on the *Elements* as well as on Ptolemy. Pappus produced one, fragments of which are found in the marginal notes, or scholia, in our manuscripts of Euclid. A further interesting fragment, on the irrational numbers of Book X and Apollonius' further treatment of them, has been preserved in an Arabic version. This work, too, was continued by Theo, who produced an edition of the *Elements* with certain additions of his own which he considered useful in instruction. His text prevails in all our manuscripts except one. On the other hand his corresponding edition of the *Data* is in very few manuscripts, for such a work was only suited for more advanced students. He probably also completed the collection of smaller works which formed the bridge between Euclid and Ptolemy (the so-called smaller astronomical course—the 'middle books' of the Arabians), and not only edited for this purpose the *Optics* and the *Phenomena* of Euclid, but composed the *Catoptrica*, which is falsely ascribed to Euclid. He further wrote two commentaries on the *Hand-tables* of Ptolemy, one short, the other very full.

Theo's daughter, Hypatia—who fell a victim to Christian fanaticism in Alexandria—published commentaries on the *Arithmetic* of Diophantus and the *Conics* of Apollonius. She belonged to the Neo-Platonist school, whose chief representatives, both in Athens and in Alexandria, were interested in many ways in astronomy and mathematics. Porphyrius (*c.* 300) had written on mathematical questions; his pupil, Iamblichus, who was particularly interested in Pythagoreanism, left behind him a philosophical introduction to mathematics and a commentary on the arithmetic of Nicomachus. The more important of the Neo-Platonists, Proclus (fifth century), wrote a commentary on Book I of the *Elements*, which, in spite of a lot of mysticism and symbolism, contains much valuable historical information, mainly drawn from Geminus. His commentary on Plato shows familiarity with mathematics; and it can still be shown that he and his school studied Ptolemy and conducted astronomical observations. His pupil and biographer, Marinus, wrote a small instruction to the *Data*. Simplicius, the admirable Aristotelian commentator, one of the seven professors who emigrated to Persia when Justinian closed the University of Athens in 529, exhibits an understanding for exact science and even seems to have commented on Euclid. Eutocius of Ascalon, like Simplicius, a pupil of the Alexandrian Neo-Platonist Ammonius, published the *Conics* of Apollonius and some works of Archimedes with notes of some historical value. It is only in his edition that Apollonius has survived in Greek. His edition of Archimedes was republished by Isidorus of Miletus, one of the architects of S. Sophia in Constantinople, and so preserved for us. Isidorus and his still more important colleague, Anthemius of Tralles, paid considerable attention in general to early Greek mathematics and mechanics—partly at least owing to the gigantic task of vaulting S. Sophia. Isidorus commented on Hero's work on vaults. Anthemius wrote on curious machines. Among other things he deals with burning-glass, and criticizes with technical knowledge the fabulous tales about the burning of the Roman ships at Syracuse by Archimedes. The treatise on solid geometry, which has been foisted into the *Elements* as Book XV, is the work of a pupil of Isidorus.

Medicine, too, was particularly active in the time of the Roman Empire. The evil consequences of civilization made its uses the more evident, and brought its practitioners a rich harvest of wealth and honour.

In the first century two new schools arose beside the old Alexandrian schools. A pupil of Asclepiades, Themiso, of Laodicea, was the founder of the *Methodist* school, which derived all diseases from the ordinary conditions of the body, without maintaining the atomic theory of Asclepiades. Their theories led to a regrettable neglect of special symptoms; but the clarity of their system attracted many adherents. Their most important member was Soranus of Ephesus (second century), of whose extensive writings only fragments and late translations have survived. He treated of all departments of medicine; of medical history; and of the lives and tenets of the older physicians. He was specially famous as an obstetrician, and the remains of his works on diseases of women and obstetrics justify his reputation, although he owes a good deal to his predecessors. His work is not only the fullest and best of the ancient works on this subject, but is full of interest for the history of civilization. Soranus not only discusses actual parturition, the different positions of the foetus, and the methods of correcting abnormal positions, but also gives minute direction for the treatment of new-born babies—swaddling clothes and cradle, the choice of a nurse and her duties, how best to teach a child to stand and walk—and in general for the care both of the mother and the child. For the act of parturition—which is to be supervised by a midwife with two experienced assistants—he recommends a special kind of chair, with pierced seat: failing this the mother should sit astride on the knees of a strong woman; only in cases of difficult labour may she lie on a hard bed. The midwife is forbidden to look at the patient's private parts, for she may easily be overcome with shame. Artificial removal of the foetus by means of forceps, and the destruction of the child in the womb, in order to save the mother, are fully dealt with. For these operations the presence of a doctor is assumed. Abortion or artificial prevention of conception are permitted to the physician only if child-birth is likely to be fatal to the mother, owing to injury to her reproductive organs, or the like. The question whether permanent virginity is harmful for a woman is thoroughly discussed, and ultimately answered in the negative on general grounds. It is admitted that the natural course is for a mother to nurse her own child (except in the first few days when the milk is vitiated by the strain of delivery), but evidently a wet-nurse was the normal thing, and is even to be preferred if the mother does not come up to the standards demanded of a good nurse. The child should be suckled at regular intervals, and special warning is given against the bad habit of putting it to the breast the moment it cries, in order to quieten it. Apropos of this a careful explanation is given of the various causes of crying: at times it serves as a good exercise for the lungs, but must not be allowed to go on too long. The child should not be weaned till it is a year and a half or two years old, and preferably in spring. Rules are given whereby the midwife can judge whether the new-born child is likely to live: from which we can infer that the old practice of exposing feeble children still subsisted. On the whole we are left with a very favourable impression of the high level of obstetrics and the care of children at this time.

Scientifically, the school of the *Pneumatists* is of greater importance. Their founder, Athenaeus of Attaleia in Asia Minor, follows in his physiology the general lines of the then dominant Stoic philosophy. From it he borrowed his theory of the *pneuma*, or breath of life, the state of which determines health and disease. The later development of the school shows a tendency to eclecticism (as in contemporary philosophy), which must have had a good effect in practice. This is very evident in the most eminent and best-known representative of the school, Archigenes of Syria (*c.* 100). His works were numerous and much used by all later physicians, but are now lost. His teaching, however, can be reconstructed to a large extent, partly from the numerous citations in Galen and others, partly, and especially, from the surviving compilation of Aretaeus of Cappadocia (probably second century), who owes everything of value to Archigenes, and himself contributed only an imbecile stylistic form and an artificial Ionian dialect. His accounts of diseases, which go back to Archigenes, are distinguished by their truth to nature, their keen observation, and their vividness of description. Well known, for example, is his description of the horrible disease elephantiasis, till then almost unknown in the West. In therapeutics he pays special attention to diet. In this field the school did the greatest service; it made the most thorough investigation into the values and effects of the various food-stuffs, of wine and of mineral water. Baths, too, played a great part in their therapeutics, especially cold baths, but sun baths were used as well. Archigenes, however, also wrote an important work on medicines, in which, *inter alia*, he included hair-dyes—out of consideration for his aristocratic lady patients. He was very much the man of the world, and his medical letters of advice to distinguished friends are characteristic. His views on fevers and the pulse long remained the standard; the latter he developed in over-elaborate detail.

The same fate which overtook Archigenes has overtaken the rest of first-century medical literature; and, with the exception of some minor works of Rufus of Ephesus, an important physician of Trajan's time, we are thrown back on extracts and citations in later writers. This is doubtless due in the main to one man, Galen,[1] the Ptolemy of Greek medicine. His influence, like Ptolemy's, lasted on far into the Renaissance, and this world-wide domination, again like Ptolemy's, was due not to the intrinsic merits of his writings, but to the fact that he arranged and collected in convenient form the material of his forerunners, with the result that their original works could be dispensed with.

Galen was born in Pergamum in 129 and died in Rome about 200. He had a careful education. His father, Nicon, had wide intellectual interests, which embraced philosophy, and throughout his life the son interested himself in philosophy, and wrote on it. Some of his philosophical treatises are extant; the majority are lost. The range of his production was enormous. He wrote as many as 150 medical works covering the whole range of the subject. Of these 80, some of them considerable works, survive. This copiousness was only made possible by a relatively small degree of independence, a shocking discursiveness, and a garrulity which never boggles at repeating itself. Personally he is not attractive; particularly offensive are his childish vanity and his snobbery. Nevertheless—quite apart from his importance in the history of medicine—we must not underestimate his real services. No doubt the best of him is borrowed plumes; but, like Ptolemy, he is no mere transcriber or bookworm. He instituted independent researches, mainly in anatomy, and, most important, he had a large practice conducted with skill and success, and this contact with life saved him from drowning in his own ink-pot. He is not without the scientific sense, and his writings undoubtedly did much to raise his profession to a higher level. There was need enough for this at a time when influential schools refused to hear of scientific training of doctors and sup-

[1] [The name Claudius Galen is a Renaissance invention.]

ported the crudest empiricism, and when the Romans had allowed the instruction of doctors—whose aid they so badly needed—to sink so low that it was only in Alexandria that anatomy could be studied from a real human skeleton—a survival there of better times. For the history of Greek medicine, Galen is invaluable; he is the main source of our knowledge of medical literature after Hippocrates. Even his garrulous bragging is, historically, a gain, for we owe to it some most interesting pictures of his practice, of the doings of medical men in Rome, and of the life and culture of the time.

After a period of study and travel in Smyrna, Corinth, and Alexandria, Galen secured a post as gladiatorial surgeon in his native city. Many memories and experiences of his practice here are recorded in his later writings. At the same time, in spite of precarious health, he continued his studies with eagerness, and even embarked upon his literary activities. A few years later he decided to make his fortune in Rome. A few successful and well-advertised cures brought him very quickly a considerable position and a distinguished practice. Meanwhile he published papers on anatomy and physiology, and wrote several polemics against his colleagues, for he made enemies on every side. This was due, according to himself, to his need of earning his daily bread; but he himself was obviously to some extent to blame. By nature he was contentious, and the devices he used, by his own admission, to enhance his reputation are at times rather crude. He was on the point of being presented to Marcus Aurelius by a distinguished patient when he suddenly left Rome; and it seems he never was able to clear himself of the accusation that he did so to escape the plague, which was then advancing from the East. Shortly after his return to his native city, however, he was summoned to the court, and during the campaign against the Marcomanni (from which he had the good fortune to be given leave of absence) he was entrusted with the care of Commodus, the heir to the throne. The rest of his life, some thirty years, was spent in Rome, in tireless activity as doctor and writer. The great bulk of his longer works belongs to this period. They are devoted mainly to the practice of medicine, and deal at length with pathology, therapeutics, dietetic, and materia medica; from now on, surgery, even in his actual practice, falls into the background.

The physiology of Galen is based in general on the Hippocratean theory of humours. (He was well acquainted with the Hippocratean writings, on many of which he had written thorough commentaries, entering even into questions of language and textual criticism.) His doctrine of the various physiological 'fundamental forces' which by the wise dispensation of Nature control the body, exercised a great influence on later generations. In therapeutics, baths and diet play their part—he warmly supports the air-and-milk cure at Stabiae—but besides these, drugs are employed to a shocking extent. Some of these ghastly prescriptions, composed of the most repulsive and poisonous ingredients, send a shiver down one's back, and one is tempted to ask how any one ever thought of such enterprising mixtures, and how many more patients they killed than they cured. The limit is reached by the *theriac*, an antidote composed of some seventy ingredients, among them stewed adders and opium. This was compounded by Galen himself for the use of the Emperor. He wrote two treatises which are meant to demonstrate the rational grounds of its composition and use, but there is a flavour of superstition in the use of snakes. Even apart from this, Galen does not always keep clear of the miraculous cure; he firmly believes in the intervention of Asclepius even in his personal relations; in particular he attributes to him—coupled with his own dietetic—his complete recovery from his youthful ailments.

These are the darker passages: there are many other more attractive features in his activities. In practice he often displays great thoroughness, presence of mind, and decision. He fully recognized the importance of anatomy, and, since dissection of the human body was no longer permitted, he seized eagerly on any chance opportunity of instructing himself in the inner structure of the body, and urges his pupils to do likewise. He was an industrious dissector of animals, especially apes, and even practised vivisection. In this way he discovered a number of new details in anatomy. He insists that dissection of animals is not to be taken as an indication of the human anatomy without further proof; but in spite of this he allowed himself to base some hasty conclusions on this evidence, conclusions which later cost much labour to eradicate.

After Galen there is a further serious decay of originality in Greek medical literature. No doubt in practice the Greek physicians continued in some sort adequate to the tasks imposed on them, but literature has nothing to show but compendia and collections of extracts, written confessedly to gather into convenient form all that need be known. The most comprehensive of these is that of Oribasius (fourth century), who was the body-physician of Julian the Apostate, and undertook the work at the Emperor's request. (From the large work he himself prepared a short epitome.) The book, which unfortunately is not completely preserved, rescued many important fragments of the earlier literature from oblivion. The sixth century yields the compilations of Aëtius and Alexander of Tralles—a much more independent writer, who wrote especially on the diseases of the eye. Practical treatment of the eye and the preparation of eye-salves had long been in the hands of specialists—throughout antiquity the physicians were apothecaries as well. This series of compilations—which are not without their merits—is closed by the work of Paulus of Aegina (seventh century). It is admittedly intended to supersede Oribasius, whose large work was too extensive for the practical doctor, and whose epitome was too slight. The author himself had an imposing practice, and had not abandoned all attempt to exercise his critical faculties. For centuries his work was the handbook of the better doctors, and his clear account of surgery is of value if only as the sole systematic survey which has survived. For in surgery as well as medicine, antiquity has made astonishing progress. From this time onwards, Greek medical literature can be compared to nothing but successively weaker and weaker decoctions from the old tea-leaves.

The development of pharmacology was of service to botany. The chief work is the *Materia Medica* of Dioscorides of Cilicia (first century). No less than 600 medicinal plants are described; and the book had enormous influence throughout the Middle Ages, and, in translation, upon the Arabians and upon the West. In some of the early MSS. admirable drawings from the work of Crateuas are appended. Zoology, on the other hand, was in poor case. The so-called *Physiologus*, an Alexandrian production of about the second century, displays in its accounts of fabulous animals and their theological symbolism all the lack of criticism and the prejudice of the Middle Ages. It had an astonishing vogue, moreover, was translated into many languages, and largely influenced mediaeval art. A better work is *The Nature of Man* by Nemesius, Bishop of Emesa (fourth century); but even it is a wholly unoriginal compendium. It was in turn excerpted by Meletius in Greek, and was early translated into Latin.

In the Byzantine Empire, where the continuity of tradition was never broken by barbarian conquest, and where, therefore, civilization never sank so low as in the West of the Middle Ages, the treasures of the past were preserved with a genuine piety and used as far as they could be used. After the time of the Iconoclasts, whose oriental fanaticism had seriously threatened the ancient culture and the profane literature of Greece, the University of Constantinople was restored and reorganized by the philosopher and mathematician Leo (ninth century). With this is connected the literary renaissance under the vigorous rule of the so-called Macedonian house, to which we owe the preservation—and the most beautiful manuscripts—of many works, among them some of scientific interest. Among the encyclopaedic compilations of the Emperor Constantine Porphyrogenitus are some which are of importance for the sciences with which we are concerned: a collection on management of land, excerpts from the veterinary writers, the medical and the zoological encyclopaedias, which contain a quantity of ancient material. But the political and economic conditions of the Empire were unfavourable, and with Constantine the tide began to turn. Nevertheless, science never quite died out, and as soon as the general situation improved it begins to be studied again. Astronomy was almost always cultivated, if only for the determination of Easter, and in the fourteenth century Persian influence put new life into it. So in the eleventh century a change came over arithmetic as soon as the Indian position-system, with zero, came to be known. (It so happens that our first systematic account of this system does not come till the thirteenth century, in the arithmetic-book of Maximus Planudes.) The exact sciences were always lectured on at the University; even after the fall of Constantinople, advanced instruction continued. Beneath the surface of rigid routine the Byzantines had kept the sacred fire alight.

And much need there was. The West had only inherited the meagre legacy of Rome, and made of it what barbarism and the Church permitted. It was not until the eleventh century, when men made up their minds to sit at the feet of the Arab infidels in Spain, that a revivifying breath of the Greek spirit blew over the parched fields of science. This renaissance was at its best in Sicily and South Italy, among the mixed populations of the Normans and the Hohenstaufens where the Greek original sources had already been discovered and exploited. But the fall of Manfred was the end; and it was not for two hundred years that the mind of the West found the freedom and the strength not only to accept but to carry farther the Greek science transmitted to them by Byzantium. All the founders of modern science, Galilei, Copernicus, Giordano Bruno, Newton, and Vesalius, learnt from the Greeks not only particular results, but also and above all the very meaning of science.

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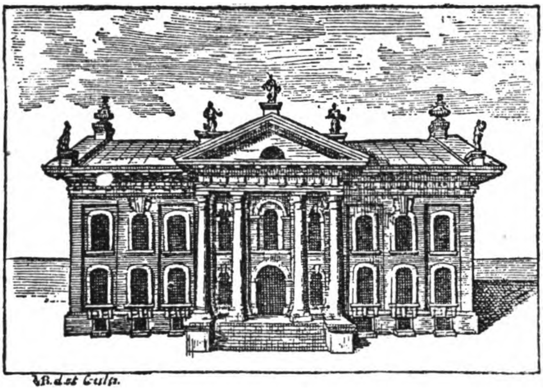

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